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Advanced Computational & Engineering Services

Computational Modeling of Wave Propagation in a Geophysical Domain

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Overview

- Objective:
  Demonstrate the capability of COMSOL Multiphysics to accurately solve wave propagation problems in geophysics
- Motivation:
  Reduce reliance on custom software and supercomputers by obtaining solution using commercially available software on high-end desktop computer
- Approach:
  - Develop closed-form solutions for
    - Point source in an infinite body
    - Point source on the surface of a semi-infinite body (Lamb's problem)
  - Develop using solid mechanics module w/ COMSOL
    - Same formulation as acoustics module
    - Three-dimensional
    - Axisymmetric
    - Plane strain
  - Comparison w/ experimental data
    - Hammer blow on surface

Closed Form Solution - Displacement

- Elastic Wave in an Infinite Body

$$4\pi\rho\ u_{ij}(\mathbf{x},t)=\frac{(3\gamma_i\gamma_j-\delta_{ij})}{r^3}\int_{r/\alpha}^{r/\beta}\tau\ f_0(t-\tau)d\tau+\frac{\gamma_i\gamma_j}{\alpha^2 r}f_0\left(t-\frac{r}{\alpha}\right)-\frac{(\gamma_i\gamma_j-\delta_{ij})}{\beta^2 r}f_0\left(t-\frac{r}{\beta}\right)$$

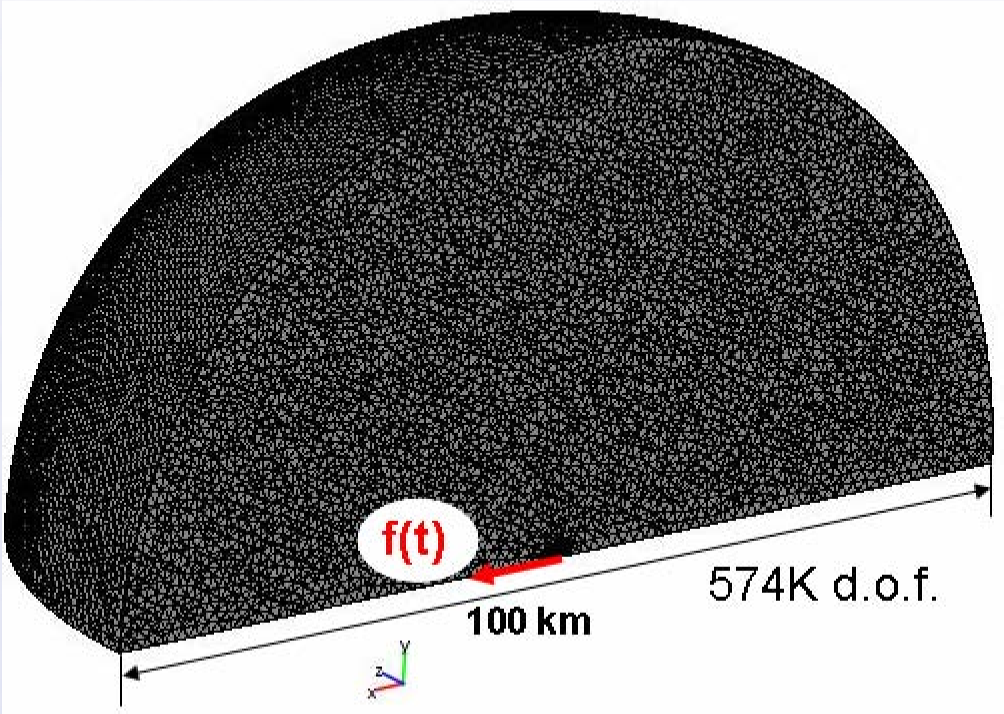
- Semi-Infinite Body (Lamb’s Problem- Fixed Poisson ratio)

$$\begin{Bmatrix} w(t) \\ u(t) \end{Bmatrix}=\frac{\sigma_{33}}{\pi^2\mu r}\left(\frac{\alpha}{\beta}\right)^2\frac{r}{\beta}\int_{-\infty}^{\tau}\left.\frac{df}{dt}\right|_{t=r\tau'/\beta}\begin{Bmatrix} G(\tau-\tau') \\ R(\tau-\tau') \end{Bmatrix}d\tau'$$

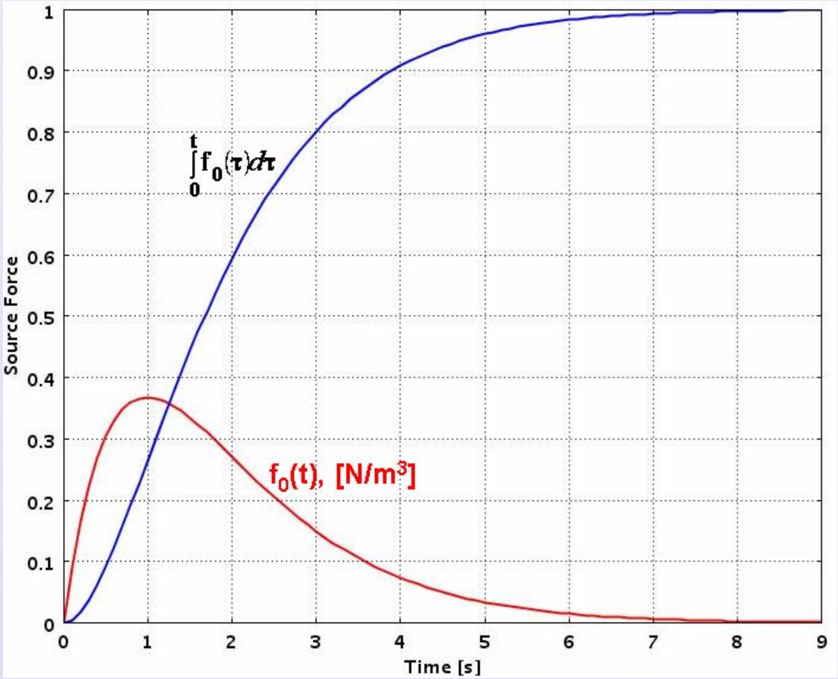
$$G(\tau)=\begin{cases} 0, & \tau<1/\delta \\ -\dfrac{\pi}{96}\left[6-\dfrac{(3\sqrt{3}+5)^{1/2}}{(\gamma^2-\tau^2)^{1/2}}+\dfrac{(3\sqrt{3}-5)^{1/2}}{(\tau^2+\sqrt{3}/4-3/4)^{1/2}}-\dfrac{\sqrt{3}}{(\tau^2-1/4)^{1/2}}\right], & 1/\delta<\tau<1 \\ -\dfrac{\pi}{48}\left[6-\dfrac{(3\sqrt{3}+5)^{1/2}}{(\gamma^2-\tau^2)^{1/2}}\right], & 1<\tau<\gamma \\ -\pi/8, & \tau>\gamma \end{cases}$$

