Turing Pattern Growth Modes

Noah H. Somberg, Christopher Konow,
Irving R Epstein, Milos Dolnik

Brandeis

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Why aren’t cows spherical?

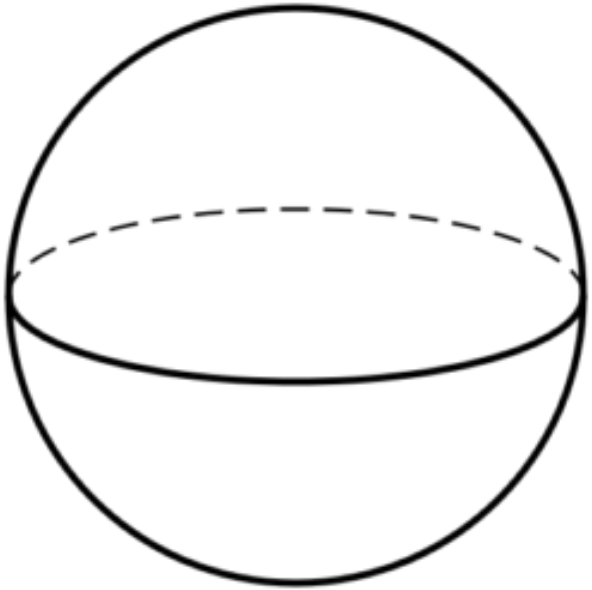
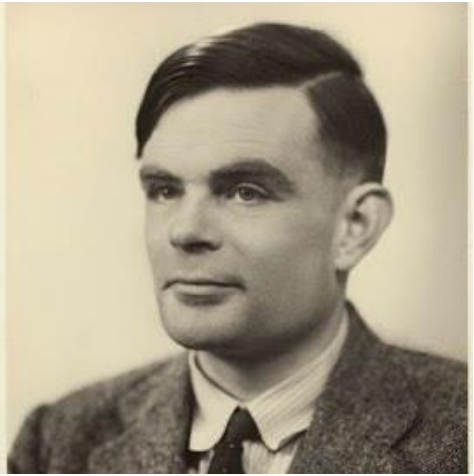
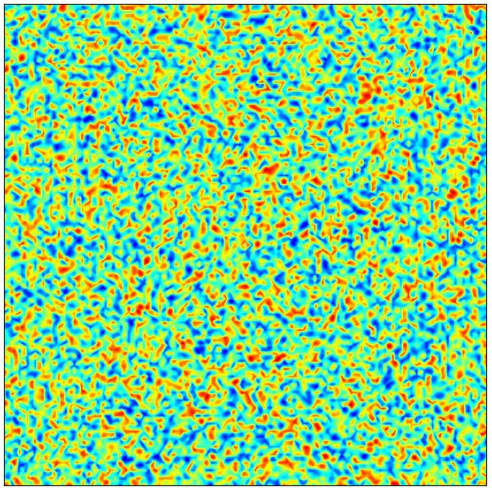


Figure 1. Not a cow.

