Time domain construction of acoustic scattering by elastic targets through finite element analysis

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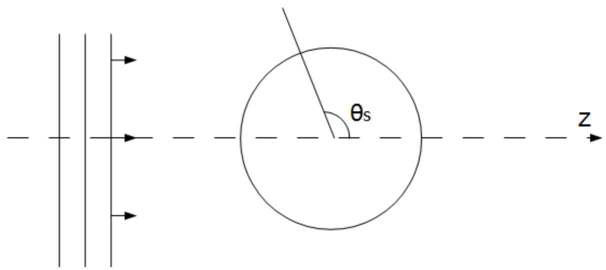
Background: Acoustic target scattering problems

- Applicable to problem of underwater target detection/classification

- Very few known analytic solutions
  - Sphere, infinite cylinder…

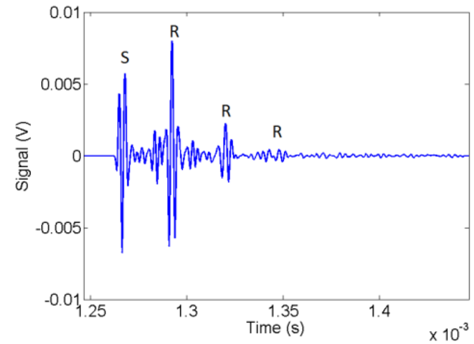


- Modeling techniques include
  - FEM, Ray theory, Kirchhoff approx., propagation models...

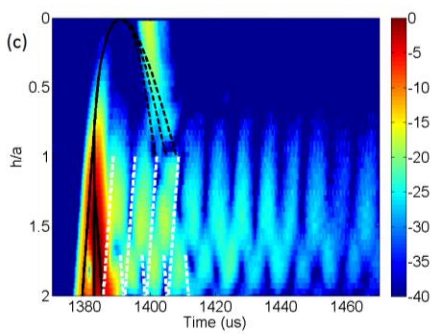
- Most data-model comparisons appear in the frequency (F.D.) rather than time domain (T.D.)

Advantages of a time domain model

- Experimental data naturally obtained in time domain
  - In situ data/model comparison

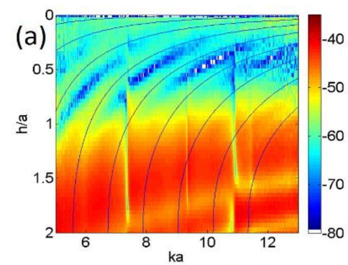


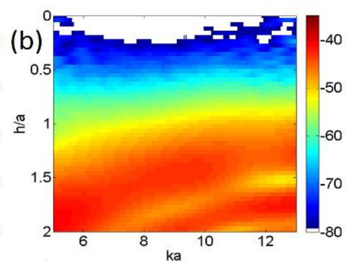
- T.D. useful for separation/isolation of distinct arrivals which often interfere in F.D.



Time domain

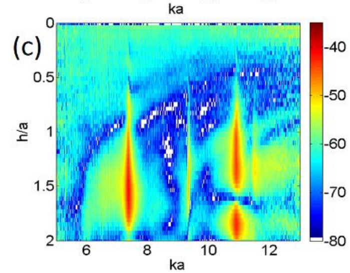
Full

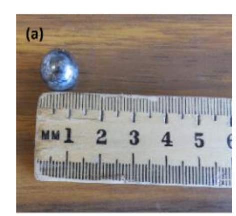




Early

Late



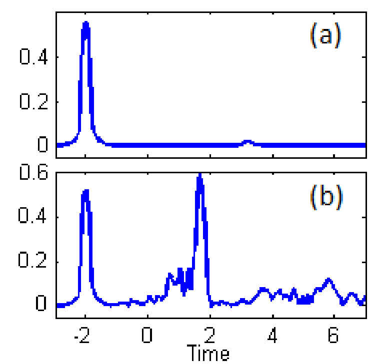


[Gunderson and Marston, JASA **140**, 3582-3592 (2016)]

