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# Turing Pattern Growth Modes

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Irving R Epstein, Milos Dolnik



**Brandeis**

**COMSOL  
CONFERENCE**  
2019 BOSTON

# Why aren't cows spherical?

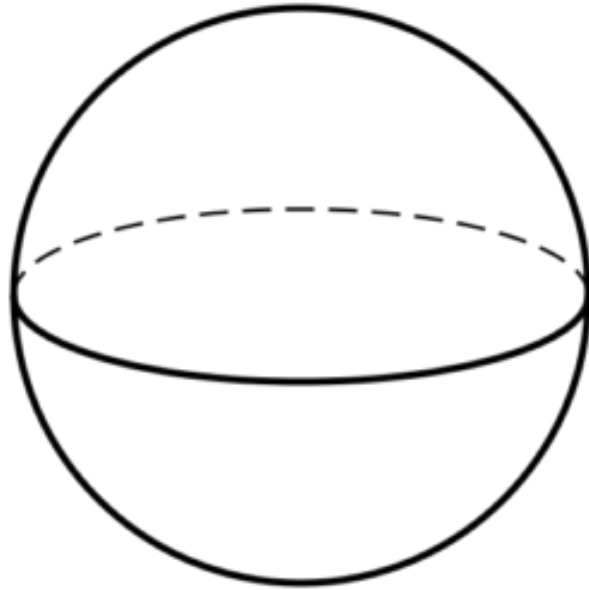
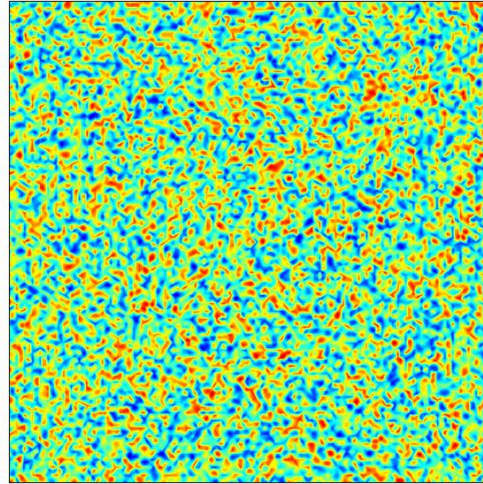


Figure 1. Not a cow.

