



A Simplified Model for the Evolution of a Geothermal Field

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Abstract

The problem is to understand how a geothermal field can evolve from a water dominated state (i.e. when the geothermal fluid is mainly in liquid phase) into a vapour dominated one. A first answer to this question is given by a simplified mathematical model of the dynamics of a **geothermal field** in which the geothermal fluid is entirely composed by pure H_2O . We considered a one-dimensional geometry and we developed a dynamic model that presents a clear interface between the gas phase (which occupies the upper part of the basin) and the liquid phase (which occupies the lower part). Due to the process of evaporation or condensation, the interface changes its position over time and it is therefore modeled with a free boundary, whose dynamics is governed by Rankine-Hugoniot conditions. We obtained a **free boundary problem** of Stefan type. Solving the problem by COMSOL Multiphysics, it is possible to get informations on the evolution of the interface.

Introduction

A geothermal reservoir is a complex permeable medium, located deep in the earth's crust, consisting in fractured hot rocks in which the geothermal fluid, composed primarily of water, can flow. The latter is responsible of the heat transfer and it can achieve the ground surface, due to natural way (geyser, "fumarole", etc.) or due to industrial purposes (geothermal wells). According to the predominant phase of the geothermal fluid the geothermal reservoirs can be divided into *water dominated* or *vapor dominated*. In general, a water dominated geothermal field evolves into a vapour dominated one. Examples of this evolutionary path are encountered in nature, as in the geothermal field of Larderello in Italy. Understanding this evolution is important because it allows to have information on the age of the basin and its ability to be exploited for the production of geothermal energy.

Thus, the following questions naturally arise:

- Which are the main factors affecting the evolution of a geothermal field?
- What is the geological time evolution of a water-dominated basin into a vapor dominated one?

In an attempt to shed light on these questions, we shall discuss in this paper a simplified mathematical model.

