Computational Simulation of Blood Flow in Stenosed Arterial Bifurcation under Body Acceleration

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INTRODUCTION

- **Stenosis**
  - Abnormal narrowing or occlusion in the blood vessels caused by deposition of plaques of fatty material on their inner walls.
  - Reduces the size of lumen

- **Atherosclerosis**
  - An arterial disease characterized by progressive abnormal narrowing of the artery.

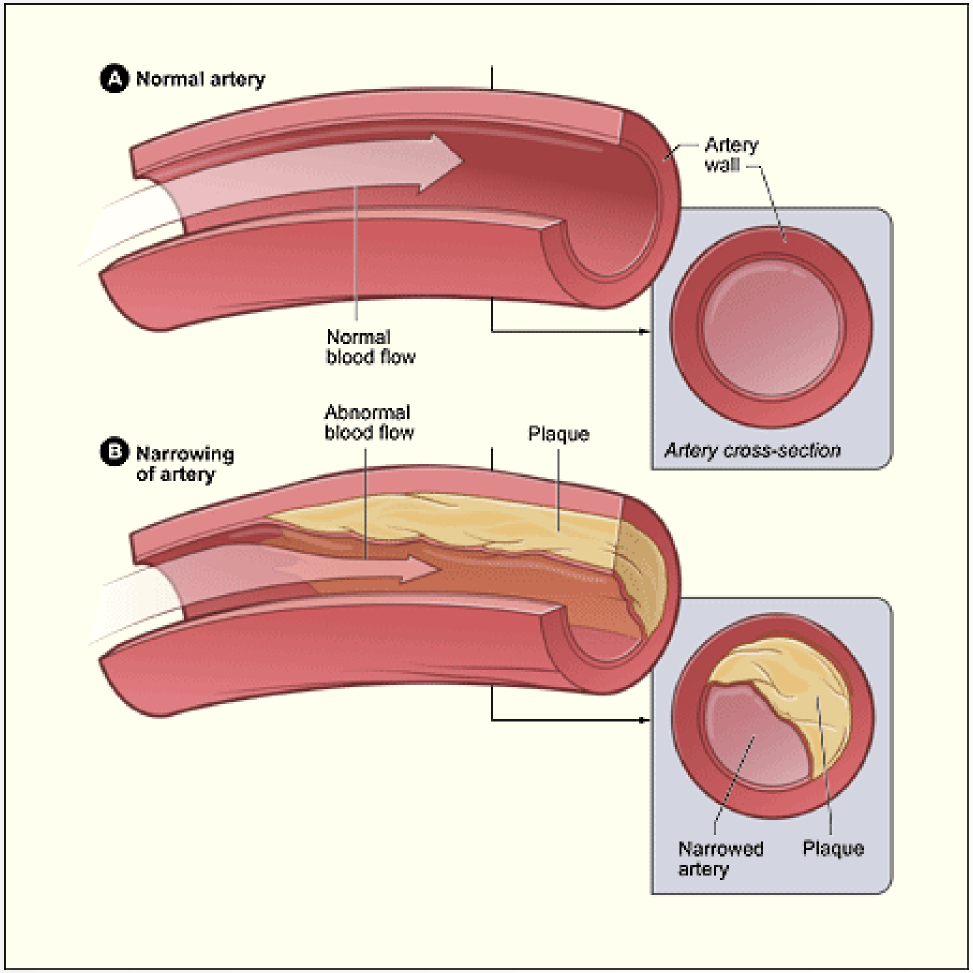


Fig. 1: (A) A normal artery with normal blood flow. The inset image shows a cross-section of a normal artery. (B) An artery with plaque buildup. The inset image shows a cross-section of an artery with plaque buildup. [1]