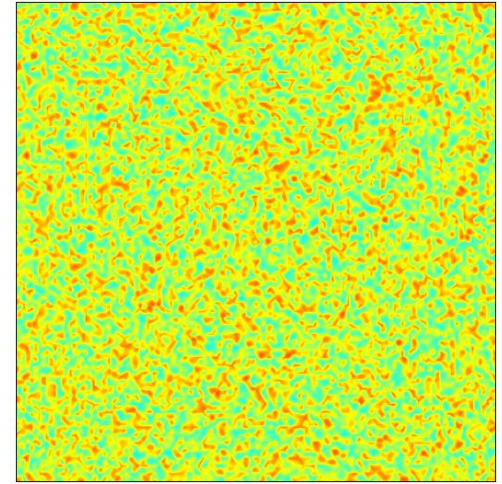
# Turing Patterns



Alan Turing



Random  
initial conditions



Turing pattern  
develops

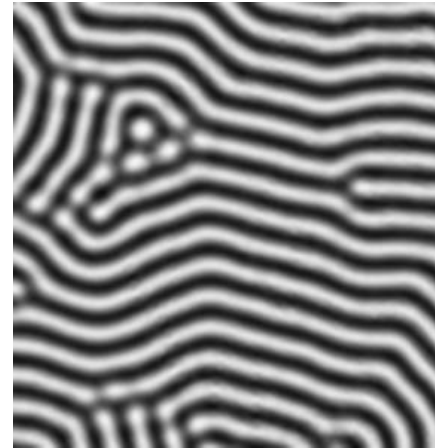
# Turing Patterns in Nature



Leopard spots



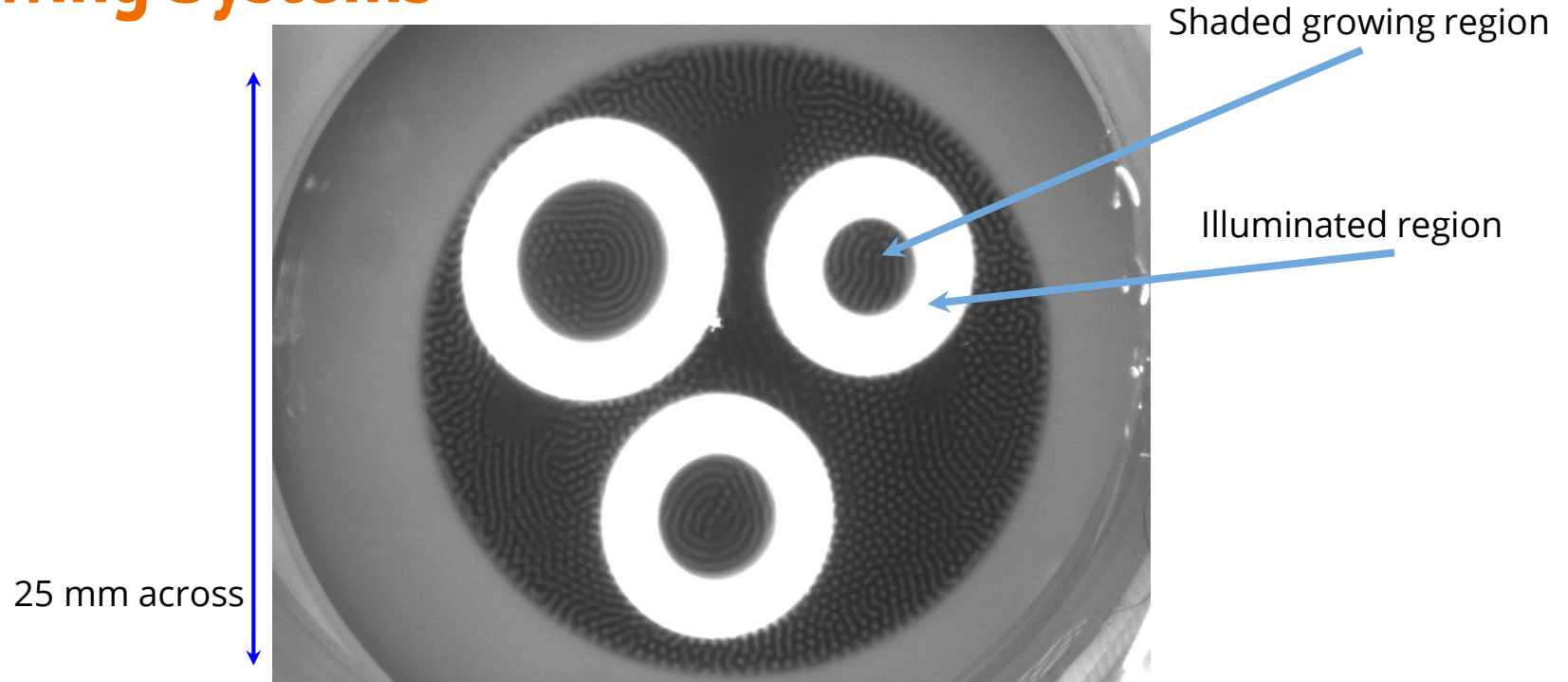
Puffer fish skin



Chemical systems

1. Leopard: Patrick Giraud (edited to fix white balance) [CC BY-SA 3.0] [https://commons.wikimedia.org/wiki/File:Namibia\\_Etoshia\\_Leopard\\_01edit.jpg](https://commons.wikimedia.org/wiki/File:Namibia_Etoshia_Leopard_01edit.jpg)
2. Giant Puffer Fish: Chiswick Chap [CC BY-SA 3.0] [https://commons.wikimedia.org/wiki/File:Giant\\_Puffer\\_fish\\_skin\\_pattern.JPG](https://commons.wikimedia.org/wiki/File:Giant_Puffer_fish_skin_pattern.JPG)

# Growing Systems



Turing patterns in the Chlorine Dioxide-Iodine-Malonic Acid (CDIMA) reaction are inhibited by light.