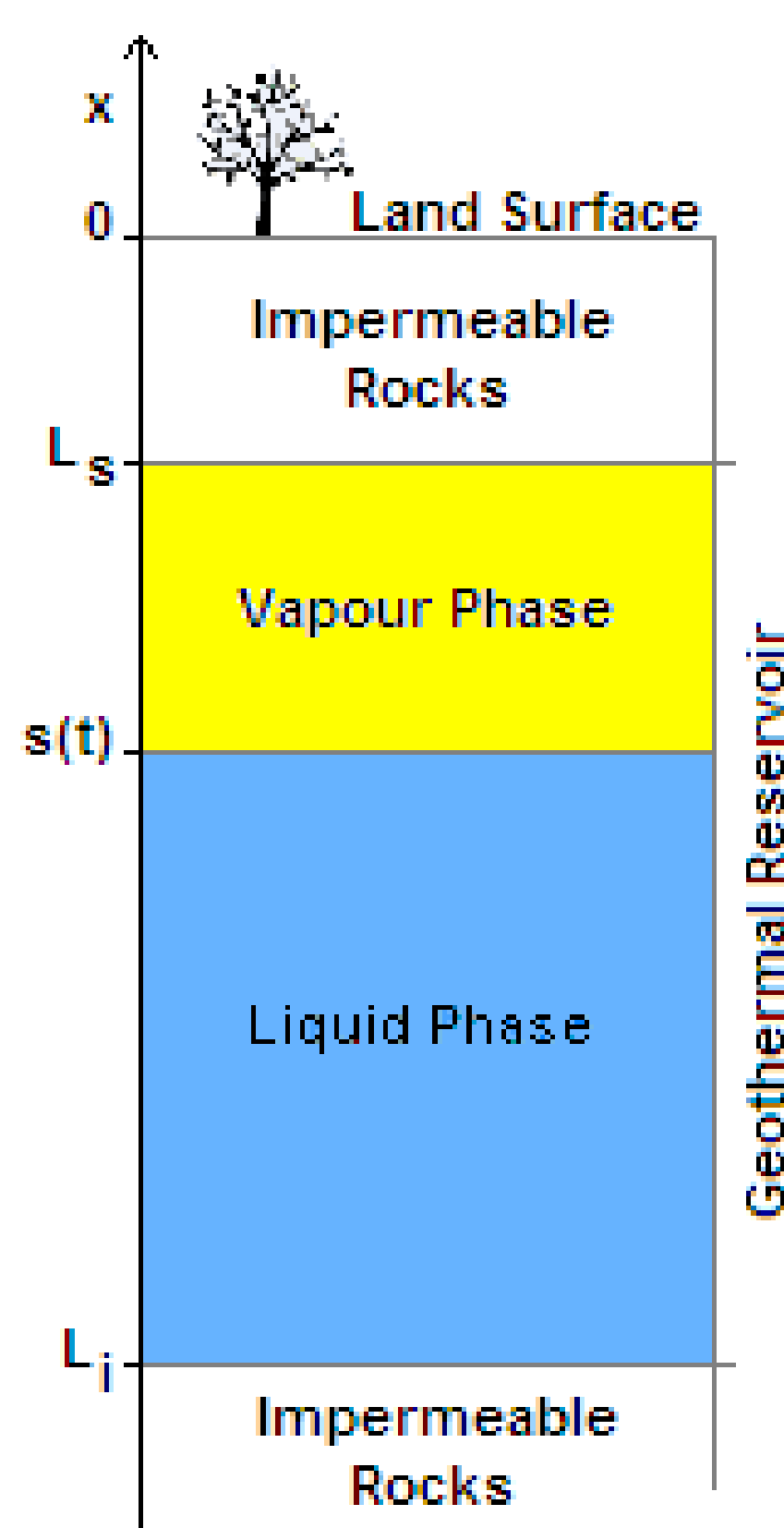
Acknowledgements



References

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Mathematical Model and Governing Equations



in the variable domain $(s(t), L_s)$

$$\left\{ \begin{array}{l} \frac{\partial P_v}{\partial t} - \frac{KT}{\phi\mu_v} \frac{\partial}{\partial x} \left[\frac{P_v}{T} \left(\frac{\partial P_v}{\partial x} + \frac{g}{r} \frac{P_v}{T} \right) \right] = 0, \\ P_v(x = L_s) = P_s, \\ P_v(x = s(t)) = P^*(s(t)), \\ \dot{s} = \frac{P^*(s(t))}{rT\rho_l} \frac{K}{\phi\mu_v} \left(\frac{\partial P_v}{\partial x} + \frac{P^*(s(t))}{rT} g \right) \Big|_{x=s(t)}, \\ P_v(t = 0) = P_{in}(x), \\ s(t = 0) = s_{in}, \end{array} \right. \quad (1)$$

Let us consider the geothermal fluid contain only pure H_2O in the states gaseous and liquid. Suppose further that there is a clear separation between the two phases, through a boundary $s(t)$ which may change over time. Let us assume a one-dimensional geometry as in Figure. According to the experimental results, we set a constant with time temperature $T(x)$, increasing linearly with depth. The model consists of the following system of equations for the vapour pressure P_v

where g , r and P^* are the acceleration of gravity, the constant of perfect gases and the pressure of saturated vapour respectively. Moreover, we supposed the porosity ϕ , the viscosity of vapour μ_v and the permeability K to be constants. We notice that (1)₁ comes from the balance of the mass [1], (1)₄ arises from continuity of mass flow and momentum through the interface [2][3], (1)₂ e (1)₃ are boundary conditions of Dirichlet type and (1)₅ e (1)₆ are the initial conditions.

Numerical Method and Use of COMSOL Multiphysics

- To simulate the movement of the spatial domain, we exploited the technique of *deformed mesh*.
- We used the package **ALE for Moving Mesh** to simulate the free boundary problem.
- The key point is that, instead of generating a new mesh for each configuration of the boundaries, we use a technique that

perturbs the mesh nodes conforming them with the moved boundaries.

- The ALE method is an intermediate between the Lagrangian and the Eulerian method, and it combines the best features of both.
- The successful implementation accounts for COMSOL Multiphysics potential to solve free boundary problems.

Results and Conclusions

The model allows to describe the variation of field of vapour pressure w.r.t. time and the dynamics of the interface, if a fixed pressure (below the saturated vapour pressure imposed on the free boundary) is imposed at the top boundary. First Figure shows the expected pressure values in Pascal along the geothermal reservoir ($x = 0$ is for the top level and $x = -1$ is for bottom level). We can see that the pressure field is defined on a domain growing with time. This fact is due to the adding area occupied by the vapour and then to the lowering interface as we see from second Figure.

A first conclusion is that **the evolution of the geothermal reservoir closely depends upon the pressure at the top of the reservoir, which is**

linked to natural events (like geysers or "fumarole" in Larderello) or the human exploitation for the production of energy (geothermal wells). Thanks to simulations **we obtained an approximate way to quantify the environmental impact due to a certain exploitation.**

Moreover, results show that while the characteristic time of diffusive phenomenon is of the order of decades, as supported by geological revelations, **the characteristic time of the movement of the interface is of the order of thousands of years.** Thus, the time evolution of a water-dominated basin into a vapor dominated one, time within which the reservoir fluid evaporates completely, is a period that spans millennia.