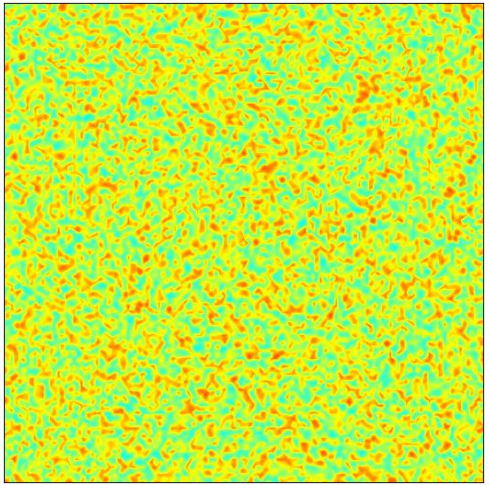
Turing Patterns



Alan Turing



Random
initial conditions

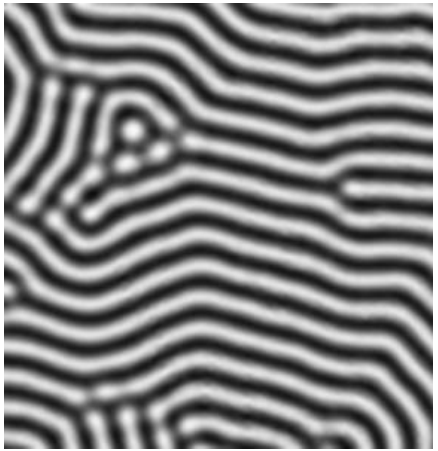


Turing pattern
develops

Turing Patterns in Nature

Leopard spots

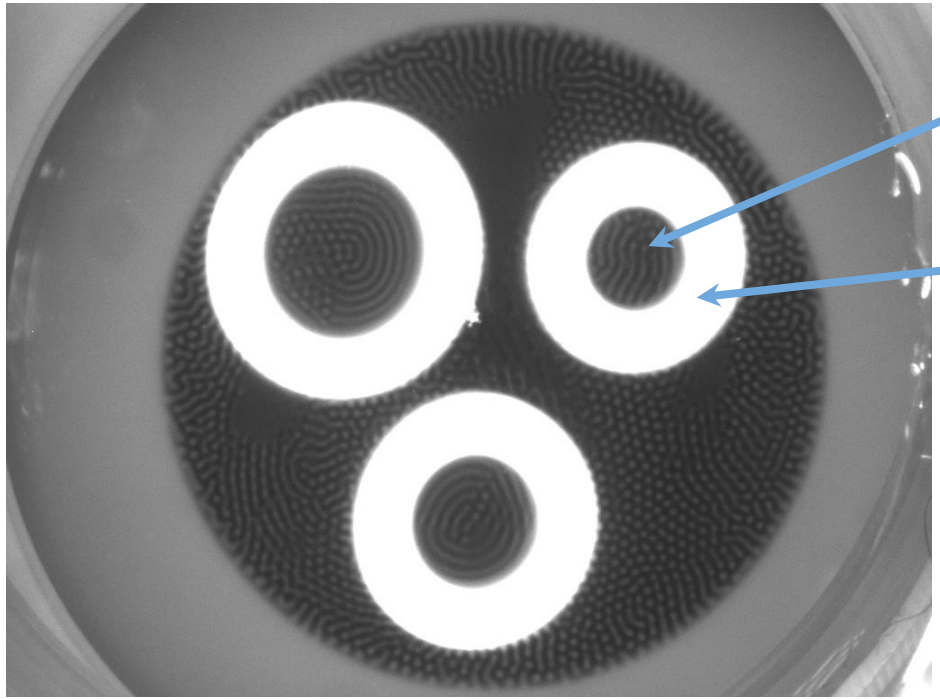
Puffer fish skin



Chemical systems

1. Leopard: Patrick Giraud (edited to fix white balance) [CC BY-SA 3.0] https://commons.wikimedia.org/wiki/File:Namibie_Etosha_Leopard_01edit.jpg
2. Giant Puffer Fish: Chiswick Chap [CC BY-SA 3.0] https://commons.wikimedia.org/wiki/File:Giant_Puffer_fish_skin_pattern.JPG

Growing Systems



Turing patterns in the Chlorine Dioxide-Iodine-Malonic Acid (CDIMA) reaction are inhibited by light.