# Modeling growth with COMSOL



LE Model  
Simulations



CDIMA Reaction  
Experiments

# Chemical Systems and the LE Model

Initial  
concentrations

Activator

$$\frac{\partial u}{\partial \tau} = a - u - \frac{4uv}{1 + u^2} - W + \nabla^2 u$$

Inhibitor

$$\frac{\partial v}{\partial \tau} = \sigma \left[ b \left( u - \frac{uv}{1 + u^2} + W \right) + d \nabla^2 v \right]$$

Activator  
complexation

Illumination

Diffusion

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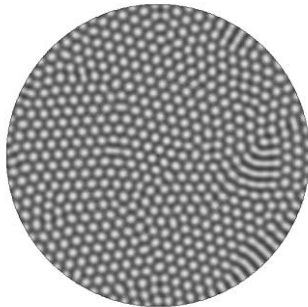
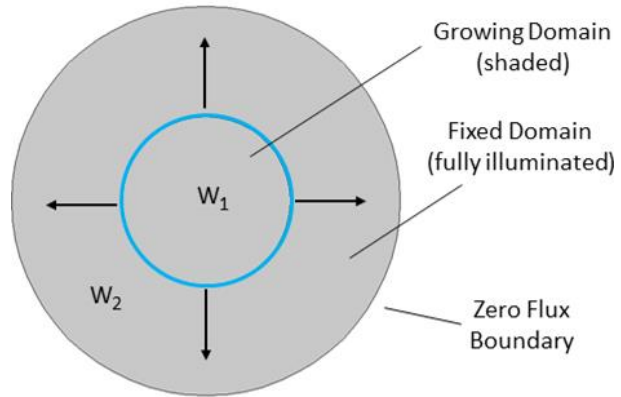
Activator  
complexation