INTRODUCTION

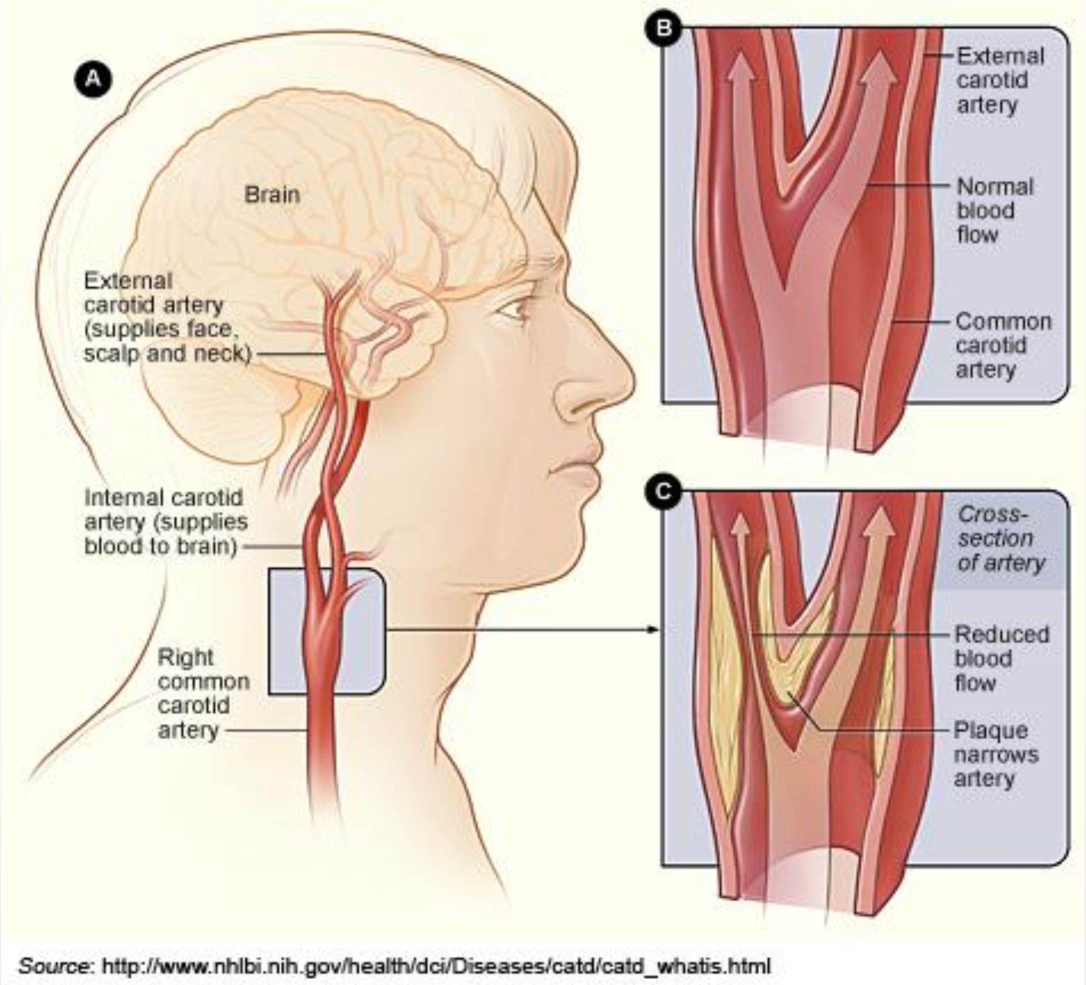


Fig. 2: (A) Location of the right carotid artery in the head and neck. (B) A normal carotid artery that has normal blood flow. (C) A carotid artery that has plaque buildup and reduced blood flow. [2]

MATHEMATICAL MODEL

- Unsteady, two-dimensional, laminar, axisymmetric flow
- Cartesian coordinates
- Incompressible, Newtonian fluid
- Rigid wall, asymmetric stenosis in the mother artery.

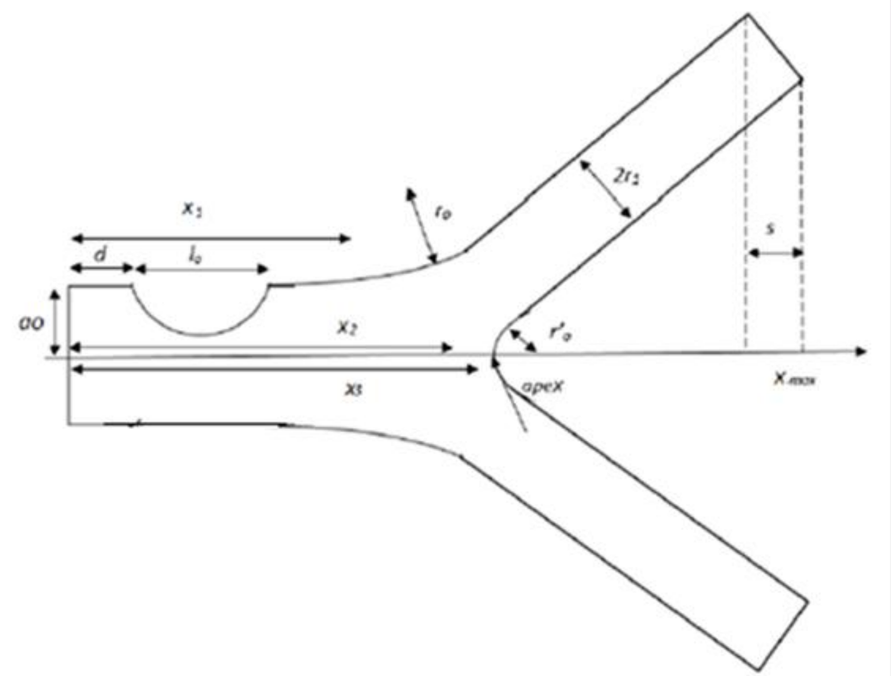


Fig. 3: Geometry of arterial bifurcation model

MATHEMATICAL MODEL: GEOMETRY

The geometry can be expressed as [3]:

- **Outer wall**

$$R_1(x) = \begin{cases} a_o, & 0 \le x \le d \text{ and } d + l_0 \le x \le x_1, \\ a_o - \frac{4\tau_m}{l_0^2}\{l_0(x-d) - (x-d)^2\}, & d \le x \le d + l_0, \\ a_o + r_0 - \sqrt{{r_0}^2 - (x - x_1)^2}, & x_1 \le x \le x_2, \\ 2r_1 \sec\beta + (x - x_2)\tan\beta, & x_2 \le x \le x_{max} - s, \end{cases}$$

- **Inner wall**

$$R_2(x) = \begin{cases} 0, & 0 \le x \le x_3, \\ \sqrt{r_0'^2 - (x - (x_3 + r_0'))^2}, & x_3 \le x \le x_4, \\ r_0' \cos\beta + (x - x_4)\tan\beta, & x_4 \le x \le x_{max}, \end{cases}$$

MATHEMATICAL MODEL: GOVERNING EQUATIONS

- **Continuity equation**

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

- **Momentum equations**

$$\rho\left[\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right] = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + \rho F(t)$$

$$\rho\left[\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right] = -\frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)$$

where

$$F(t) = A_o \cos(\omega_b t + \phi)$$

A_o is the amplitude,

$\omega_b = 2\pi f$ is the angular velocity, f is the frequency,

ϕ is the lead angle of $F(t)$ with respect to heart action.

MATHEMATICAL MODEL: BOUNDARY & INITIAL CONDITIONS

- Inlet: time-dependent pressure based on pressure gradient [3]

$$-\frac{\partial p}{\partial x} = A_0 + A_1 \cos(\omega t)$$

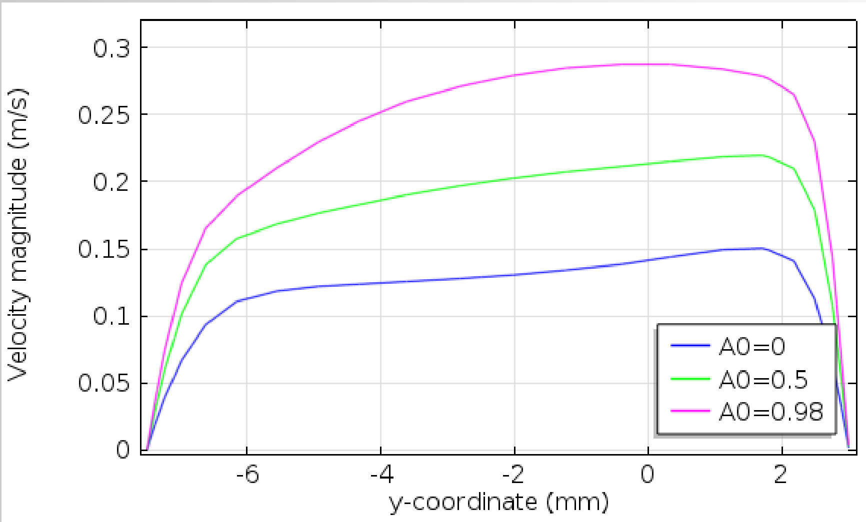
- Outlet: Traction-free condition
- Walls: No-slip condition
- Initial condition: average velocity $u = U_{ave}$

  with Reynolds number is given by [4]:

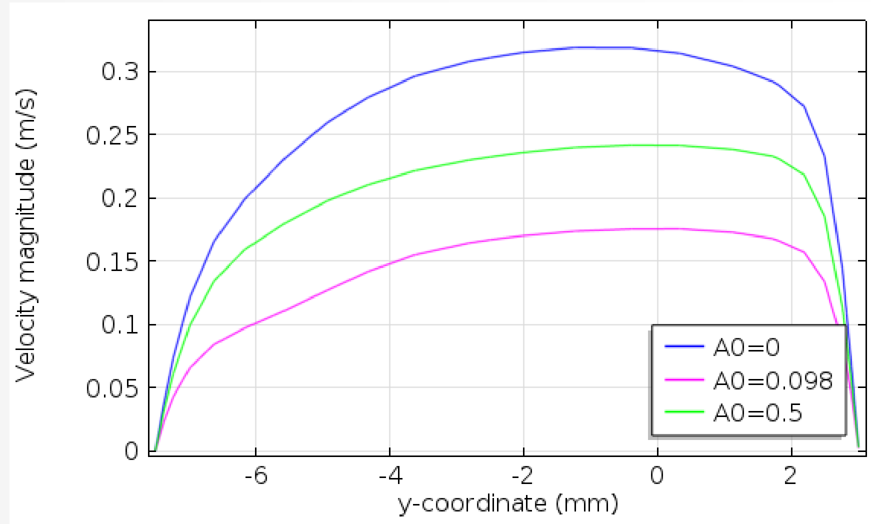
$$Re(t) = Re_{mean}\left[1 + 0.75\sin\left(2\pi\frac{t}{T}\right) - 0.75\cos\left(4\pi\frac{t}{T}\right)\right]$$

Mean Reynolds number=300.