Modeling growth with COMSOL

LE Model Simulations

CDIMA Reaction Experiments

Chemical Systems and the LE Model

Initial concentrations

Activator

$$\frac{\partial u}{\partial \tau} = a - u - \frac{4uv}{1+u^2} - W + \nabla^2 u$$

Inhibitor

$$\frac{\partial v}{\partial \tau} = \sigma\left[b\left(u - \frac{uv}{1+u^2} + W\right) + d\nabla^2 v\right]$$

Activator complexation

Illumination

Diffusion

Chemical Systems and the LE Model

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Activator complexation

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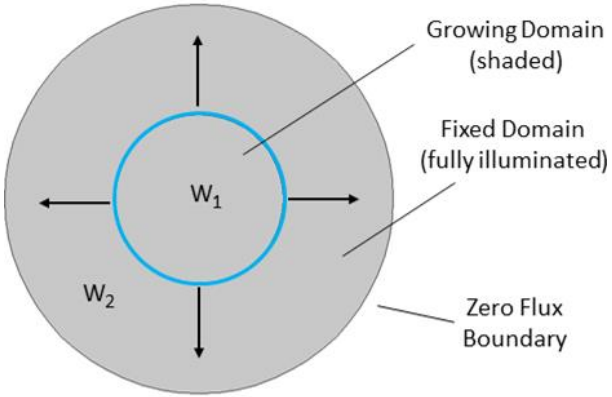
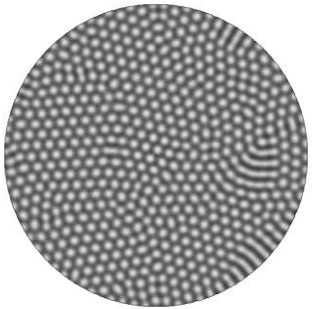
Diffusion