$$R(\tau)=\begin{cases} 0, & \tau<1/\delta \\ \dfrac{\tau}{16\sqrt{6}}\left\{\begin{aligned} &6\mathrm{K}(k)-18\Pi(8k^2,k)+(6-4\sqrt{3})\Pi[(20-12\sqrt{3})k^2,k] \\ &\quad+(6+4\sqrt{3})\Pi[(20+12\sqrt{3})k^2,k] \end{aligned}\right\}, & 1/\delta<\tau<1 \\ \dfrac{\tau/k}{16\sqrt{6}}\left\{\begin{aligned} &6\mathrm{K}\left(\frac{1}{k}\right)-18\Pi\left(8,\frac{1}{k}\right)+(6-4\sqrt{3})\Pi\left[(20-12\sqrt{3}),\frac{1}{k}\right] \\ &\quad+(6+4\sqrt{3})\Pi\left[(20+12\sqrt{3}),\frac{1}{k}\right] \end{aligned}\right\}, & 1<\tau<\gamma \\ \text{Preceding}+\dfrac{\pi\tau}{24}(\tau^2-\gamma^2)^{-1/2}, & \tau>\gamma \end{cases}$$

Point Force Solution – 3D



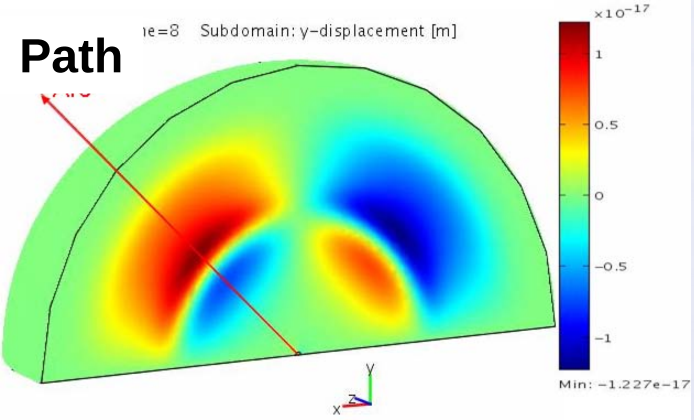
Long Duration Loading

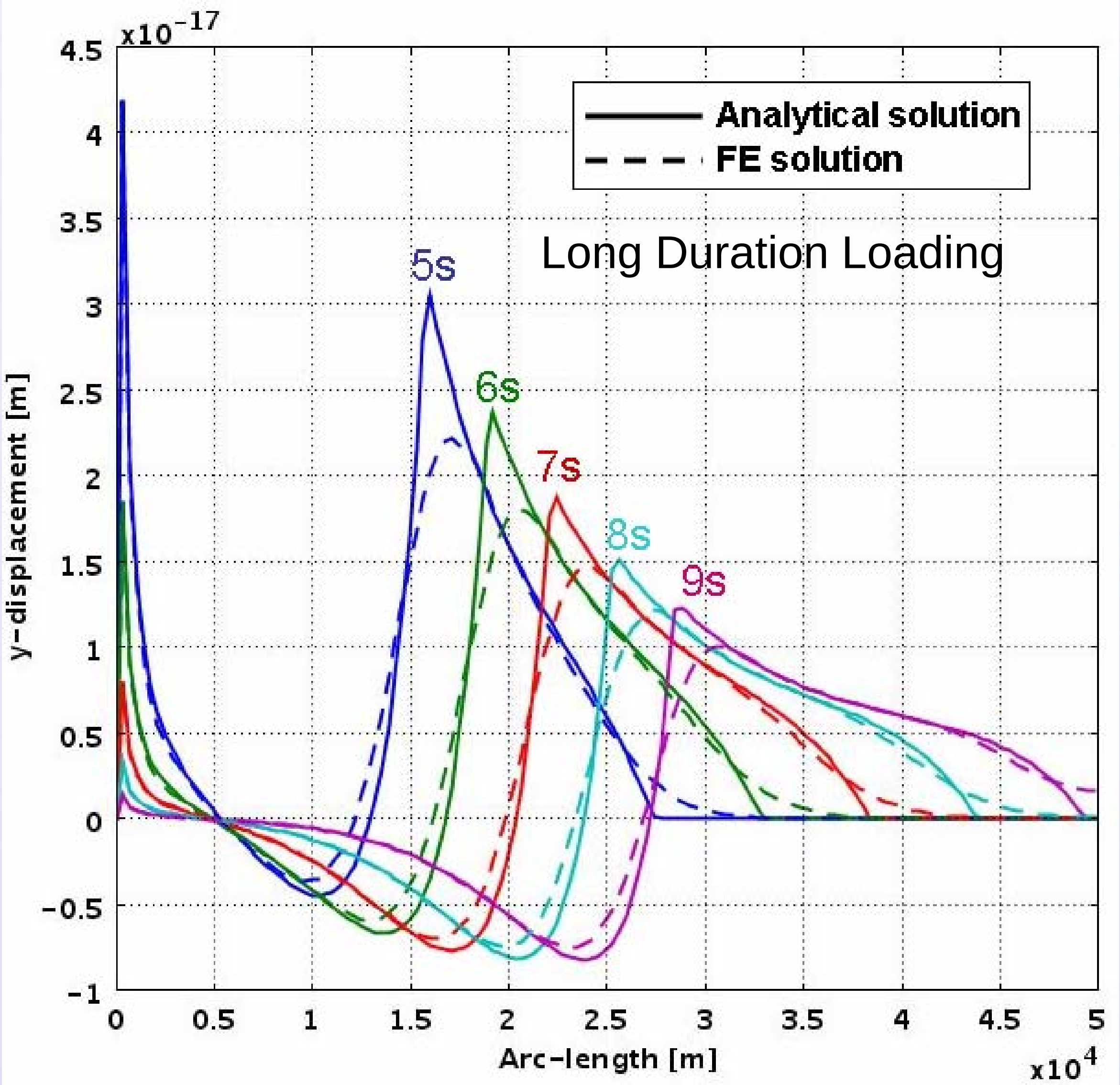


Quarter Symmetric Model

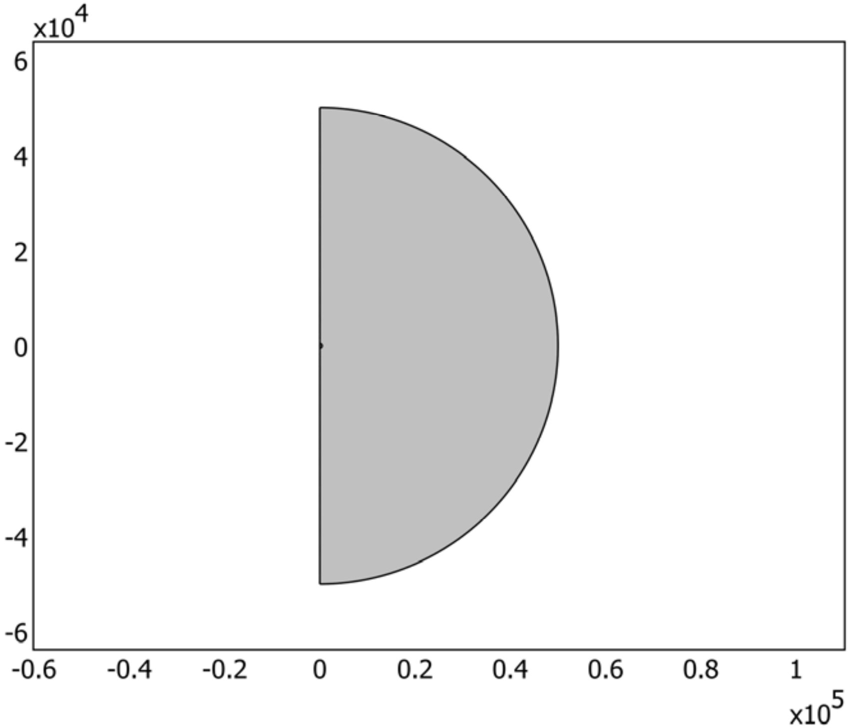
N_{DOF} = 574,000

Comparison to Analytical Solution – 3D



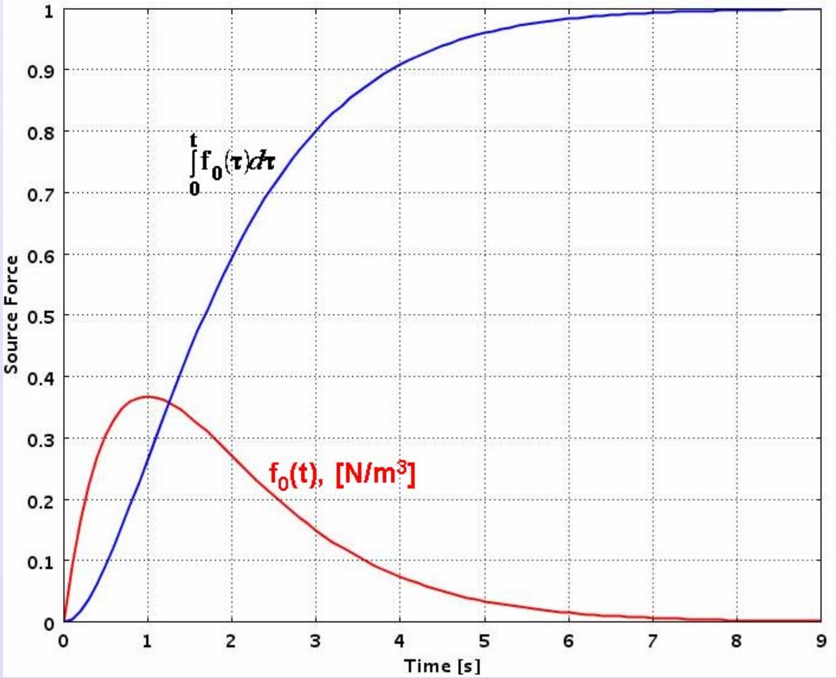


Point Force Solution – 2D



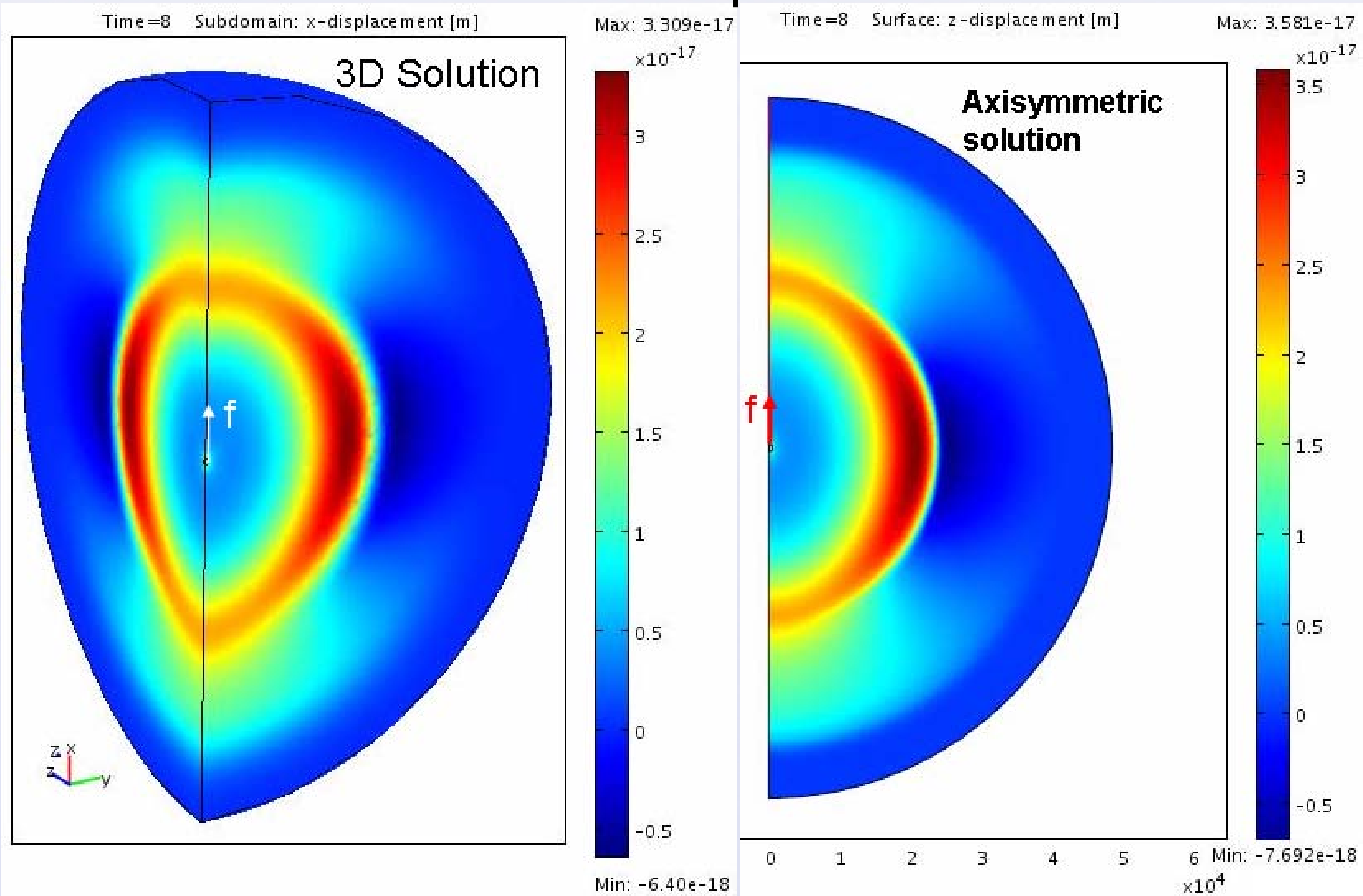