Finite element models (COMSOL)

- High fidelity, wide applicability
- COMSOL capable of solving full 3D model for any target in any environment
- Fourier synthesis can be used to transform result to time domain

  Fine frequency sampling → Long time window
  Wide frequency range → Better time resolution
- Problem: Elastic ringing



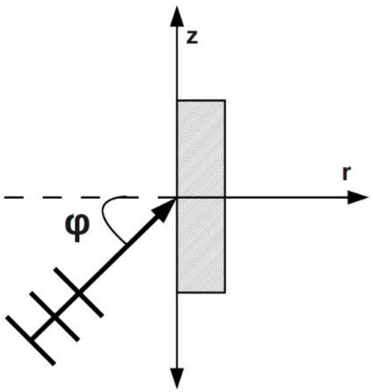
Rigid sphere

Elastic (Cu) sphere

[Exact solution: Marston, JASA **122**, 247-252 (2007)]

Axial wavenumber decomposition (AWD)

- Uses 2D geometry for 3D axisymmetric targets



1) Decompose incident field into Fourier components

$$p_{inc}(r,\theta,z) = \sum_{m=-\infty}^{\infty} [p_m(r,z)\exp(-im\theta)]$$

2) The m'th component is given by

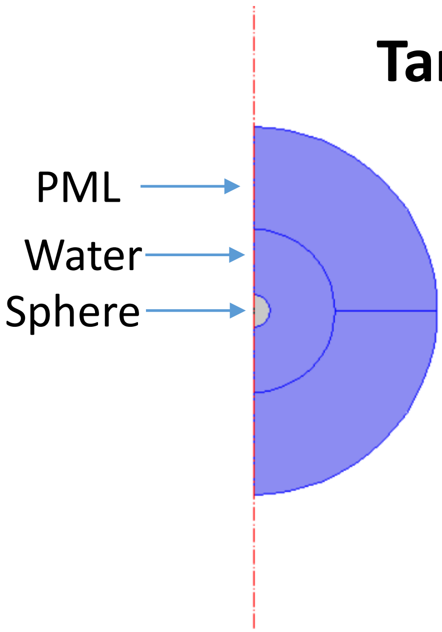
$$p_m(r,z) = (i^m)\exp(-ikz\sin\phi)J_m(-kr\cos\phi)$$

3) Solve a series of 2D COMSOL problems, each with a different incident component $p_m(r,z)$.

4) Compose full 3D scattered field, analogous with 1)

[Zampolli et. al., JASA **122**, 1472-1485 (2007)]
[Bonomo and Isakson, Proc. COMSOL Conf., Boston (2016)]

COMSOL: Problem setup



Target: User-input weak-form PDE interface

- COMSOL's built-in elastic domain interface inconsistent with AWD

$$\int_{\Omega_s}(-\omega^2\rho_s u_i\delta u_i+\sigma_{ij}\delta\epsilon_{ij})d\Omega-\int_{\partial\Omega_s}t_i\delta u_i dS=0$$

[Zampolli et. al., JASA **122**, 1472-1485 (2007)]

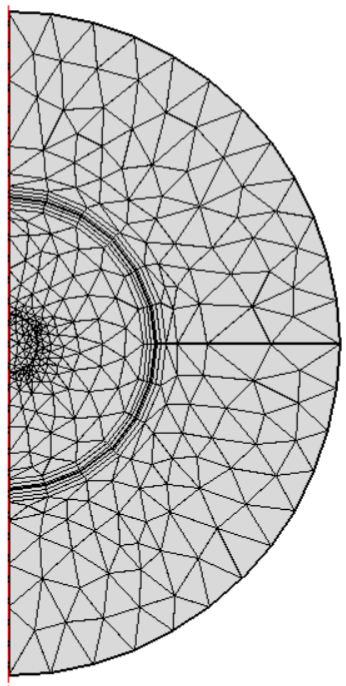
Water: Fluid: Pressure Acoustics, Freq. Domain (Acoustics mod.)