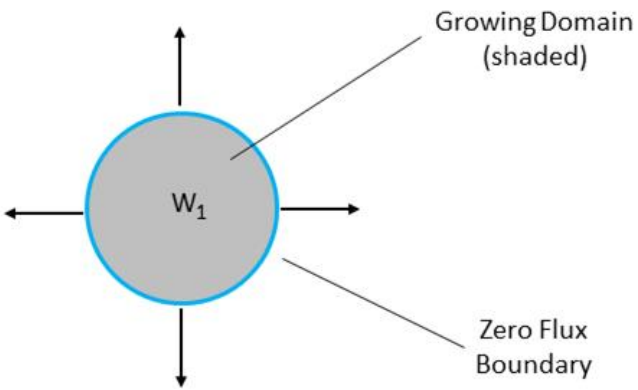
Alternative Growth

Two Domain Growth





One Domain Growth

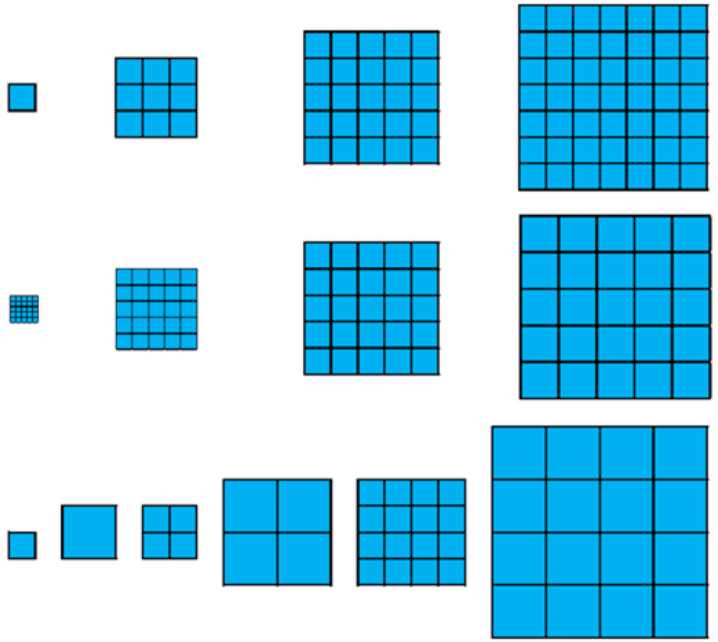


Meshing methods

Addition - Mesh elements are added to the exterior

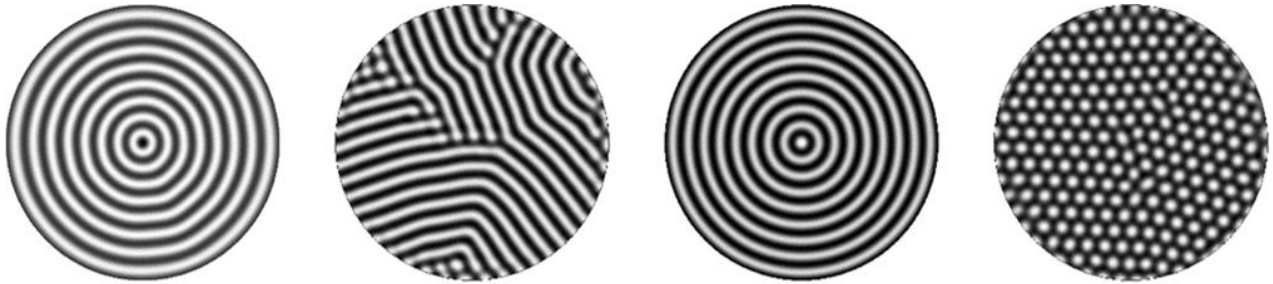
Stretching - Mesh elements increase in size

Growth and Division - Mesh elements increase to a point, then divide



Previous Results

Slow Growth | Intermediate Growth | Fast Growth | (Varying speeds)



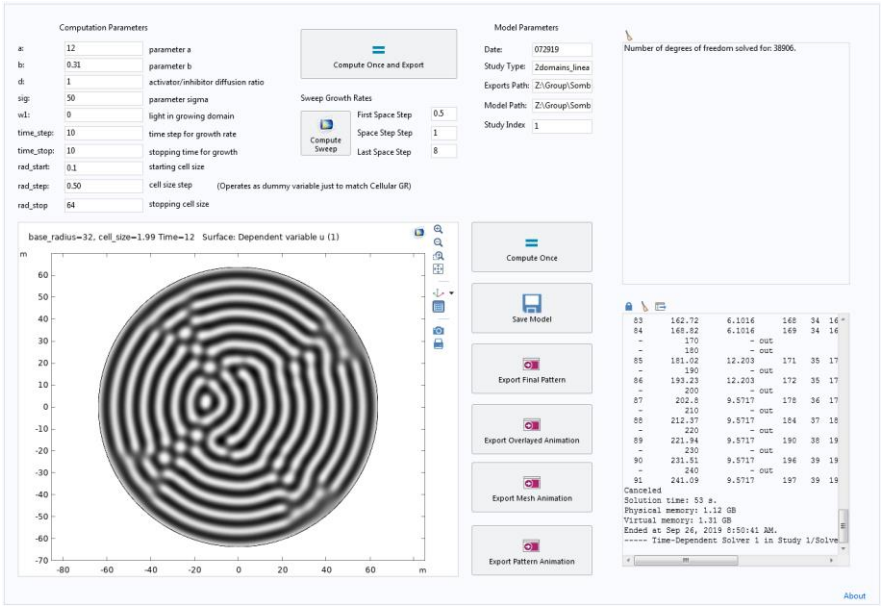
Inner Ring Growth (IRG)

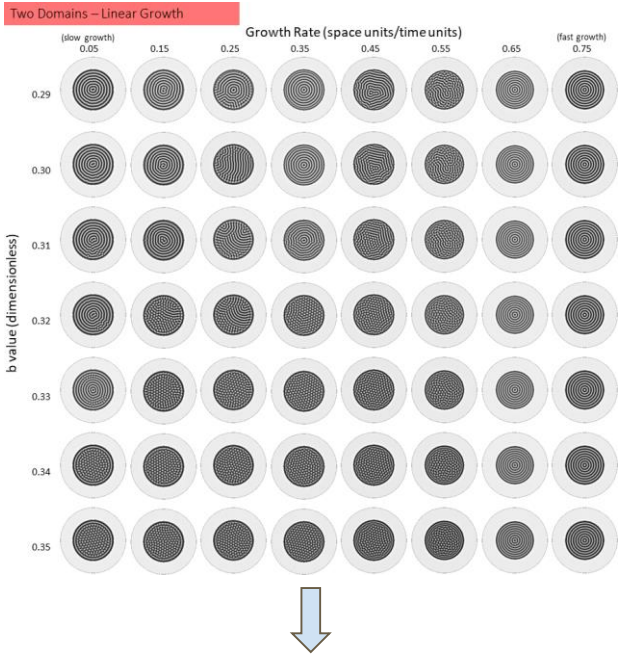
Perpendicular Pattern Growth (PPG)

