COMSOL CONFERENCE 2015 BOSTON

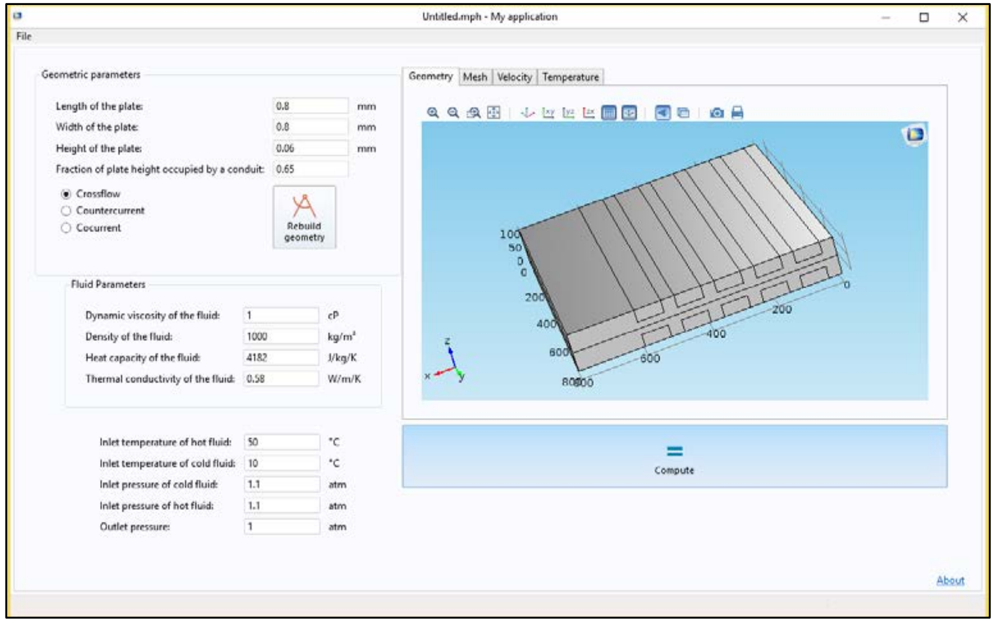
COMSOL Conference 2015 Boston

Session: Optimization and Simulation Methods

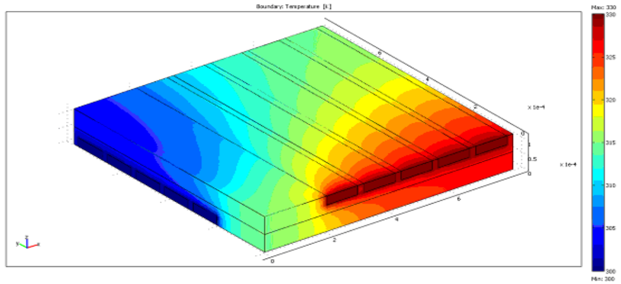
Session Chair: Jeffrey Fong, National Institute of Standards & Technology

October 8, 2015 1:00 PM – 2:30 PM

Application of COMSOL Multiphysics™ Software in Transport Phenomena Educational Processes



Micro Heat Exchanger (MHE) Application



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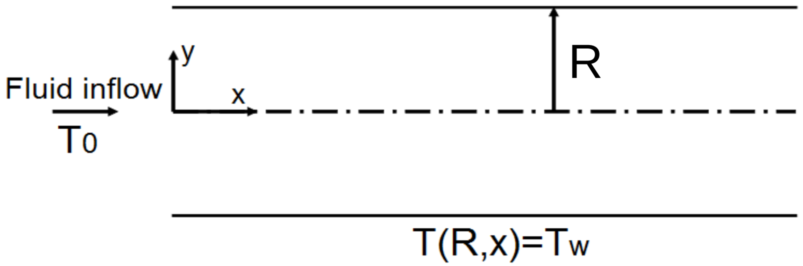
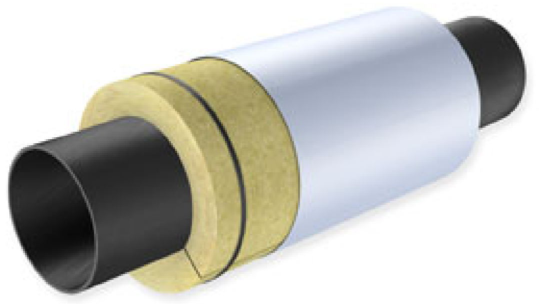
Transport Phenomena: Connecting Theory to Practical Problems