- Incident plane wave constructed through AWD, applied as a background pressure field

Perfectly Matched Layer (PML): COMSOL built-in feature

- Enforces Sommerfeld radiation condition
- Absorbs outgoing energy at near-normal incidence
- Thickness of at least one acoustic wavelength in water, λ_f

Mesh and far-field calculation



Mesh: Free triangular

- Maximum element size: $\lambda_f/6$
- Thin boundary layers at interfaces, $\sim\lambda_f/60$

Far-field calculation: COMSOL built-in tool

- Evaluated using solution at PML inner boundary

Far-field results: Exported to MATLAB through LiveLink script

Near-field results: Solution interpolated onto a pre-designed grid in MATLAB using LiveLink's “mphinterp” command

Loop structure and Fourier synthesis

Inner loop: Axial wavenumber decomposition (COMSOL)

- Loop over m from {-m_{max},m_{max}}, $m_{max} = 1.6k(\lambda_f + a)|\cos\phi|$ *
- Export results to MATLAB, interpolate (for near-field), sum over m

Outer loop: Frequency step (MATLAB/Livelink)

- Frequency range, step size chosen to balance computational time demand against needs for time domain duration, step size
- Typical frequency range was $ka = 1 - 35$

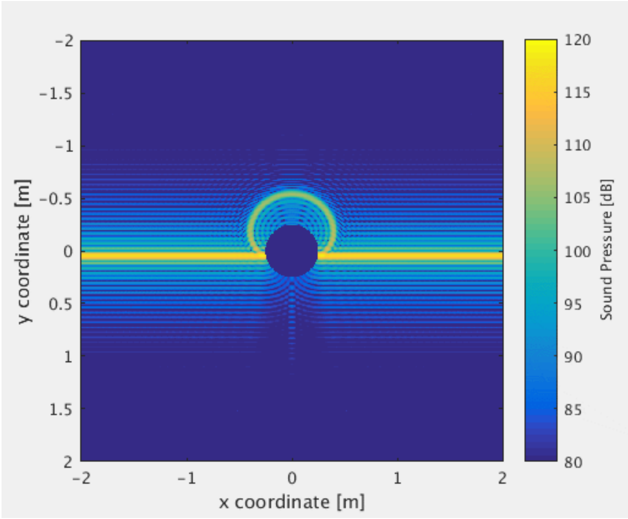
Final step: Fourier synthesis (MATLAB/Livelink)

- MATLAB's inverse fast Fourier transform "ifft" command
- Time window determined by the relations:

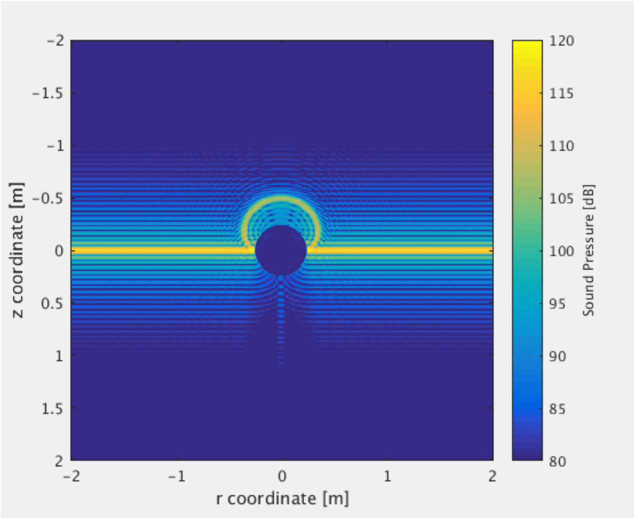
$$dt = 1/f_{max} \quad , \quad t_{max} = (N-1)dt$$