Illumination

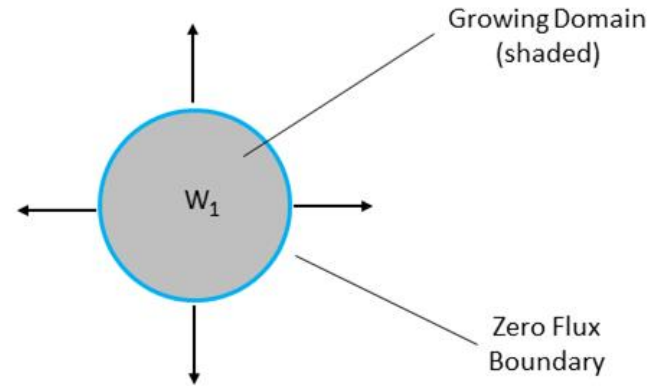
Diffusion

# Alternative Growth

Two Domain Growth



One Domain Growth

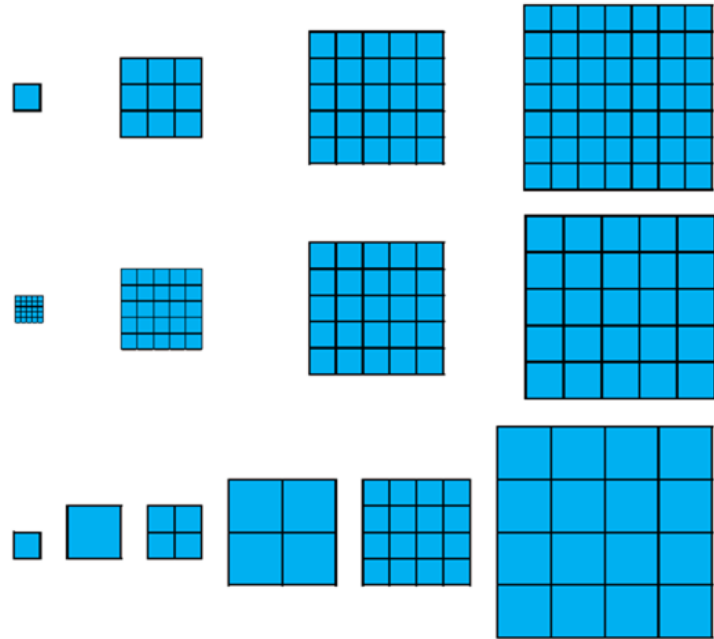


# Meshing methods

Addition - Mesh elements are added to the exterior

Stretching - Mesh elements increase in size

Growth and Division - Mesh elements increase to a point, then divide



# Previous Results

Slow Growth

Intermediate  
Growth

Fast Growth

(Varying speeds)

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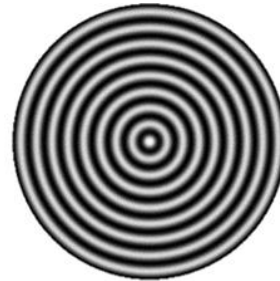
.



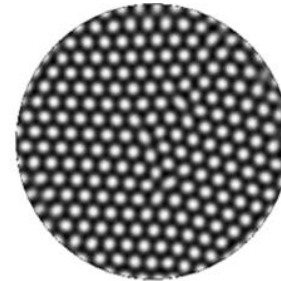
Inner Ring Growth  
(IRG)



Perpendicular Pattern Growth  
(PPG)



Outer Ring Addition  
(ORA)



Spotted Growth

# Application and Results

**Computation Parameters**

$\kappa$ : 12 parameter a  
 $\lambda r$ : 0.31 parameter b  
 $d$ : 1 activator/inhibitor diffusion ratio  
 $\sigma$ : 50 parameter sigma  
 $w$ : 0 light in growing domain

Sweep Growth Rates  
 First Space Step: 0.5  
 Space Step Step: 1  
 Last Space Step: 8

time\_step: 10 time step for growth rate  
 time\_stop: 10 stopping time for growth  
 rad\_start: 0.1 starting cell size  
 rad\_step: 0.50 cell size step (Operates as dummy variable just to match Cellular GR)  
 rad\_stop: 64 stopping cell size

Compute Once and Export  
 Compute Sweep

**Model Parameters**

Date: 072919  
 Study Type: 2domains\_linear  
 Exports Path: Z:\Group\Somb  
 Model Path: Z:\Group\Somb  
 Study Index: 1

Number of degrees of freedom solved for: 36906.

```

83 142.72 6.1016 168 34 16
84 168.82 6.1016 169 34 16
- 170 - out
- 180 - out
85 181.02 12.203 171 35 17
- 190 - out
86 193.23 12.203 172 35 17
- 200 - out
87 202.8 9.8717 178 36 17
- 210 - out
88 212.37 9.8717 184 37 18
- 220 - out
89 221.94 9.8717 190 38 19
- 230 - out
90 231.51 9.8717 196 39 19
- 240 - out
91 241.09 9.8717 197 39 19

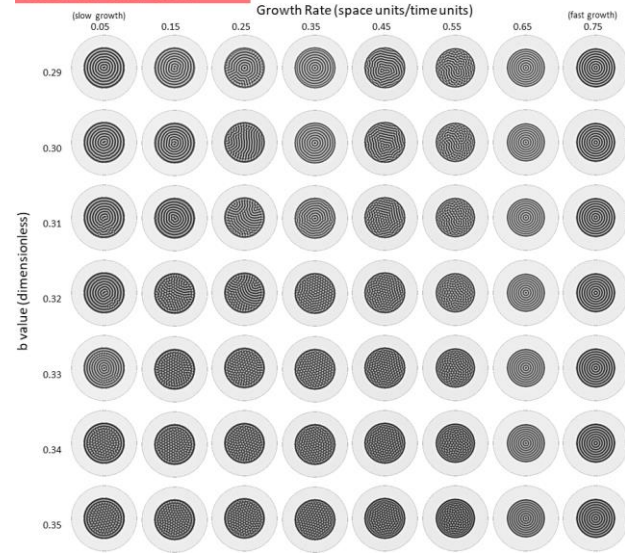
```