RESULTS : VELOCITY PROFILES

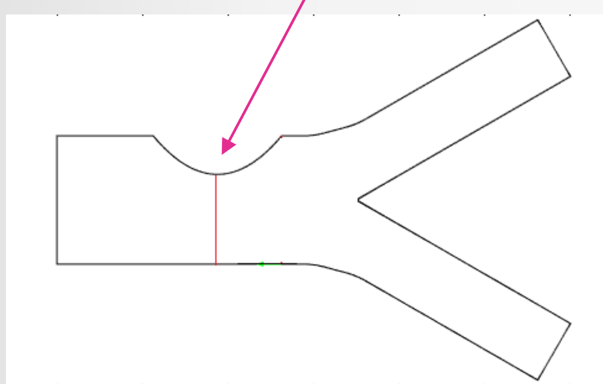


(a) t=0.2s



(b) t=0.7s

Fig. 2: Axial velocity profiles at maximum constriction for different acceleration parameter.



RESULTS: VELOCITY PROFILES

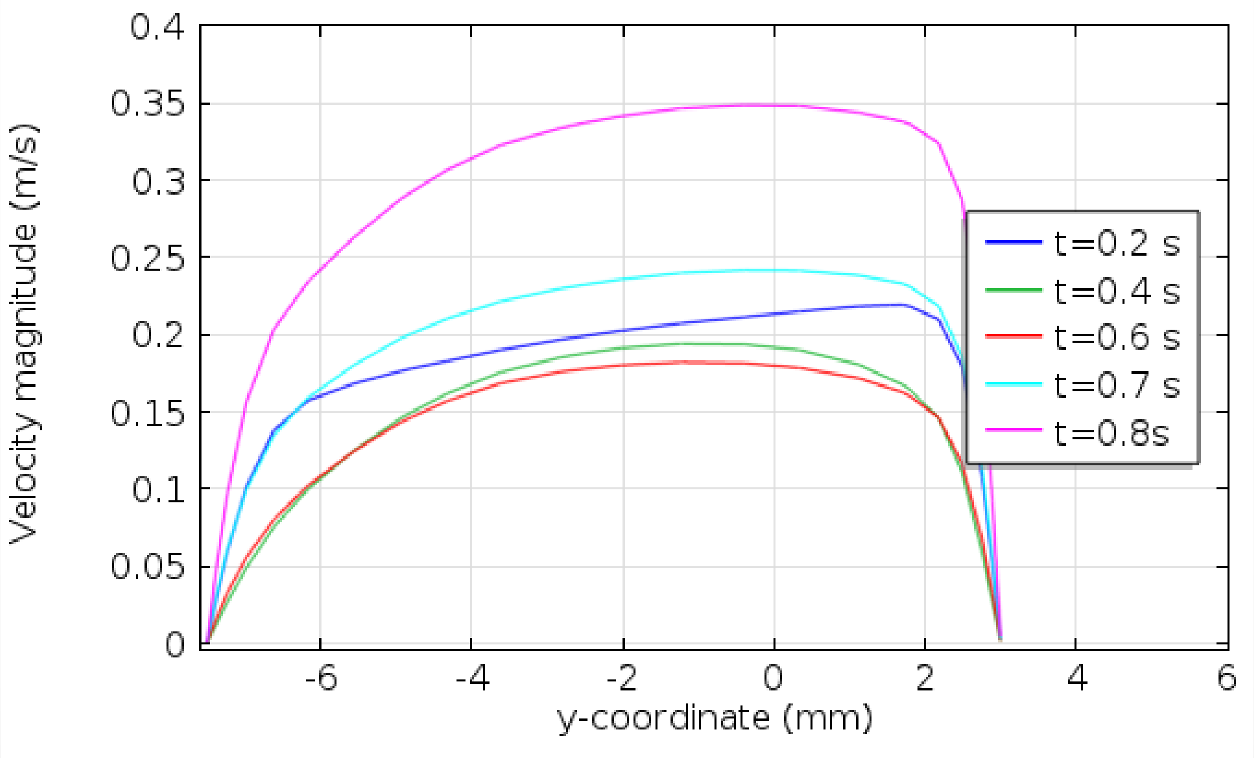


Fig. 2: Axial velocity profiles at maximum constriction at different time for A0= 0.5.