2D Model
N_{DOF} = 195,000

- Axisymmetric
- Plane Strain

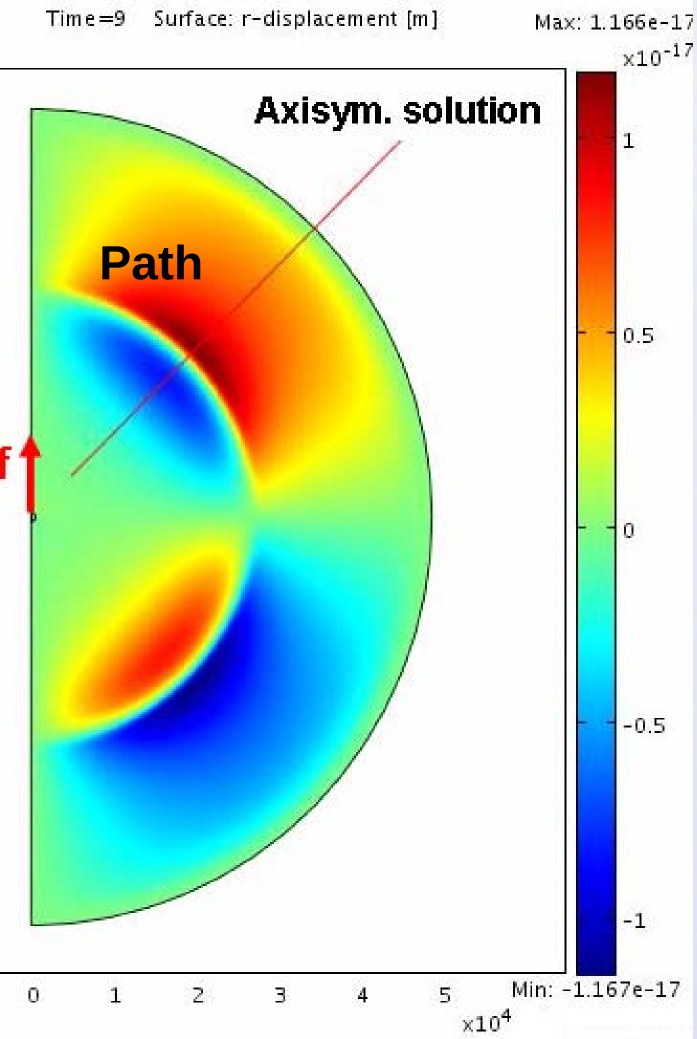


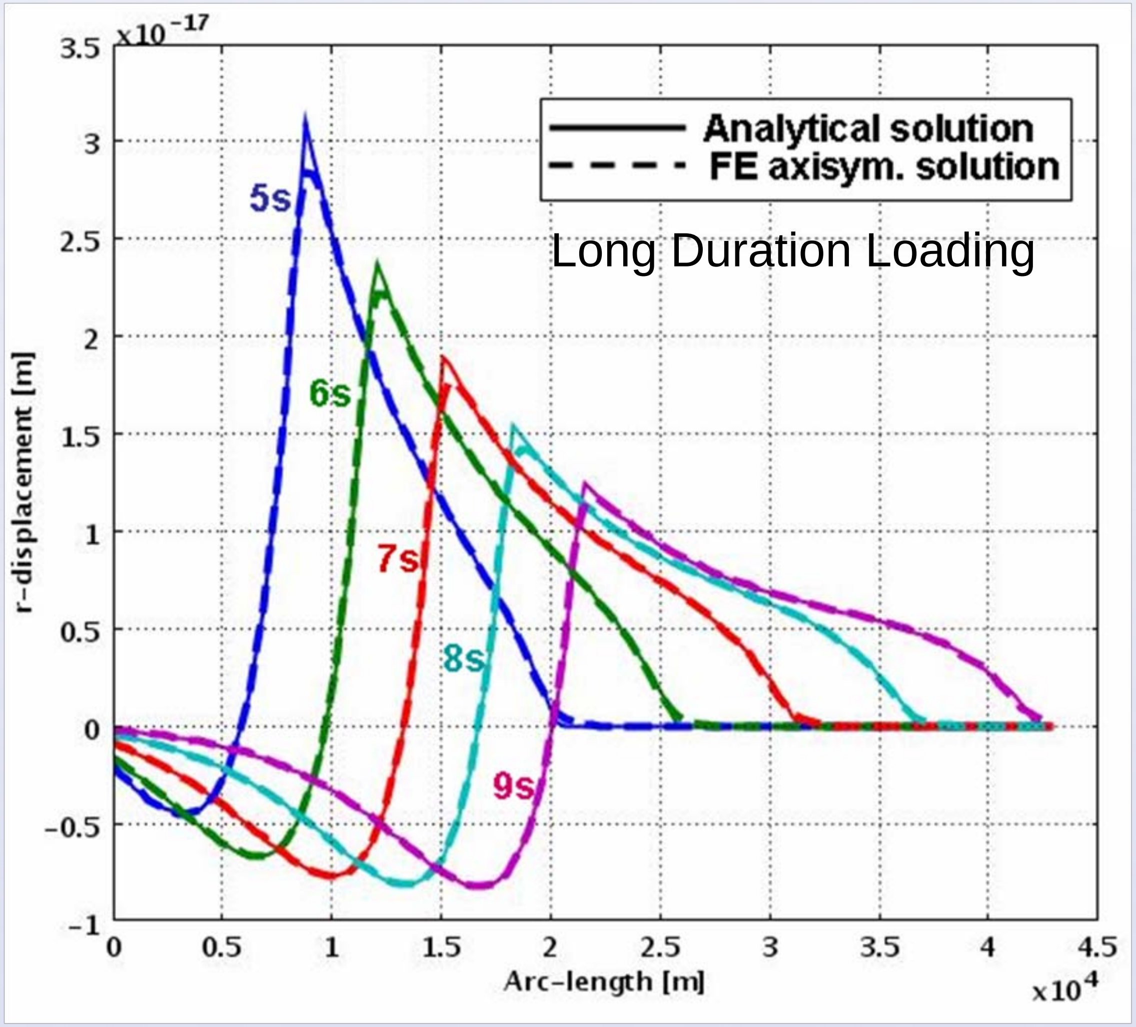
Comparison of 3D and Axisymmetric

Vertical Displacement

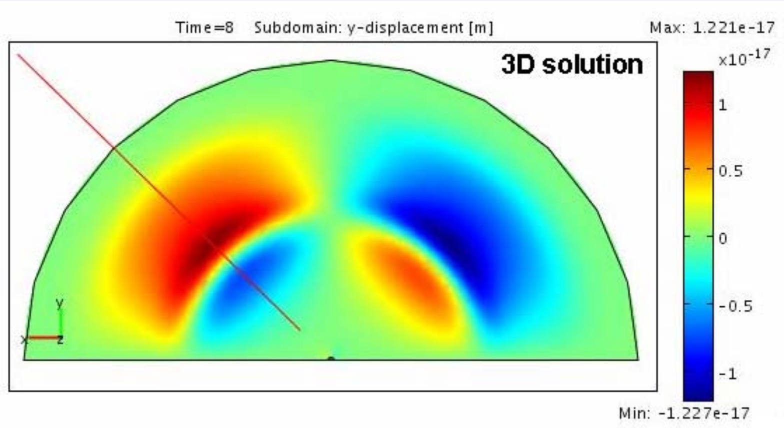


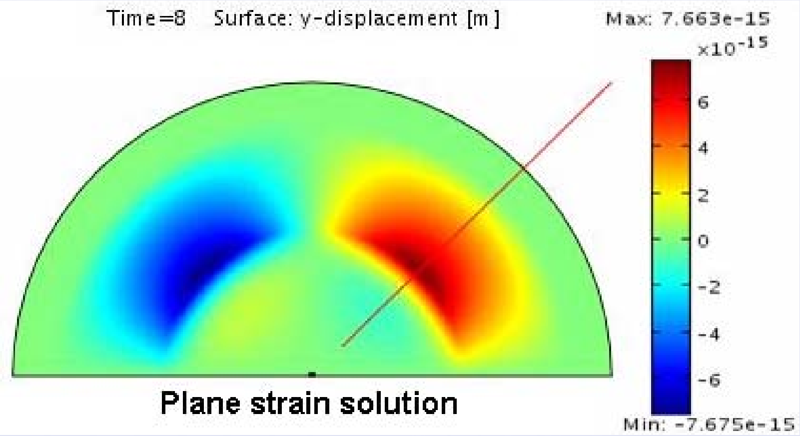
Comparison to Analytical Solution - Axisymmetric