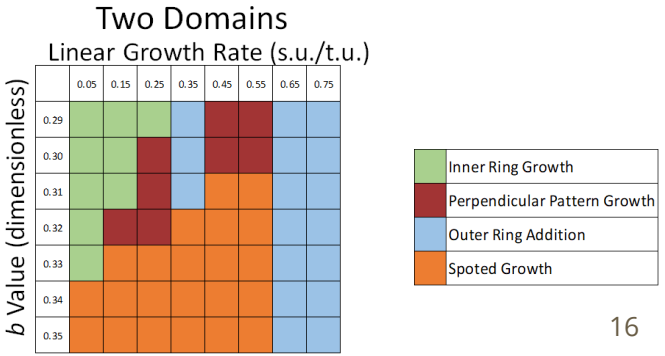
Outer Ring Addition (ORA)

Spotted Growth

Application and Results

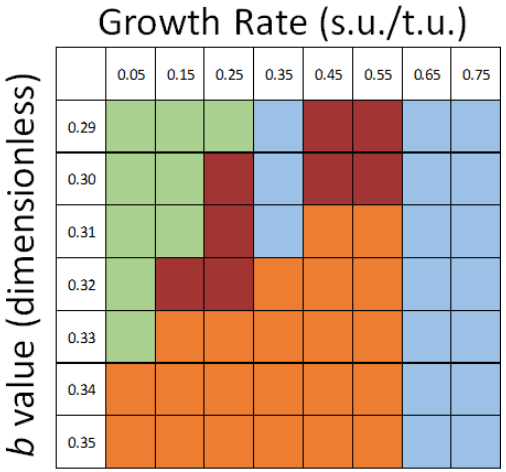


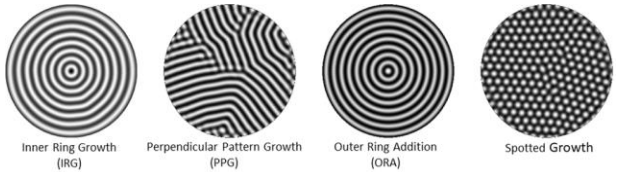


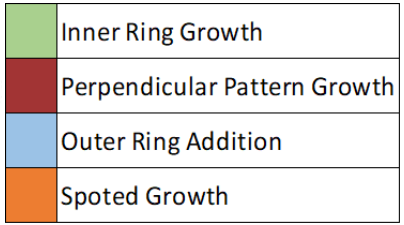


Results