RESULTS: VELOCITY CONTOURS

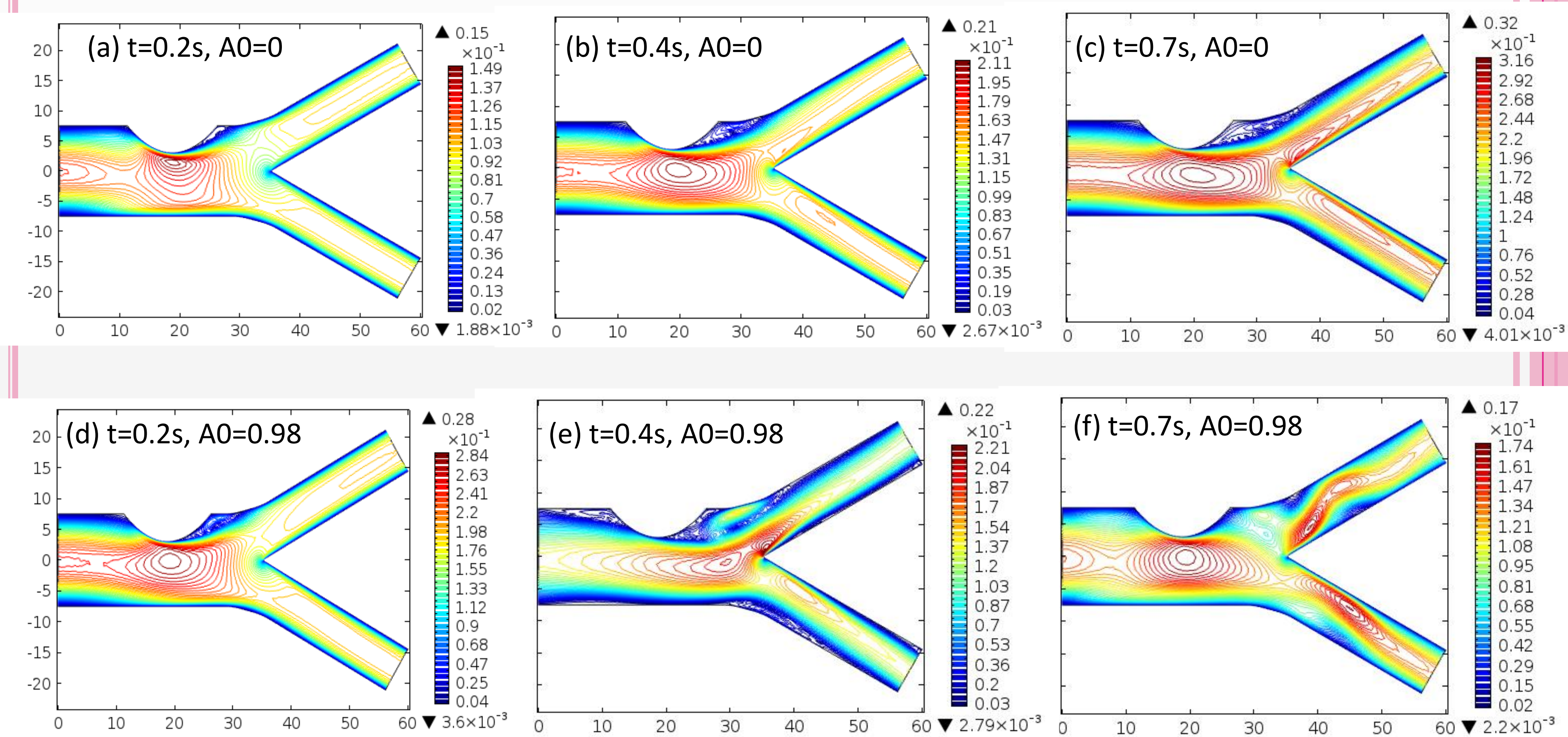


Fig. 3: Velocity contours for at t=0.2s and t=0.7s with and without body acceleration.