*[Ihlenburg, Finite Element Analysis of Acoustic Scattering, Springer-Verlag (1998)]

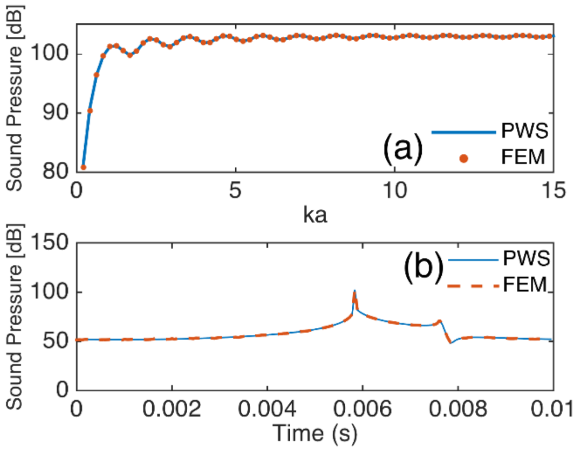
Rigid sphere near field solution



PWS (Exact)



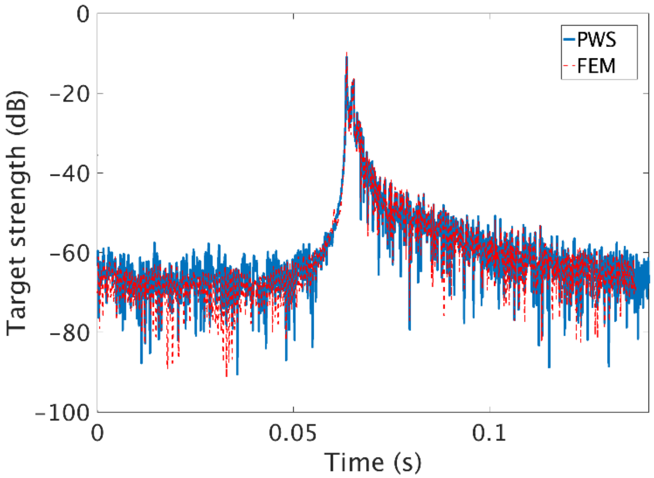
FEM



Backscattering at r = 1.5 m

Partial Wave Series (PWS):
[Morse and Ingard, *Theoretical Acoustics*, 418-419 (1968)]

Aluminum sphere far field solution

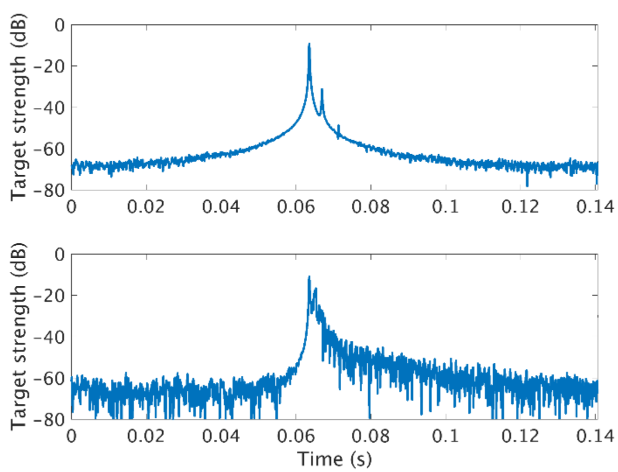


Backscattering

Partial wave series (PWS):
[Marston, JASA **122**, 247-252 (2007)]