Cancelled  
 Solution time: 53 s.  
 Physical memory: 1.12 GB  
 Virtual memory: 1.31 GB  
 Ended at Sep 26, 2019 8:50:41 AM.  
 ----- Time-Dependent Solver 1 in Study 1/Solve

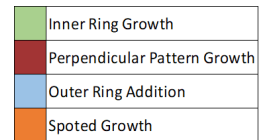
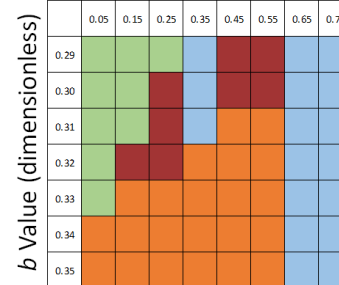
base\_radius=32, cell\_size=1.99 Time=12 Surface: Dependent variable u (1)

Compute Once  
 Save Model  
 Export Final Pattern  
 Export Overlayed Animation  
 Export Mesh Animation  
 Export Pattern Animation

## Two Domains – Linear Growth



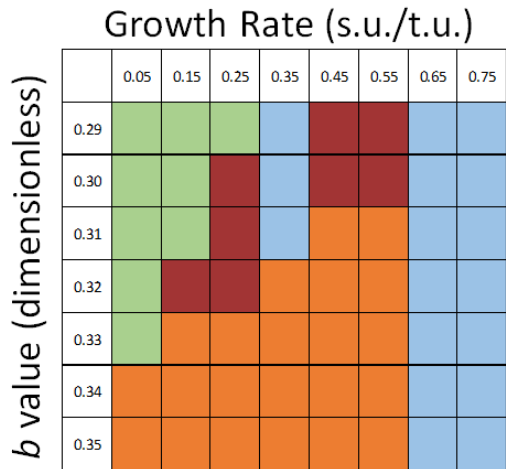
## Two Domains Linear Growth Rate (s.u./t.u.)





# Results

## Two Domain Growth



Inner Ring Growth (IRG)



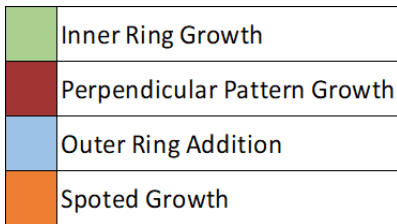
Perpendicular Pattern Growth (PPG)



Outer Ring Addition (ORA)

