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Modelling Waste Water Flow in Hollow Fibre Filters

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Introduction

Project **PURIFAST**

Advanced **PUR**ification Of **I**ndustrial And Mixed Wastewater By Combined Membrane **F**iltration **A**nd **S**onochemical **T**echnologies

LIFE +

Environmental Policy and Governance

Grant agreement n. LIFE07 ENV/IT/000439

Duration: January 2009 – December 2011

Partnership:

- *Coordinator*: Next Technology Tecnotessile (Italy)
- *Research and Technical activities*:
  University of Florence - Dep. of Civil Engineering (Italy)
  University of Florence - Dep. of Mathematics (Italy)
  IWW GmbH (Germany)
- *Manufactures industries*:
  Lavo (Italy) – Polymem SA (France) – Inge AG (Germany)
- *End-users industries*:
  Gestione Impianti Depurazione Acque S.p.A. (Italy)
  King Colour S.p.A. (Italy)

Introduction

Final goal of the project:

Demonstration of a tertiary treatment system based on ultrafiltration and sonochemical technologies for purification and reuse of textile and mixed effluents pre-treated by a biological process, to be spread among industries and public service managers located in textile clusters.

Main tasks of our activity in the project

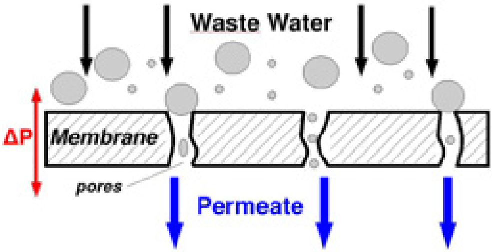
1. Modelling and simulation of filtration process at the meso-scale (i.e. single filter module)
2. Optimization of the parameters at the macro-scale (i.e. filtering plant)

Two filtering devices (based on polymeric membranes)

- Hollow fibre
- Multi-bore

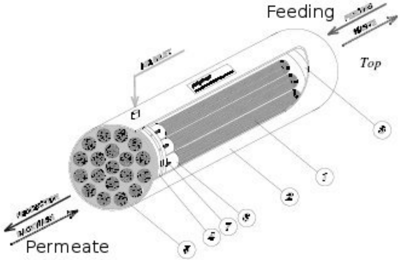
How the membrane works.

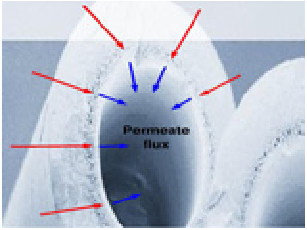
- We deal with an **ultrafiltration** process: pores diameter 0.01 – 0.1 μm
- A pressure gradient ΔP is applied.
- All the particles larger than the pore diameter are cut off.



- Several membrane configurations: flat, tubular, spiral wound, **hollow fibre**

Modelling the hollow fibre module





Each membrane module consists of a pressure vessel housing a number of membrane bundle *U-shaped* (i.e. one potting at the bottom end); each bundle consists of a series of hollow fibre membranes.
A *dead-end* filtration process consisting of two steps:

- *Production*: outside/in filtration based on a pressure gradient (suction) between fibre lumen and outside.
- *Backwashing*: to clean the membrane surface (often coupled with *air scouring*)

(Courstesy of *Polymem SA* ®)

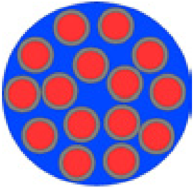
Model definition: general consideration.

The approach of coupled porous regions (double-porosity, double-permeability medium)

We identify two regions:

- The lumen region: the total space occupied by the lumina of the hollow fibres.
- The shell region: the space between the fibres.

The total membrane area is the interface between these interpenetrating media.
These media have a continuous spatially dependent source/sink.
(Ref: Labecki et al., *Chem. Eng. Sci.*, 1995).

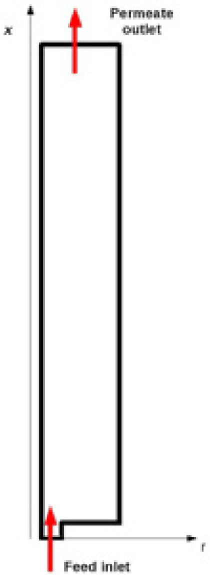


Model definition: general consideration (ctd.)