Target strength:

$$TS = 20\log_{10}[r|p_{scat}(r,\theta,z)|/(r_0|p_{inc}(r,\theta,z)|)]$$

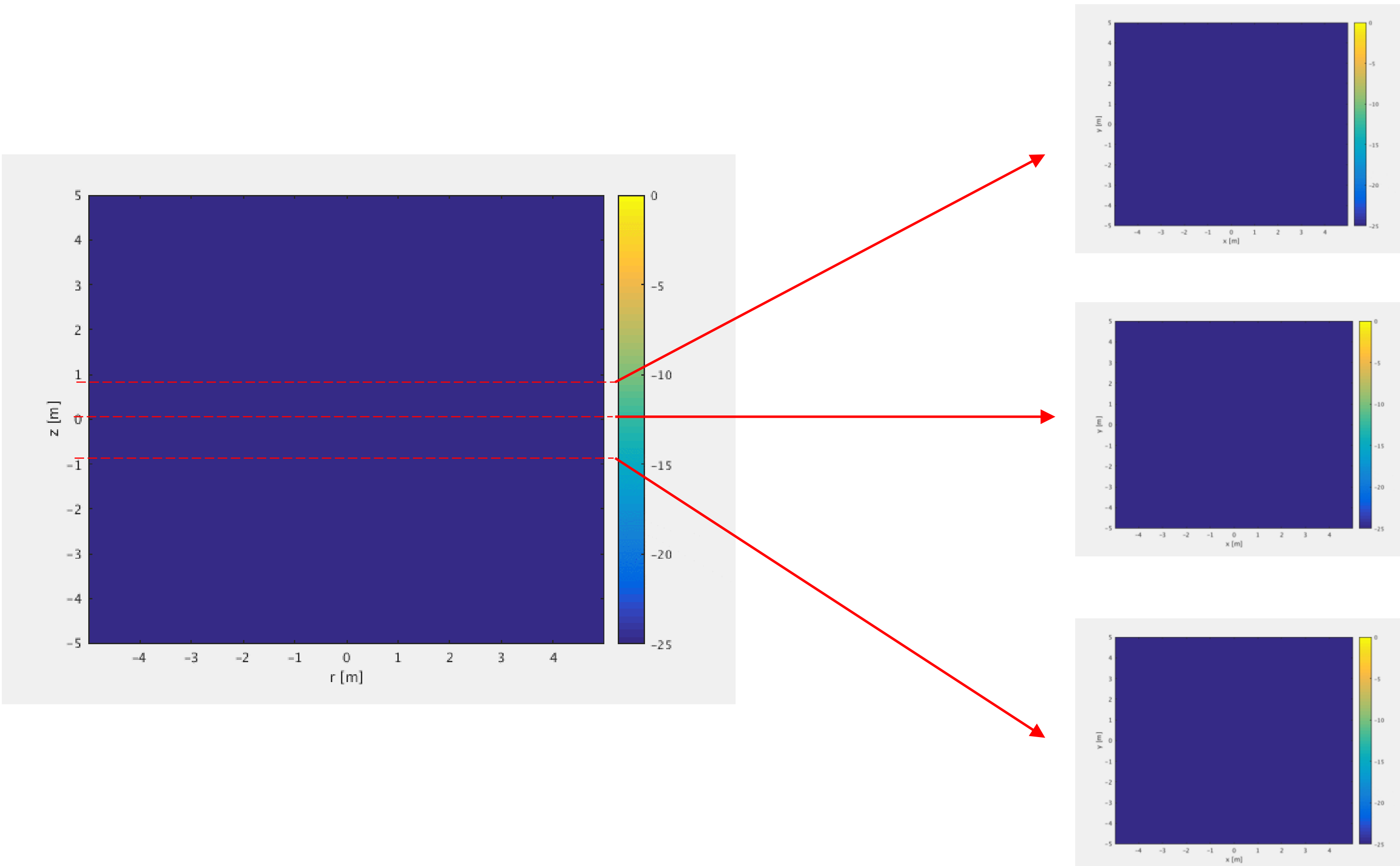


Rigid

Aluminum

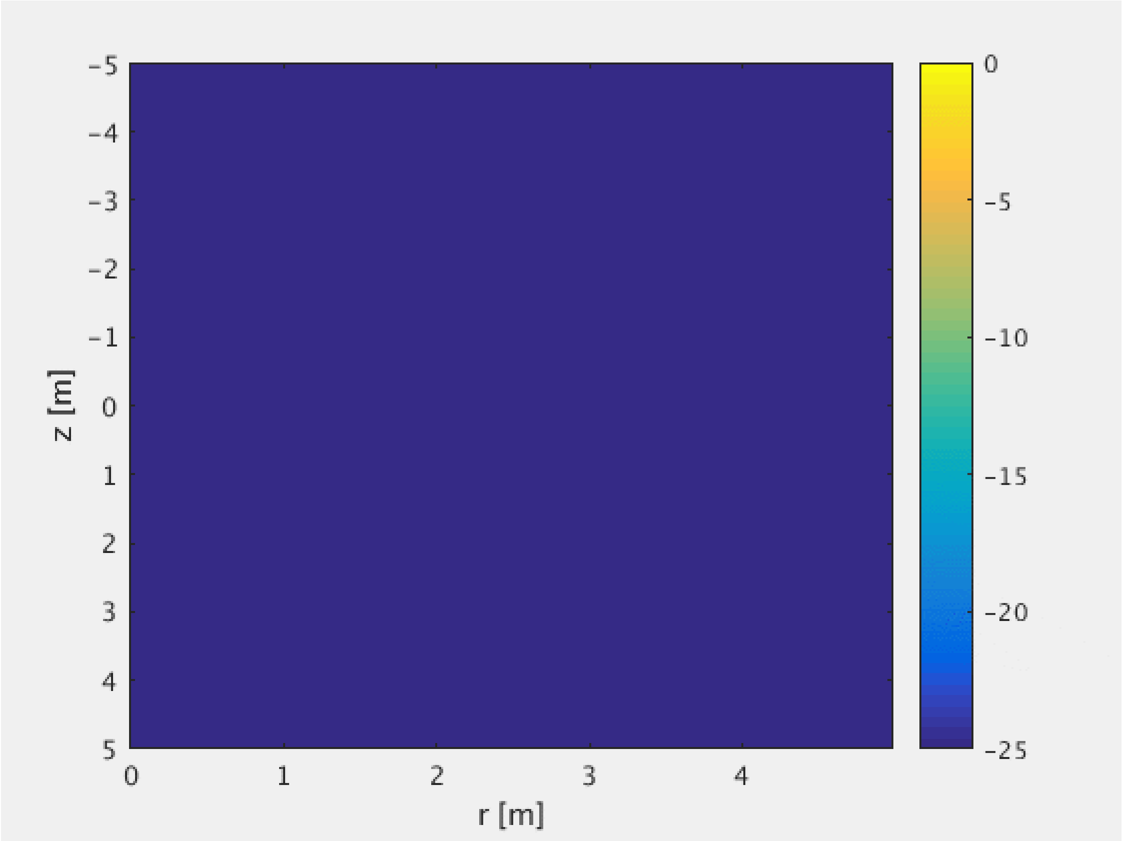
Backscattering

Aluminum sphere near field solution

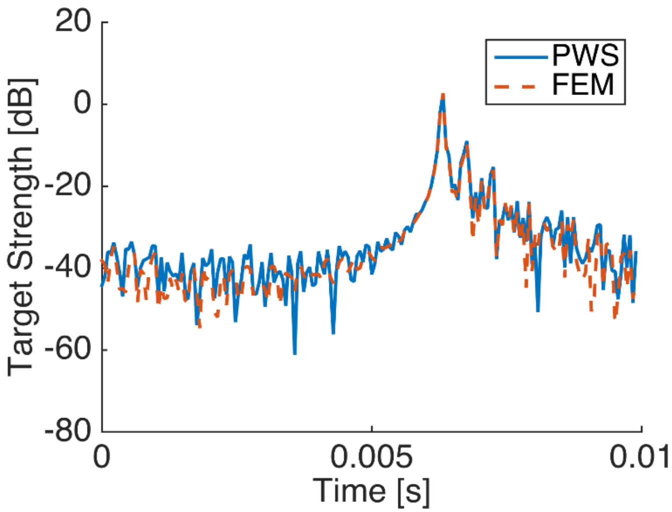


Horizontal slices

Aluminum sphere off-axis incidence