RESULTS: STREAMLINES

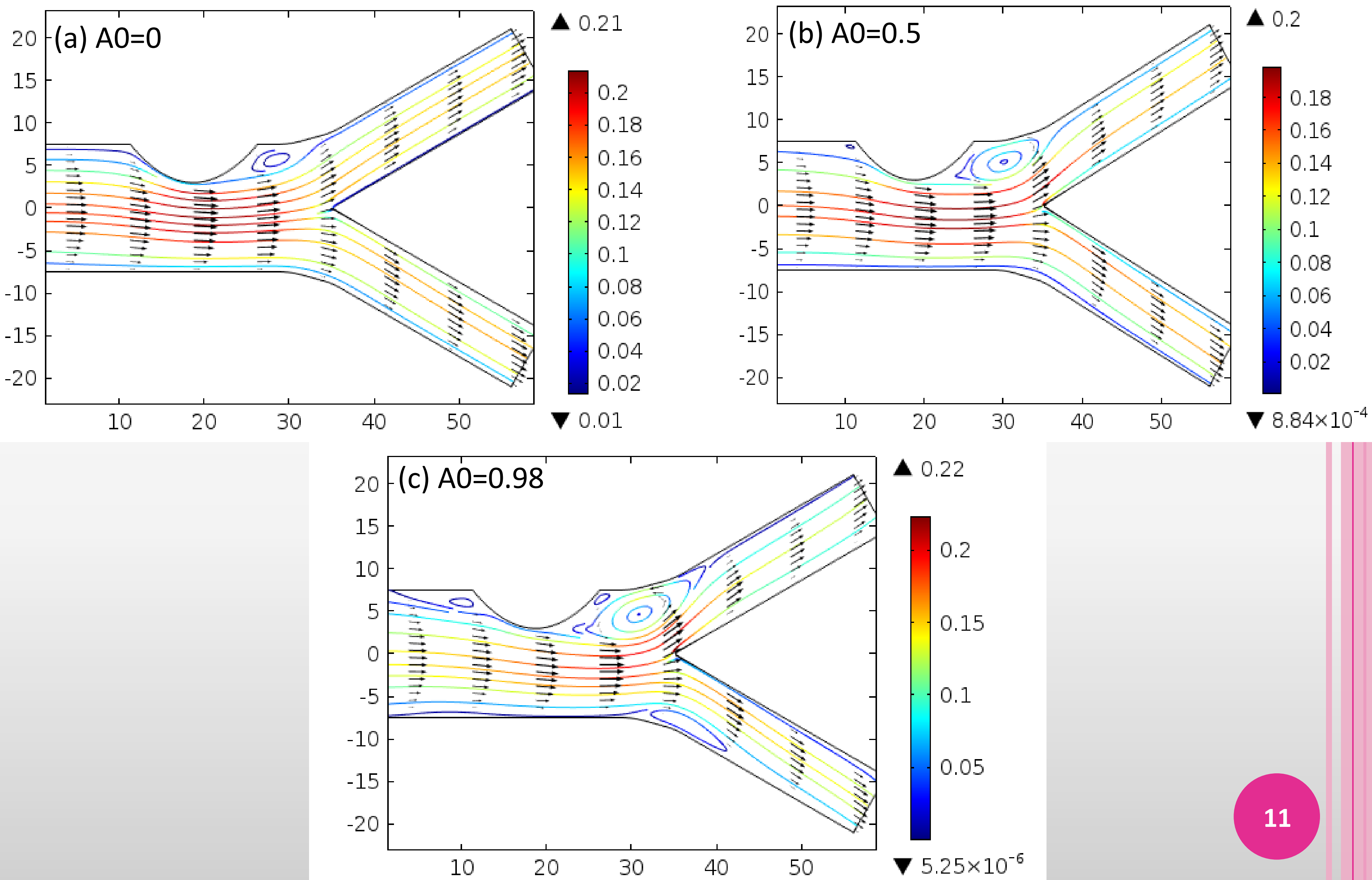


Fig. 4: Streamines and arrow surface at t=0.4s for different acceleration parameters.