Physical assumptions:

- Saturated porous media.
- Typical Reynolds number: $Re < 1 \Longrightarrow$ Darcy's law.
- The waste water has some chemical species in supension (to be filtered!). Due to the pressure gradient a portion of this pollutant is adsorberd on the outer surface of the fibre: it forms a film (s.c. *cake*) soiling the membrane (s.c. fouling process).
- Only one chemical species (for the sake of simplicity).
- A periodic backwash is set to remove the cake from the membrane surface: the water flux is inverted.

Geometry. 2D-domain: (cilindrical) axial symmetry



Governing Eqs.

Subscripts notation:
$(\cdot)_s$ is referred to the shell region.
$(\cdot)_l$ is referred to the lumen region.

Steady-state mass conservation (with constant fluid density, ρ):

$$\nabla \cdot \mathbf{v}_s = -\Gamma, \tag{1}$$
$$\nabla \cdot \mathbf{v}_l = \Gamma, \tag{2}$$

where: $\mathbf{v}$ is the Darcy's velocity and Γ is the source/sink term.

Rate of mass density loss: $\rho\Gamma = \tilde{\Gamma} = \frac{(\Delta\rho)_{filt}}{(\Delta t)_{filt}} = \rho\frac{Q_{filt}}{V_{filt}} = \rho\frac{A_{filt}}{V_{filt}} v_{filt}$

where Q_{filt} and v_{filt} are the mass rate loss and the filtration specific discharge, respectively.

How can we describe Γ ?

The filtration velocity is linked to the averaged velocity within the (porous) membrane $\Longrightarrow$ it is **proportional to the pressure difference** $(P_l - P_s)$.

Definition: *membrane resistance*

$$R_m, \quad [R_m] = L^{-1} \text{ linked to } \approx \frac{d_m}{k_m}.$$

...Thus:

$$\Gamma = \tilde{\Gamma}/\rho = -\frac{A_v}{\mu R_m}(P_l - P_s)$$

with $A_v = A_{filt}/V_{filt}$, the membrane surface area per unit volume available for filtration.

Remark: we confirmed such a relationship also by means of an *upscaling procedure*

Modelling the fouling process: the resistances in series approach

During the process, the membrane resistance is increased by the presence of the cake on the surface.
Therefore, the total resistance is

$$R_m + R_c,$$

with

$$R_c(x, r, t) = \gamma_c r_0 c_m(x, r, t) \tag{3}$$

where c_m is the mass concentration of the cake into the shell region.
(r_0: outer radius of the fibre; γ_c: constant parameter)

Modelling the fouling process: the resistances in series approach (ctd.)

The cake's growth is linked to the concentration of the pollutant species (c) in the shell region.
The variable c obeys to a classical **advection-diffusion equation**:

$$\frac{\partial}{\partial t}(\varepsilon_s c) + \nabla \cdot (c\mathbf{v}_s) = \nabla \cdot (\varepsilon_s D_s \nabla c) - \frac{\partial c_m}{\partial t} \tag{4}$$

where:
ε_s, porosity of the shell region
D_s, hydrodynamic dispersion; α, constant parameter.
The evolution of c_m is modeled as

$$\frac{\partial c_m}{\partial t} = \alpha \, \Gamma \, c \tag{5}$$

with (remember): $\Gamma = -\dfrac{A_v}{\mu\,(R_m + R_c)}\,(P_l - P_s)$

Darcy's law

Important assumptions:

1. Since typically $c_m \ll 1 \implies$ the cake affects the filtration efficency but it does not change porosity and permeability
   Therefore: **porosity & permeability are constant**.
2. In the lumina region the fibres are not directly connected with one another.
   Therefore: the radial component of permeability can be set to zero, so that **the lumen flow is 1-D**.

Thus:

$$\mathbf{v}_s = -\frac{1}{\mu}\left(\mathbf{e}_r k_{s,r}\frac{\partial P_s}{\partial r} + \mathbf{e}_x k_{s,x}\frac{\partial P_s}{\partial x}\right), \qquad (6)$$

$$\mathbf{v}_l = -\frac{1}{\mu}\left(\mathbf{e}_x k_{l,x}\frac{\partial P_l}{\partial x}\right). \qquad (7)$$