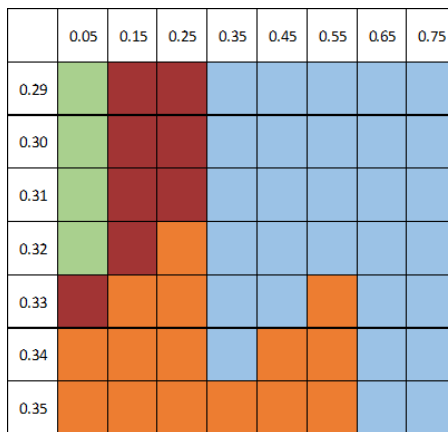


Spotted Growth

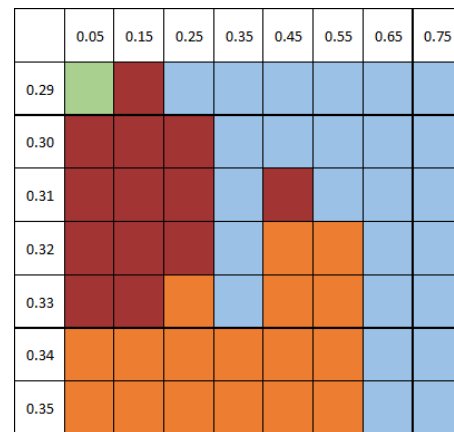


## One Domain Growth

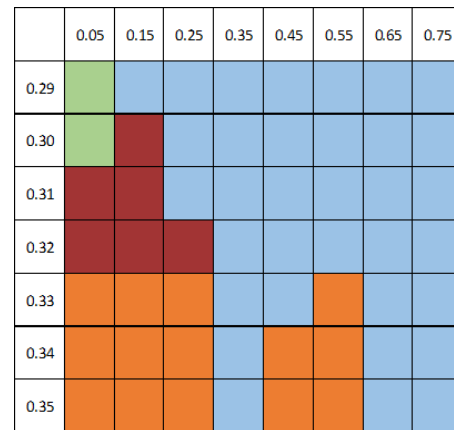
### Addition



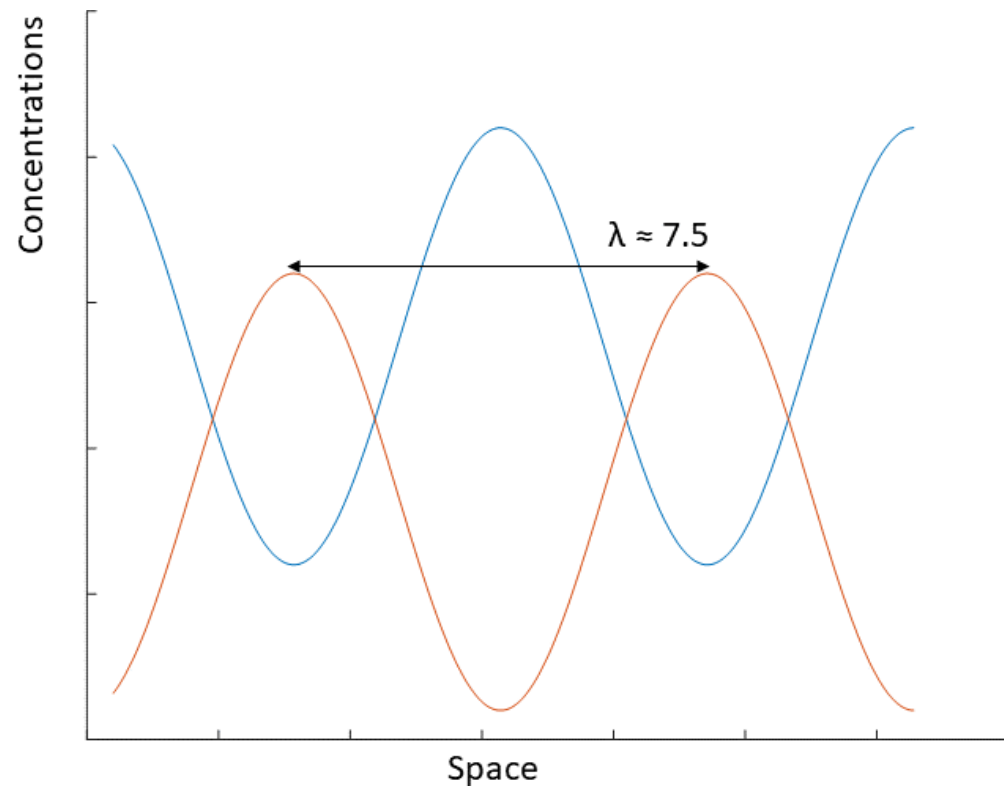