RESULTS: PRESSURE CONTOURS

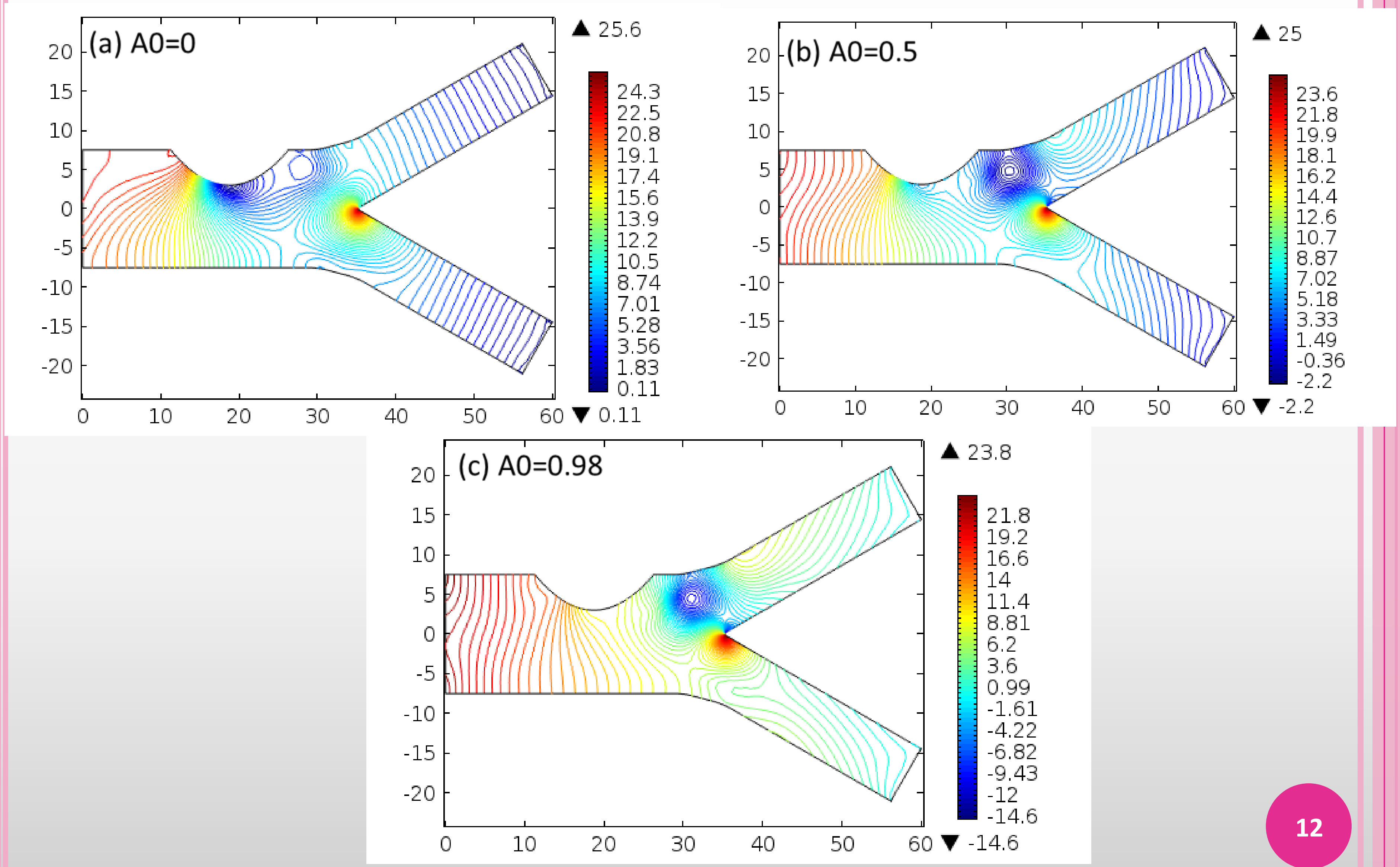


Fig. 5: Pressure contours when t=0.4s.

RESULTS: PRESSURE CONTOURS

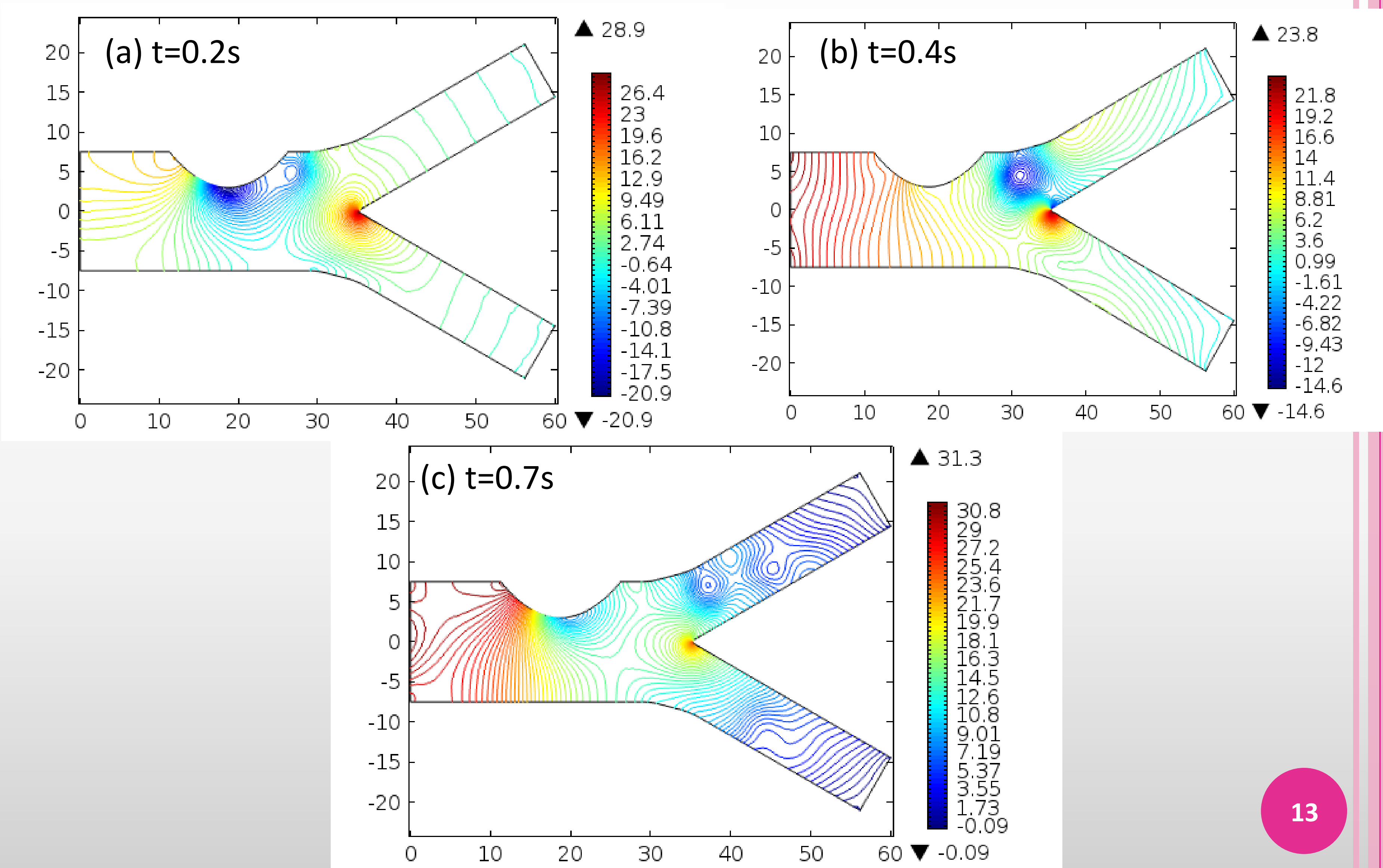


Fig. 6: Pressure contours for A0=0.098.