Summarising: the complete system

$$-\frac{1}{r}k_{s,r}\frac{\partial}{\partial r}\left(r\frac{\partial P_s}{\partial r}\right)-k_{s,x}\frac{\partial^2 P_s}{\partial x^2}=A_v\frac{(P_l-P_s)}{R_m+R_c}, \tag{8}$$

$$-k_{l,x}\frac{\partial^2 P_l}{\partial x^2}=-A_v\frac{(P_l-P_s)}{R_m+R_c}, \tag{9}$$

$$\frac{\partial}{\partial t}(\varepsilon_s c)+\nabla\cdot(c\mathbf{v}_s)=\nabla\cdot(\varepsilon_s D_s\nabla c)-\frac{\partial c_m}{\partial t} \tag{10}$$

$$R_c(x,r,t)=\gamma_c r_0 c_m(x,r,t) \tag{11}$$

$$\frac{\partial c_m}{\partial t}=\gamma\left[-\frac{A_v}{\mu}\frac{(P_l-P_s)}{R_m+R_c}\right]\ c \tag{12}$$

Similar to the previous case, but now the evolution eq. for c_m is:

$$\frac{\partial c_{m,back}}{\partial t} = -\alpha \left[\frac{A_v}{\mu} \frac{(P_s - P_l)}{R_m + R_c} \right] c_{m,back} - \beta J_{back} c_{m,back}. \quad (13)$$

where $\beta J_{back} c_{m,back}$ is the term accounting for the air scouring, with β parameter to be calibrated via experiments, $[\beta] = L^{-1}$, and

$$R_c = R_c \left(c_{m,back} \right).$$

The initial condition for equation (13) is the value of c_m at the end of the previous step.

Simulations on a pilot-module

We run the model by *COMSOL Multiphysics*® to simulate the process relative to a HF module by Polymem® (see *www.polymem.fr*)

Module specifications

Only 1 boundle of hollow fibres.
Module Length, $L = 900mm$.
Module radius, $R = 37,5mm$.
Membrane Area, $A_{filt} = 6m^2$.

Process conditions

Feed volumetric flux, $150l/h$.
Pollutant type: *chlorides*.
Pollutant concentration in the feed, $460mg/l$.
Time of production: 1 *hour*
Time of backwash: 30 *sec*.

Simulations on a pilot-module (ctd.)

Boundary conditions

Remember: we are dealing with a **dead-end filtration**.
Therefore: inlet flux (feed) equals the outlet flux (permeate).
Denote the flux with J_f, $[J_f] = LT^{-1}$.

- On the inlet boundary:
  - $\mathbf{v}_s = J_f$.
  - $\mathbf{v}_l = 0$.
  - $c = c_{in}$.
- On the outlet boundary:
  - $\mathbf{v}_s = 0$.
  - $\mathbf{v}_l = -J_f$.
  - No flux condition for c.
- Elsewhere: *no flux condition*.

Use of COMSOL Multiphysics

We solved separetely filtration and backwash. For each stage:

- The *Darcy's law - Pressure* for a saturated medium (*Earth Science Module*), to solve equations flow equations.
- The *Solute Transport* mode (*Earth Science Module*) for the transport equation.
- The *Diffusion* mode for the evolution of c_m.

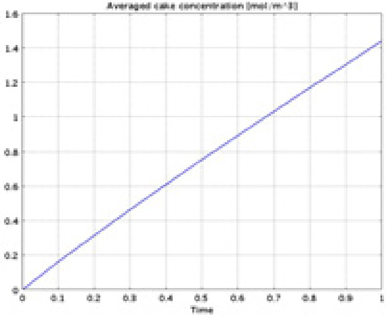
Remark: tips (and useful "tricks" ;-)

For each stage:

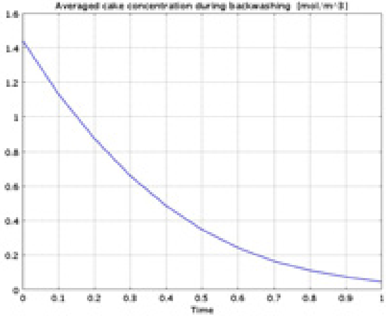
- The *Diffusion* mode was applied with a **vanishing diffusion coefficient**.
- The term Γ was defined as *Global Expression*
- The flow eqs. are stationary: nevertheless, after preliminary tests we decided to analyze them in a *transient* mode with a zero storage term.