Two Domain Growth

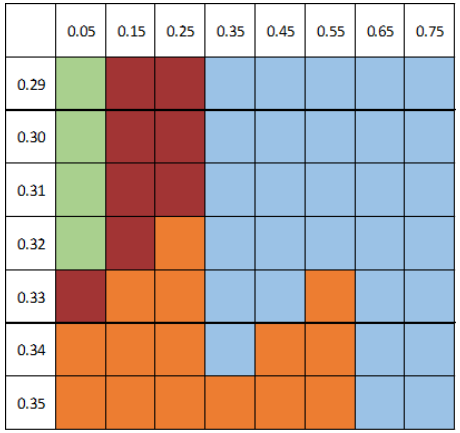




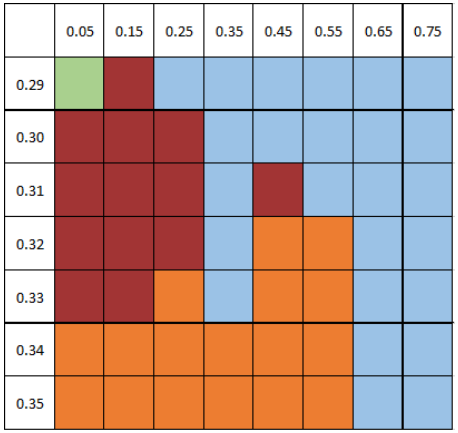


One Domain Growth

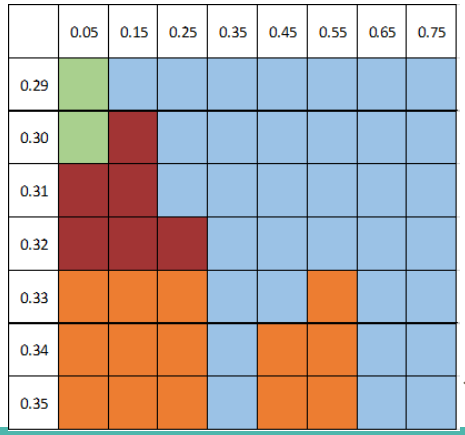
Addition



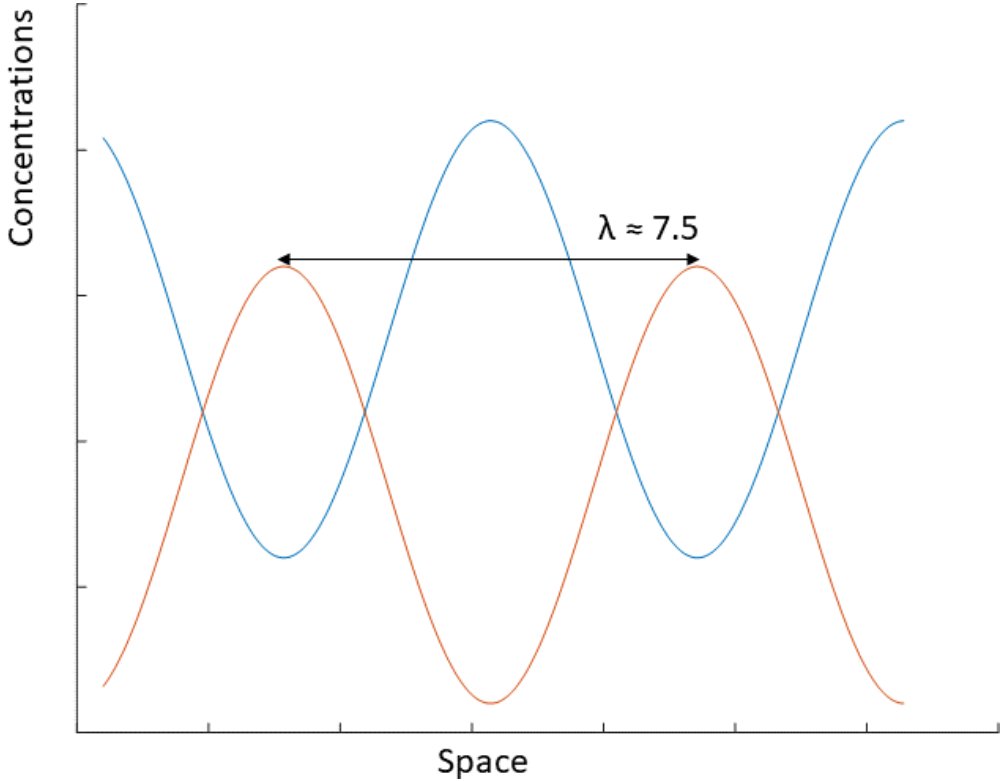
Growth and Division



Stretching