RESULTS: EFFECT OF STENOSIS

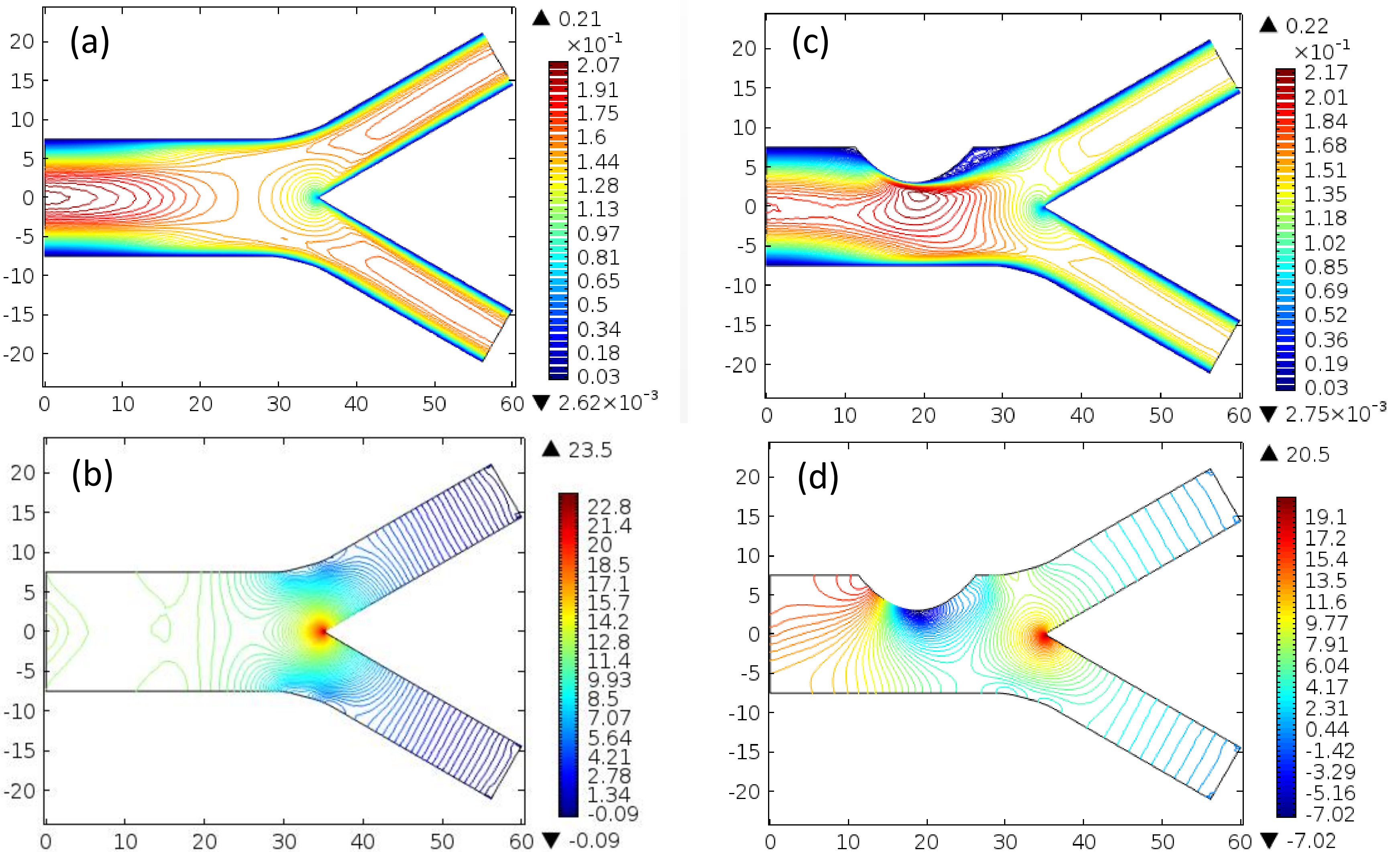


Fig. 7: Pressure contours when A0=0.5 at t=0.2s for non-stenotic artery.

REFERENCES

- [1] What Is Atherosclerosis?, http://www.nhlbi.nih.gov/health/health topics/topics/atherosclerosis.
- [2] What Is Carotid Artery Disease?, http://www.nhlbi.nih.gov/health/health-topics/topics/catd.
- [3] Chakravarty, S. and Mandal, P.K., An Analysis of Pulsatile Flow in a Model of Aortic Bifurcation. *International Journal of Engineering Science*, 1997. 35(4): 409-422.
- [4] Banerjee, M.K., Ganguly, R. and Datta A., Effect of Pulsatile Flow Waveform and Womersley Number on the Flow in Stenosed Arterial Geometry, *International Scholarly Research Network,* 2012, Article ID 853056, doi:10.5402/2012/853056