Simulation results:

Cake concentration (average over the volumetric domain)



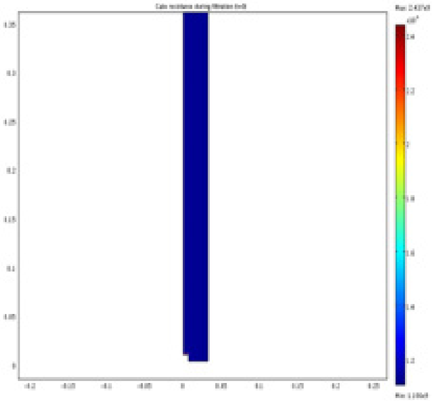
Production



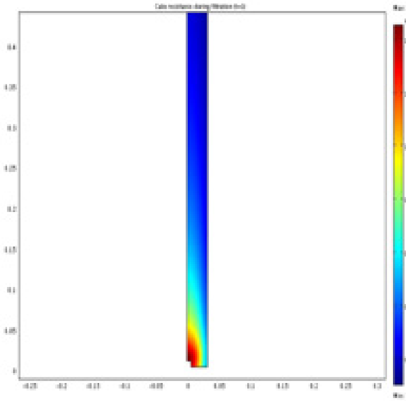
Backwash

Simulation results: cake resistance during **production**

Time scaled to 1 h.



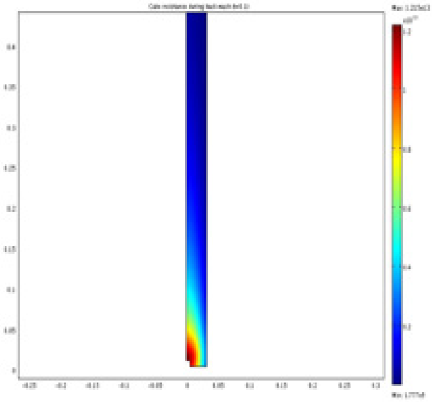
t=0



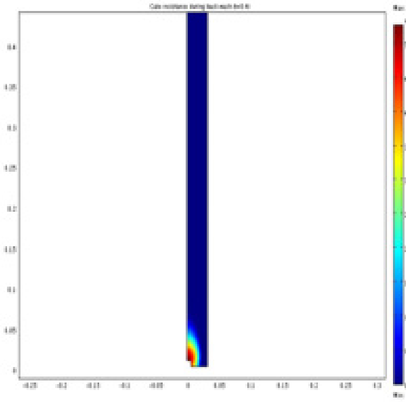
t=1

Simulation results: Cake resistance during **backwash**

Time scaled to 60 sec.

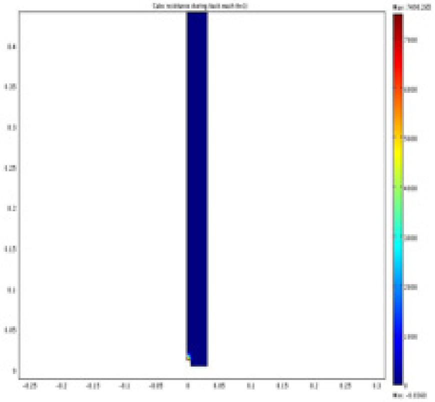


t=0



t=0.4

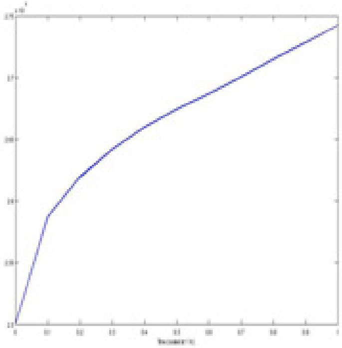
Time scaled to 60 sec.



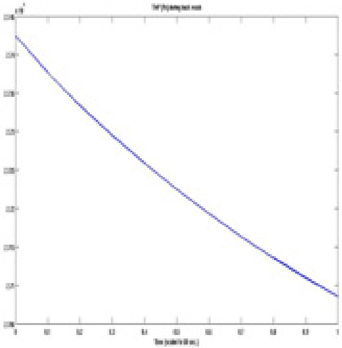
t=1

Simulation results

TMP: trans membrane pressure (average over the volumetric domain)



Production



Backwash

Simulation remarks and future work

Remarks:

(a) Typical simulation time for each cycle:

Filtration	Back wash
147.112 *sec*.	175.258 *sec*.

(b) The qualitative behaviour corresponds to what happens in reality.

Future work:

1. Calibration of unknown parameters
2. Iterate the simulation for several cycles production/back wash.

...Thank you for your attention ...