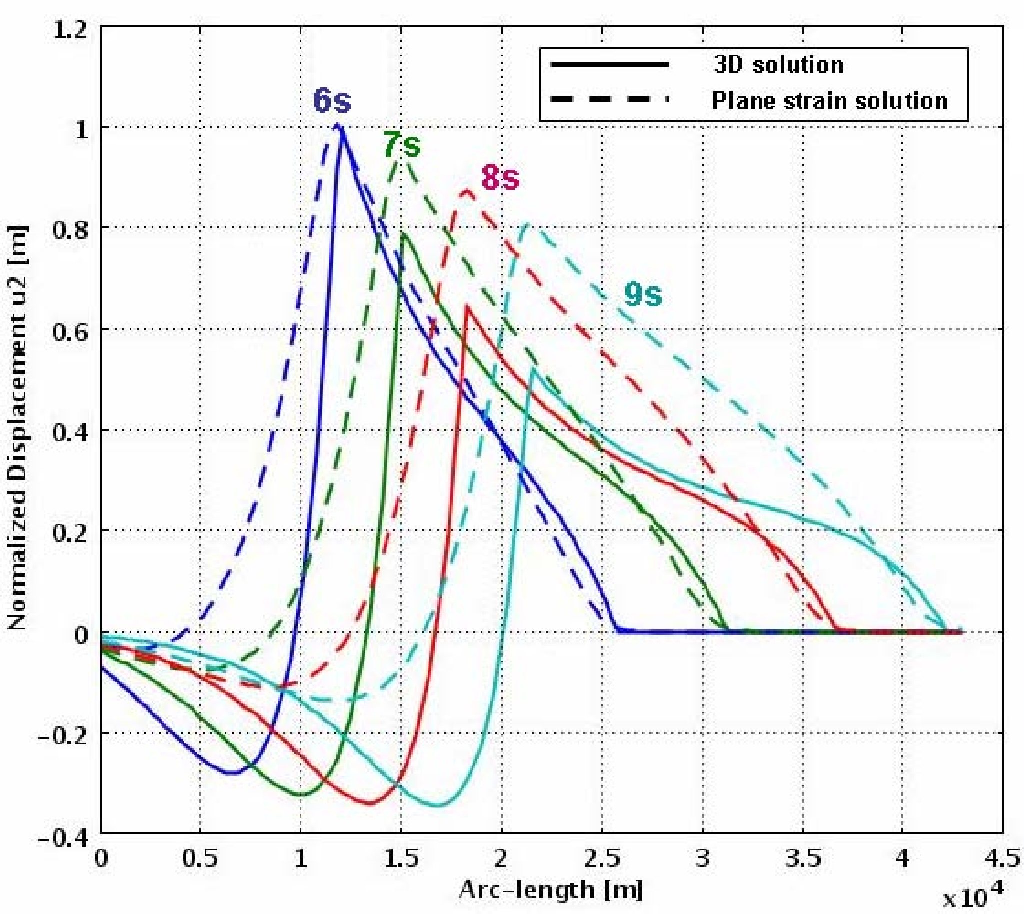




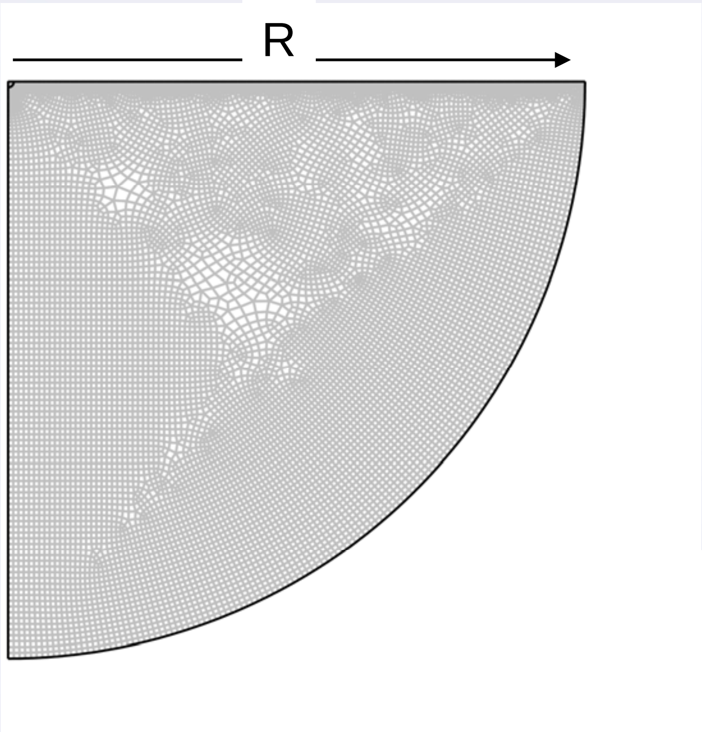
Comparison of 3D and Plane Strain





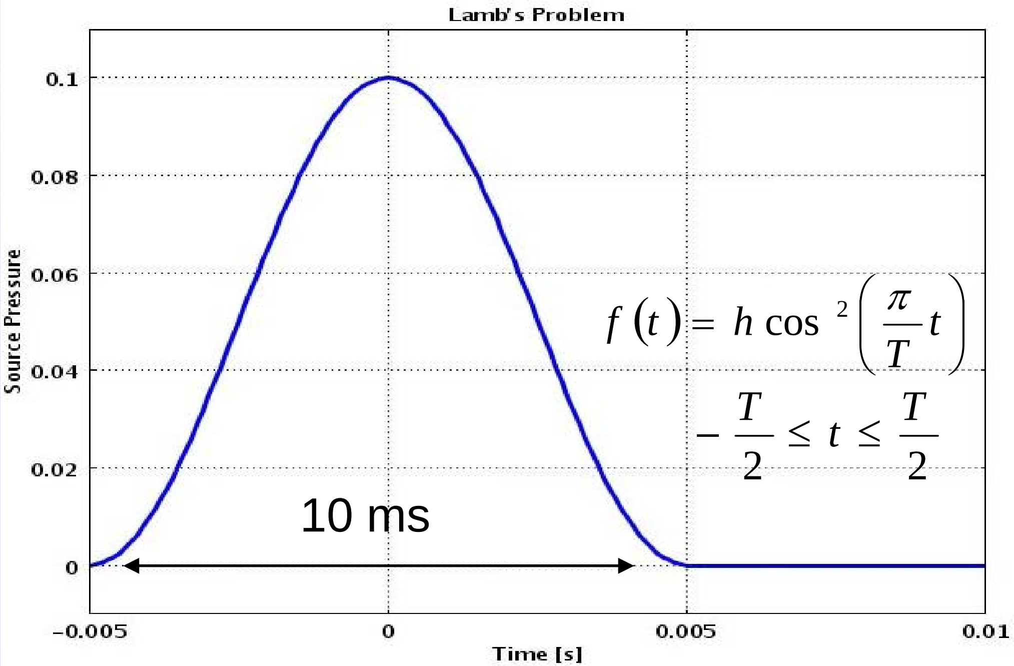


Surface Wave Problem

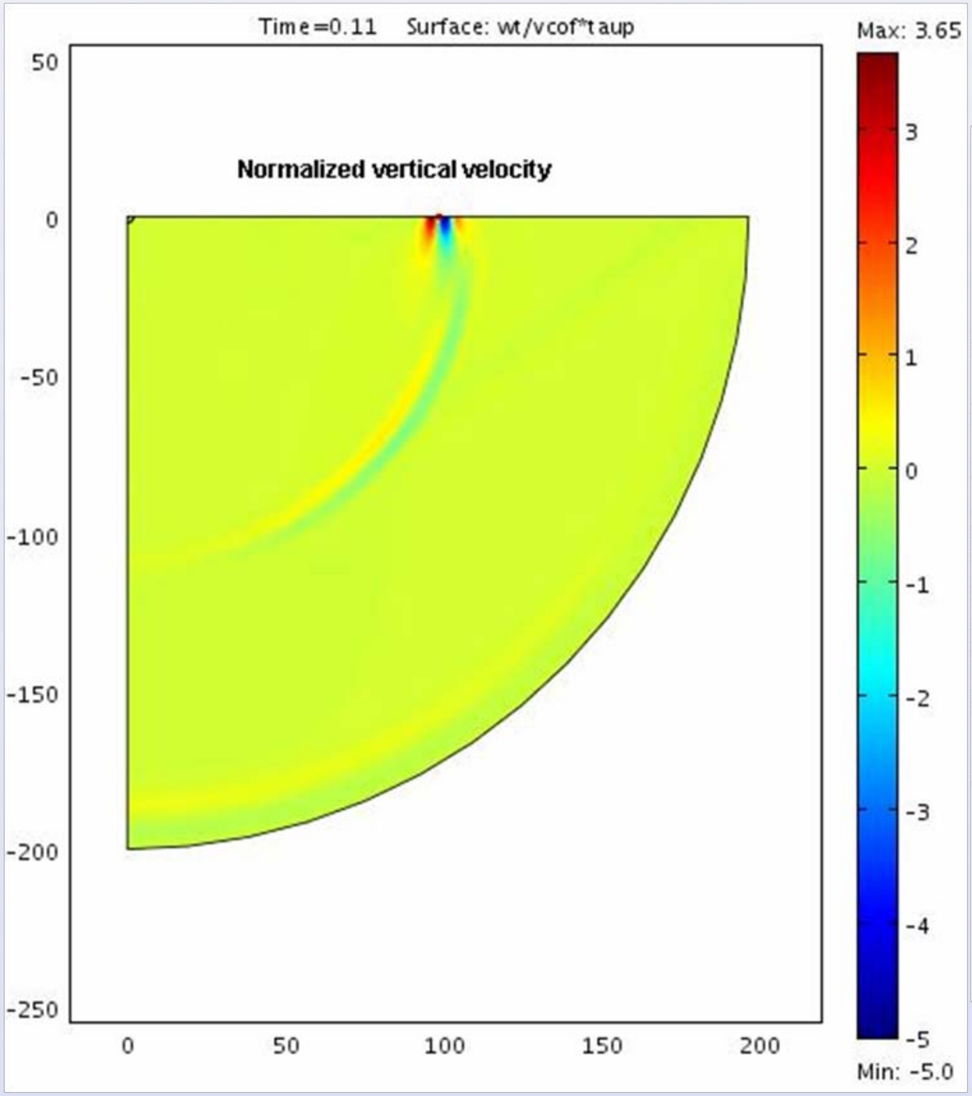


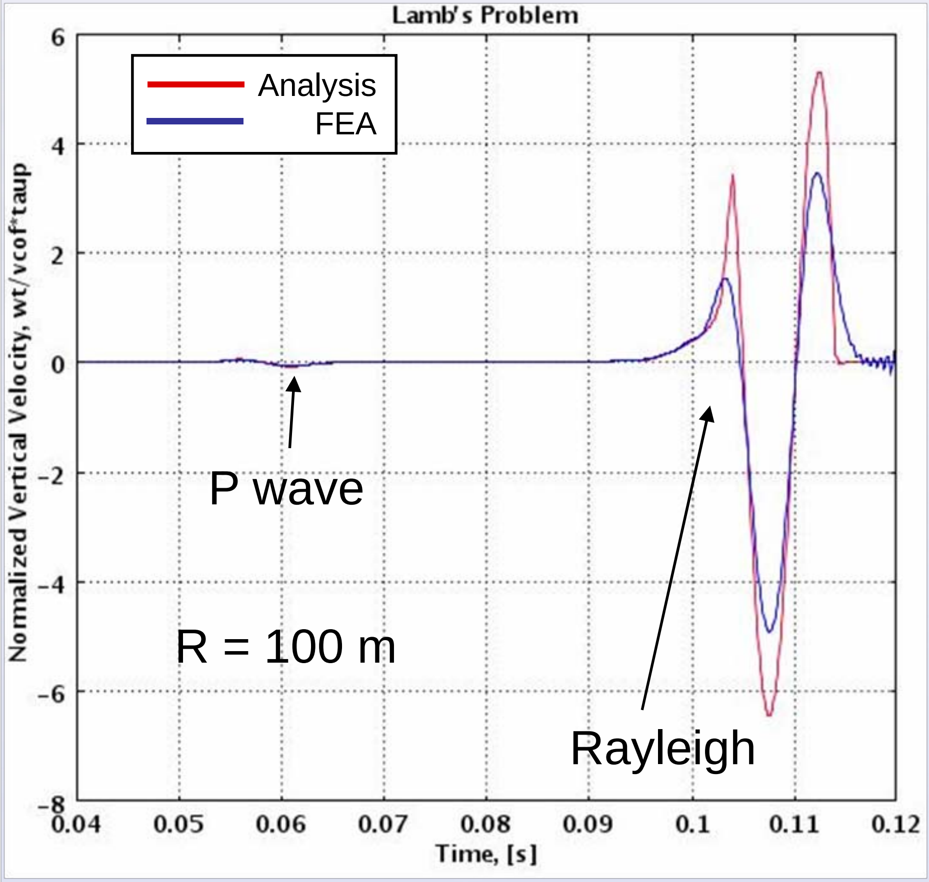
2D Model
R = 200 m
N_{DOF} = 23,000

Short Duration Loading

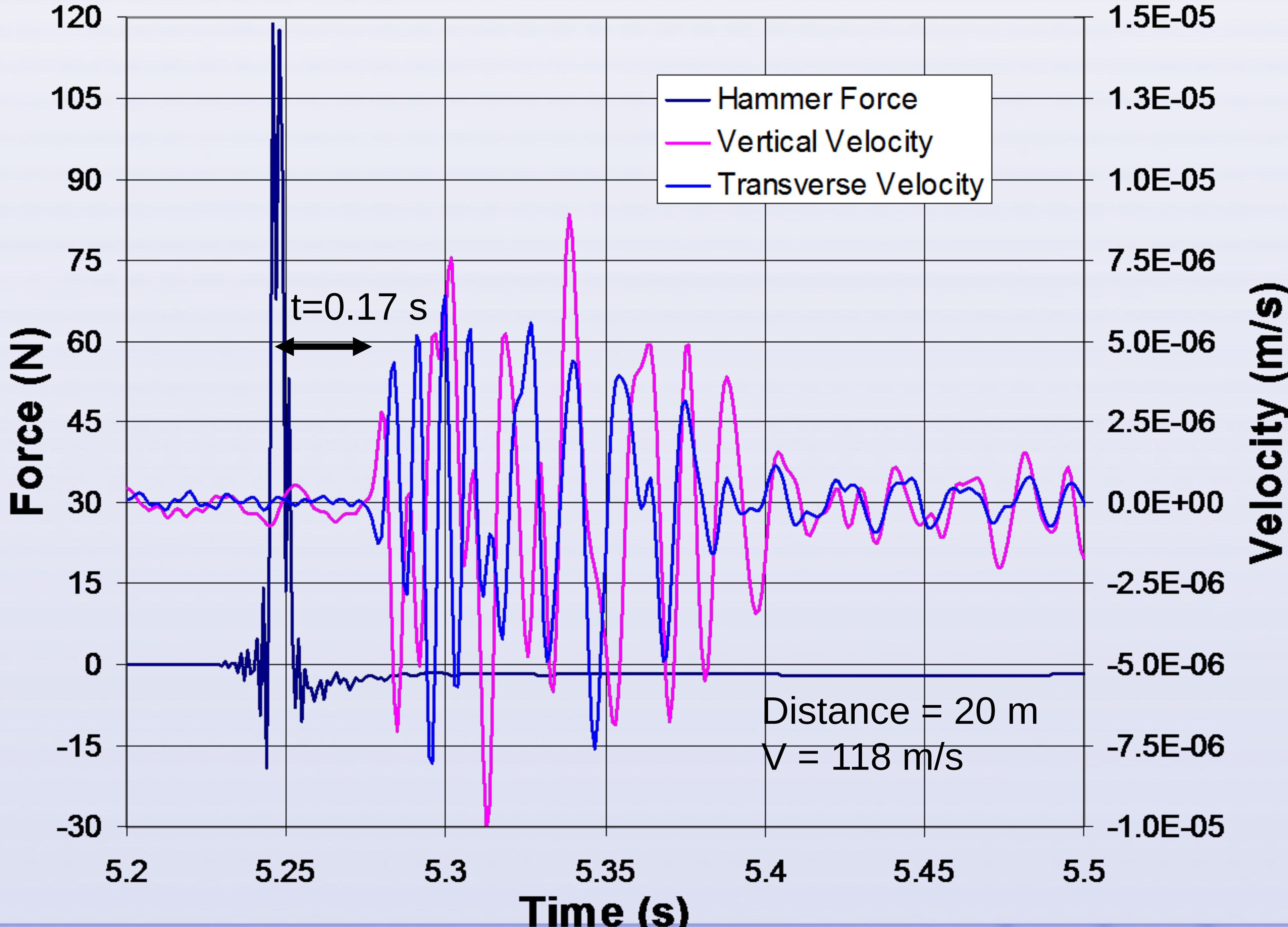


Surface Wave Problem – Vertical Velocity



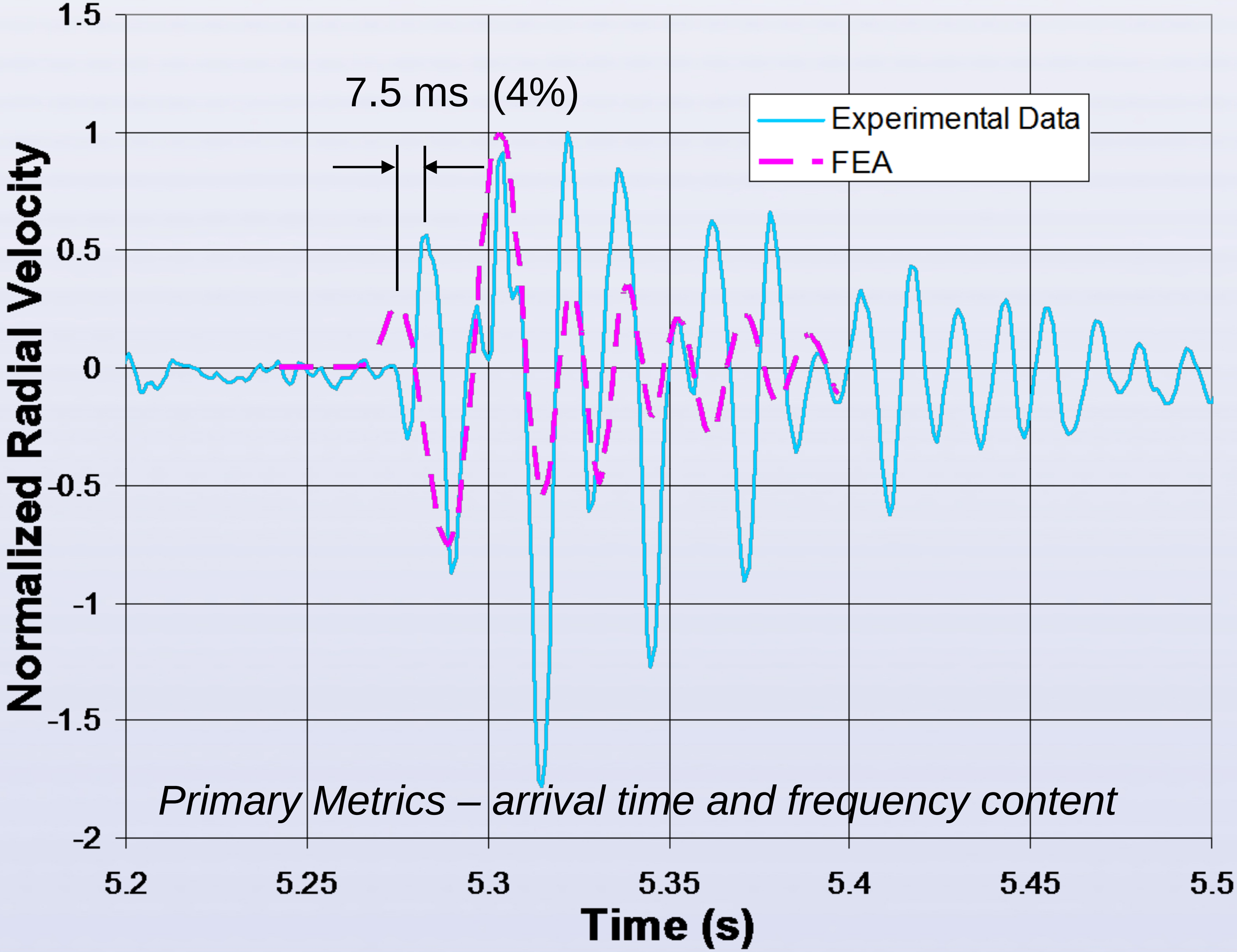


Hammer Data



120
105
90
75
60
45
30
15
0
-15
-30
Force (N)
1.5E-05
1.3E-05
1.0E-05
7.5E-06
5.0E-06
2.5E-06
0.0E+00
-2.5E-06
-5.0E-06
-7.5E-06
-1.0E-05
Velocity (m/s)
5.2
5.25
5.3
5.35
5.4
5.45
5.5
Time (s)
Hammer Force
Vertical Velocity
Transverse Velocity
t=0.17 s
Distance = 20 m
V = 118 m/s

Comparison w/ Exp Data



Summary

- Closed-form solution developed for
  - Elastic wave in infinite media
  - Elastic wave in semi-infinite media
- Computation models developed using Solid Mechanics Module
  - Three-dimensional
  - Two-dimensional
    - Axisymmetric
    - Plane Strain – not sufficiently accurate for point source
- Comparison with analytical solutions and experimental data
  - Agreement with arrival time, and frequency content

Conclusions

- COMSOL Multiphysics provides a sufficient level of accuracy for the problems of interest
- COMSOL Multiphysics provides a commercially available tool that can solve wave propagation problems on desktop computing resources