Resonance



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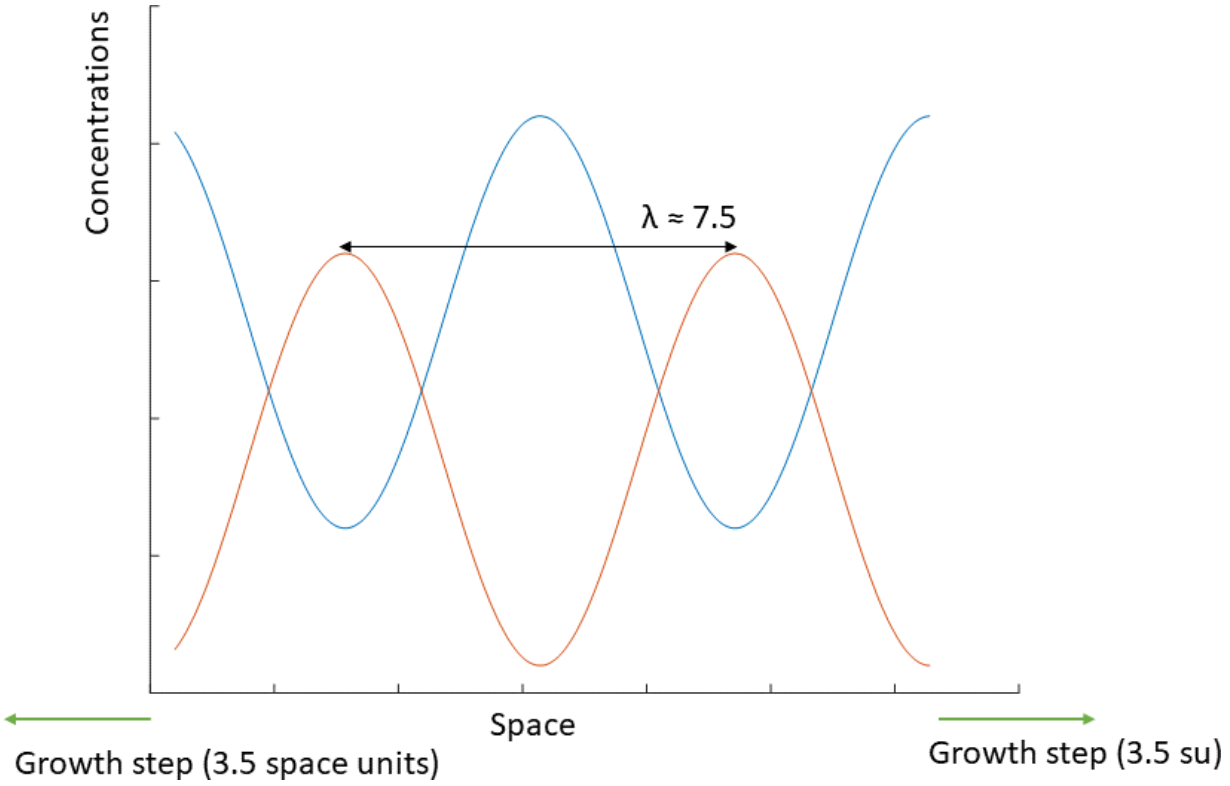
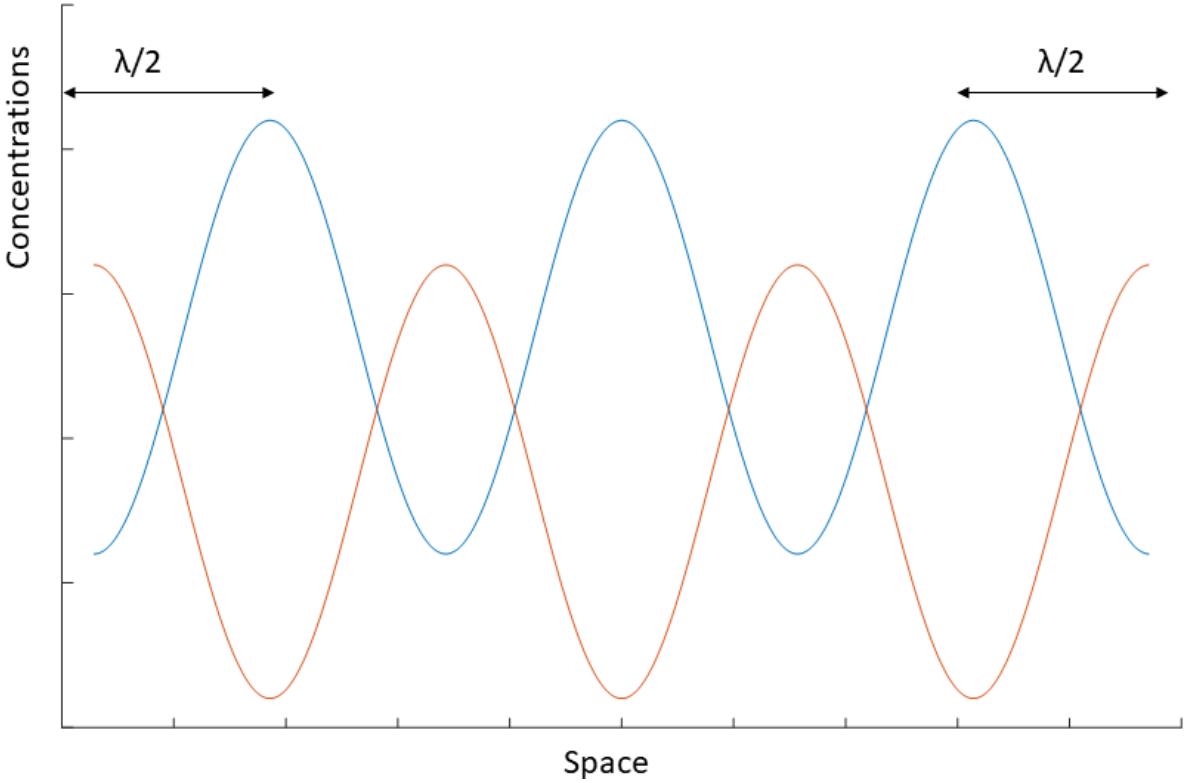
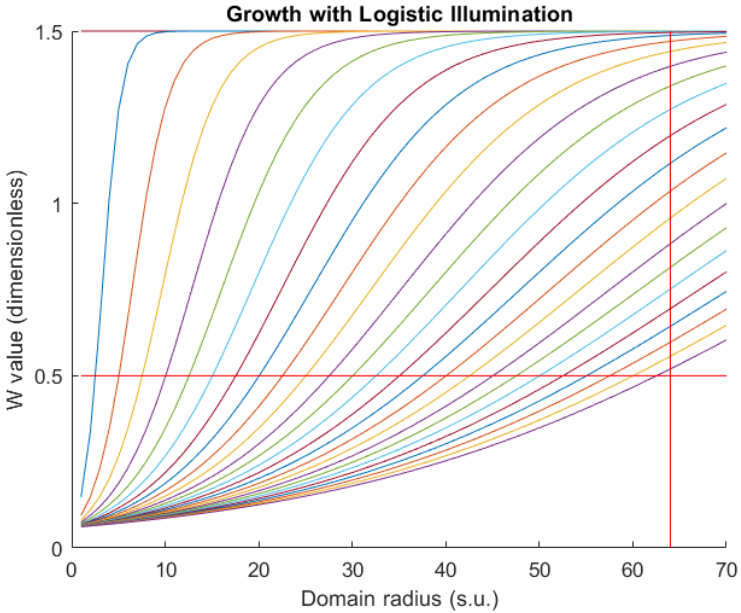

Concentrations
λ ≈ 7.5
Space
Growth step (3.5 space units)
Growth step (3.5 su)

Resonance



Growth Rate = Wavelength

Conclusion


Growth with Logistic Illumination
W value (dimensionless)
Domain radius (s.u.)
0
0.5
1
1.5
0
10
20
30
40
50
60
70

Questions?

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