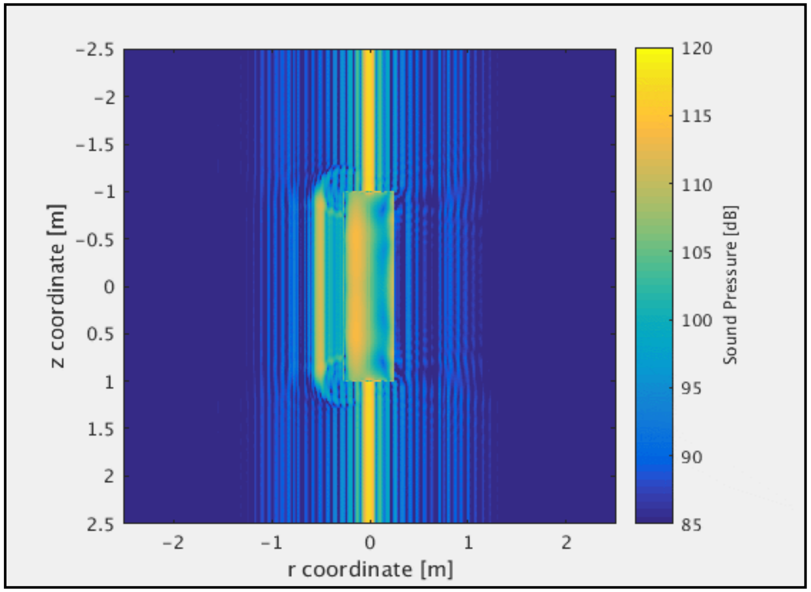
4:1 steel cylinder far field solution



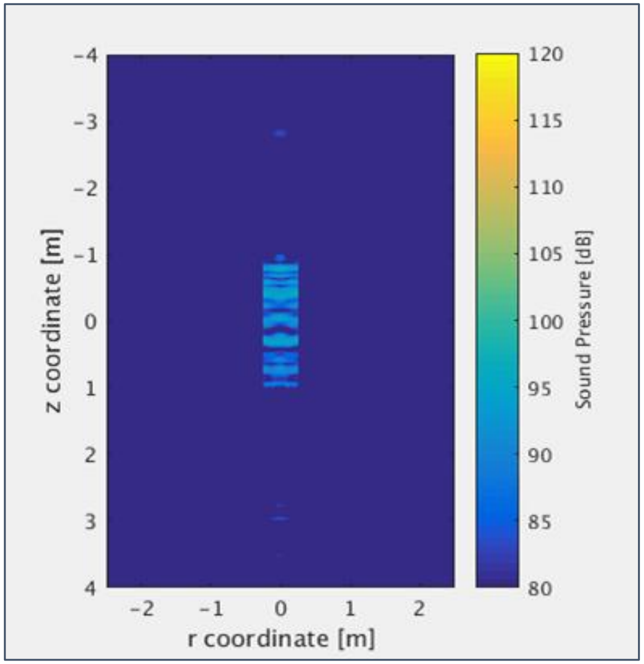
Backscattering
Broadside incidence

Partial wave series (PWS, approximate):
[Stanton, JASA **83**, 64-67 (1988)]

4:1 steel cylinder near field solution



Broadside incidence



End-on incidence

Conclusions

- Time domain models obtainable through Fourier synthesis of FEM results
- FEM results widely applicable and highly exact
  - Useful for data/model comparisons
- FEM can provide the closest thing to an exact solution

Future Work

- Introduce boundaries (seafloor)
- Apply to more complicated targets and environments → Full 3D