### Growth and Division



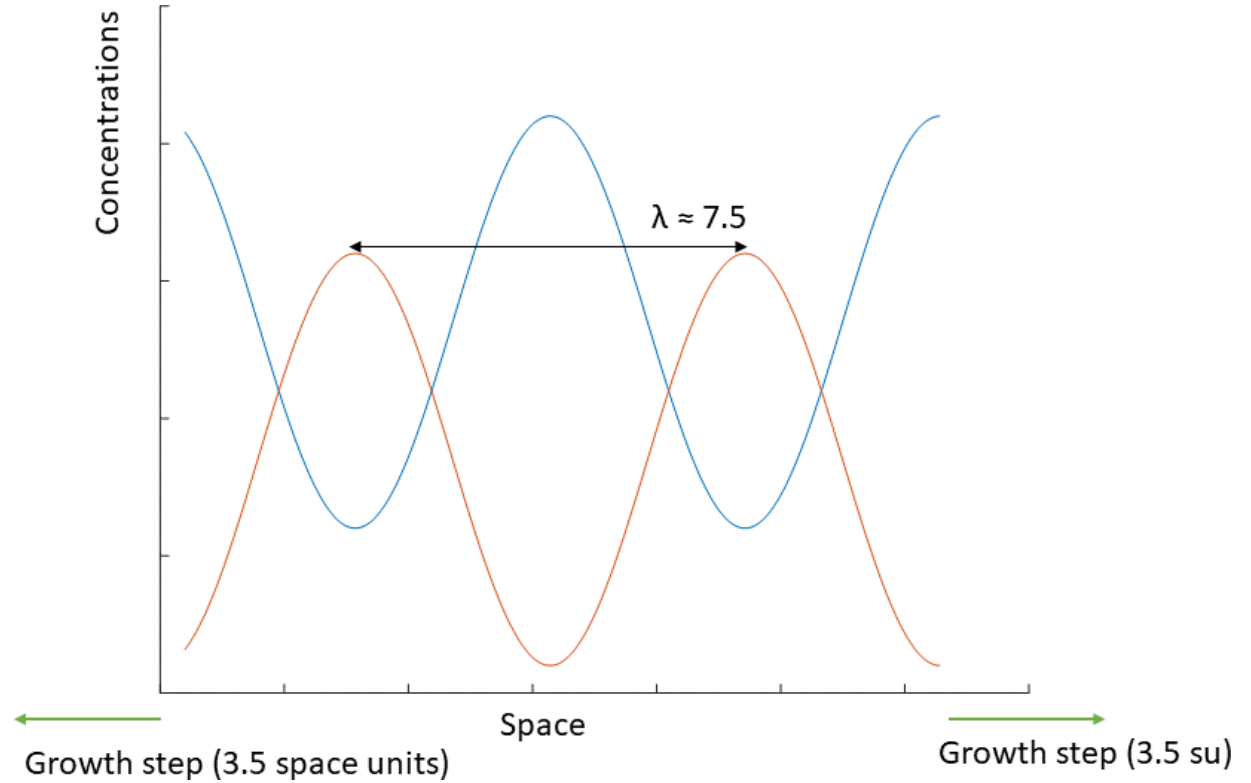
### Stretching



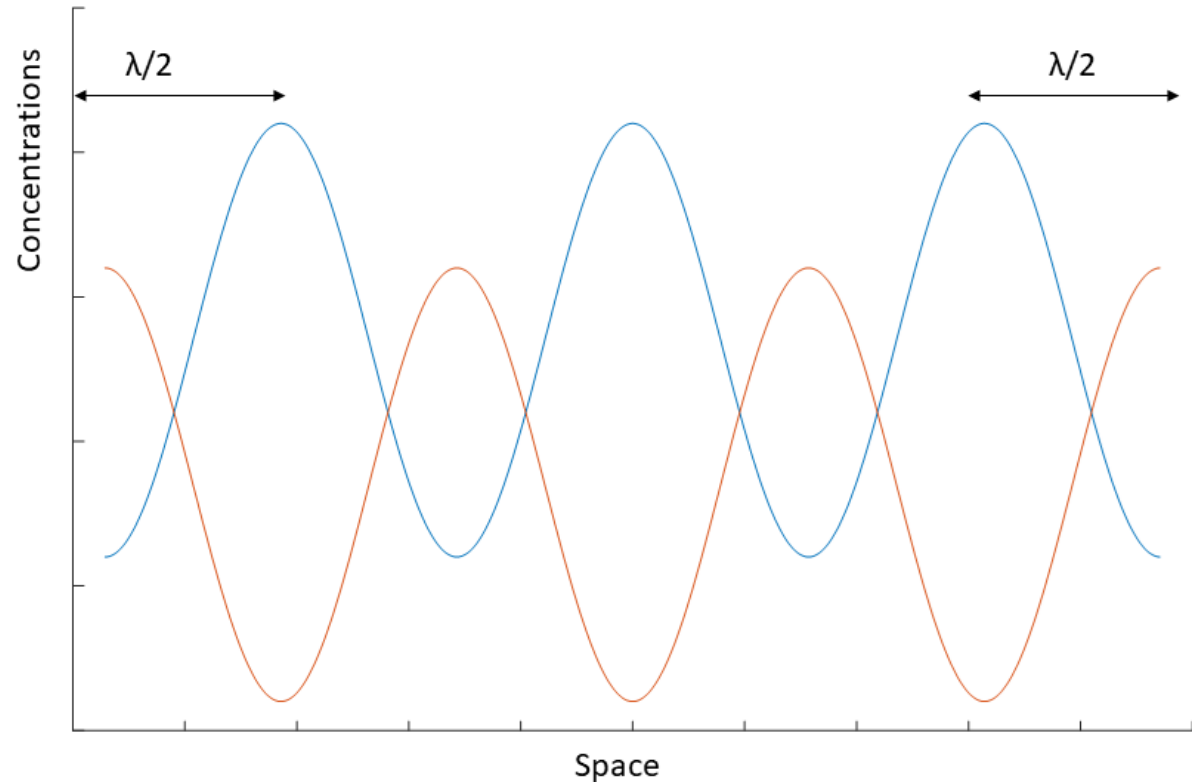
# Resonance



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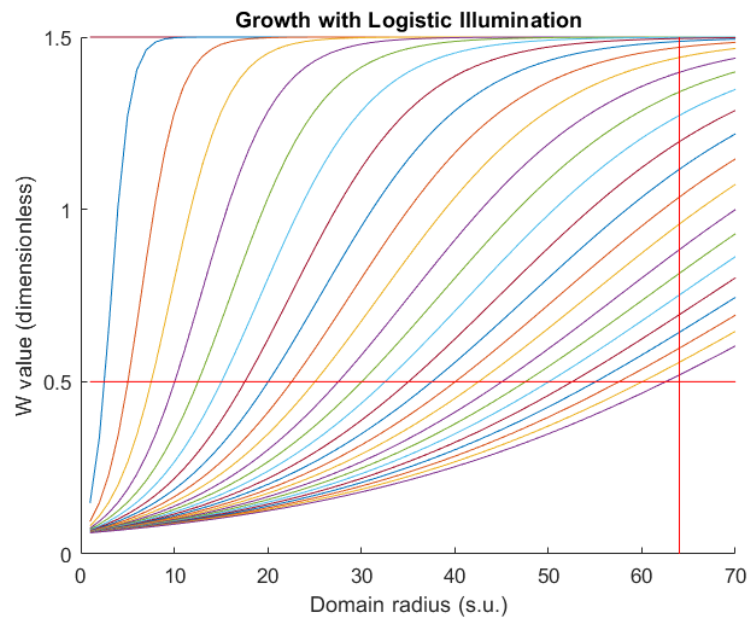


# Resonance



Growth Rate = Wavelength

# Conclusion



# Questions?

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**M.R. Bauer  
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