Graetz-Nusselt Problem

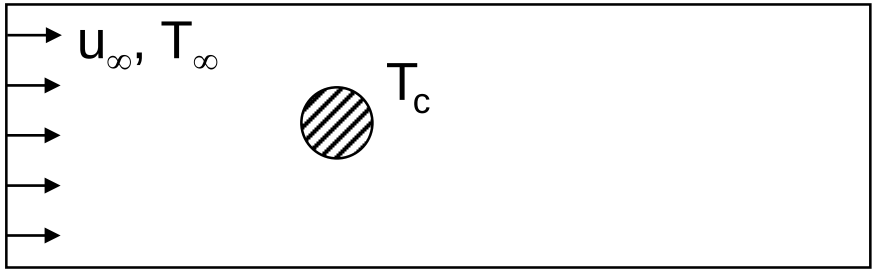


Not constant boundary conditions?
Flux specified only for some region of the pipe?
Imperfect thermal insulation?

Electrically Heated Pipe



Nonisothermal Flow Over a Cylinder



Complex arrangement of multiple pipes?
Viscous flow?
Higher Reynolds number?

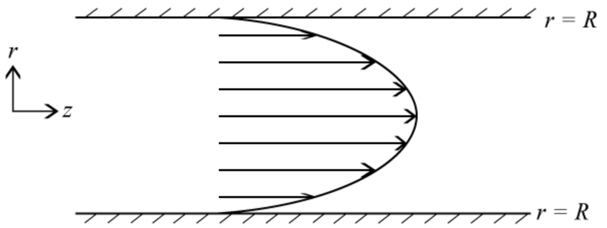
Subsea Pipeline

Examples of Typical Transport Phenomena Course Problems

Momentum transport

1-D fluid flow through a circular pipe

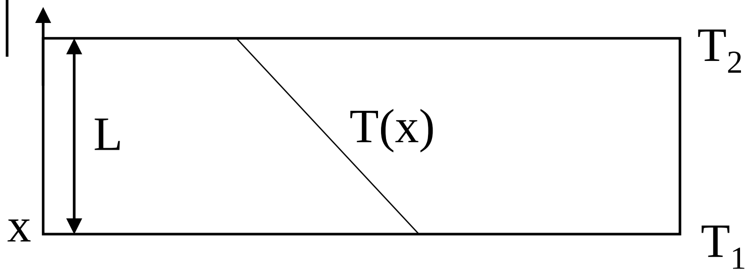
$$u(r) = u_{max}\left[1 - \left(\frac{r}{R}\right)^2\right]$$



Energy transport

Linear heat conduction through a solid wall

$$T(x) = \frac{(T_2 - T_1)}{L}x + T_1$$

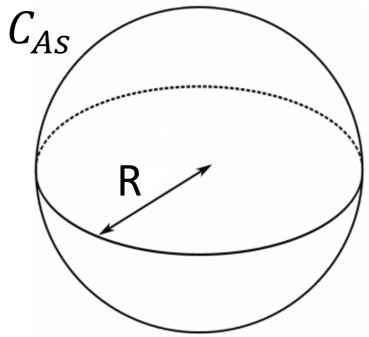


Mass transport

Fickian diffusion in isothermal spherical catalyst particle followed by a 1st order reaction.

$$\Psi(\lambda) = \frac{1}{\lambda}\left(\frac{\sinh(\phi\lambda)}{\sinh(\phi)}\right),$$

where $\Psi = \frac{C_A}{C_{As}}$ and $\lambda = \frac{r}{R}$



Pros:

- Not very complicated – Straightforward to understand
- Existence of analytical solutions
- Provides initial insight into more complex problems

Cons:

- Lack of model response visualization
- Limited applicability to real-world problems
- Transient solutions for 1-D problems are not simple to compute, *e.g.*, infinite series of complex Eigenfunctions and Eigenvalues

Motivation for Creating COMSOL Applications

Desired Problem Attributes

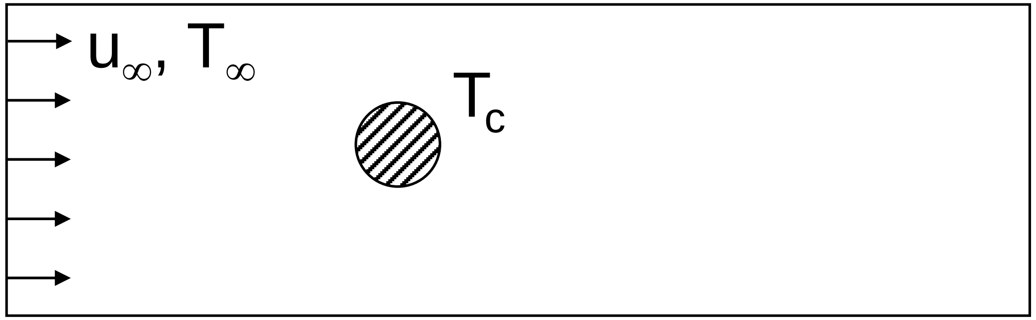
- Reinforce 1-D problems by extension to multi-dimensions (2-D or 3-D) and multi-physics.
- Contains various physico-chemical parameters that can be varied by the user.
- Solution can be generated within seconds to minutes with modern computing hardware.
- Ability to readily modify the application to account for problem variations and other derived quantities.

Applications Developed

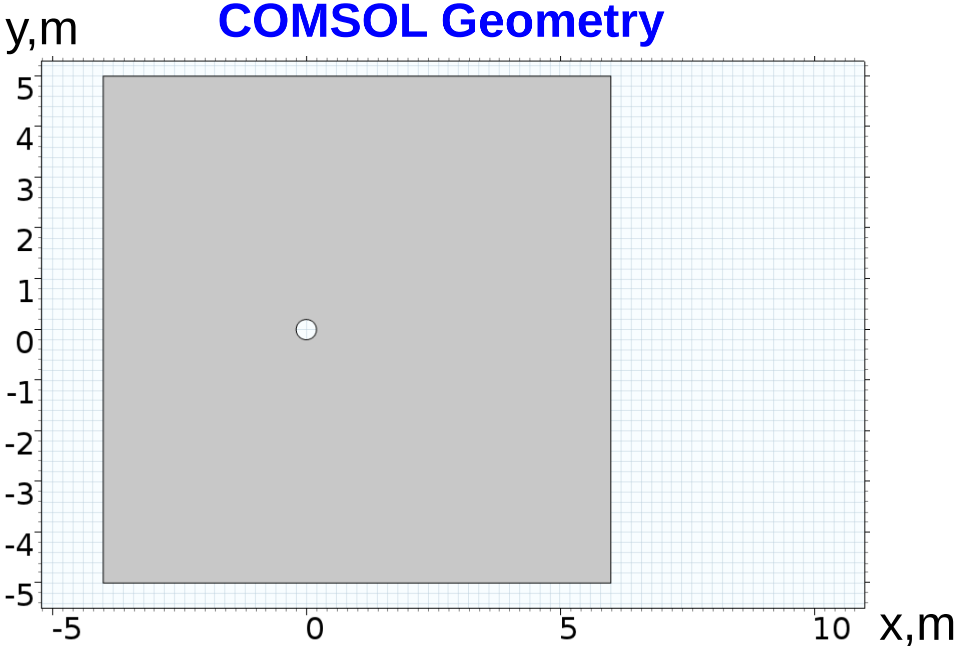
- Non-isothermal Flow Over a Cylinder
- Graetz-Nusselt Problem
- Flow Through a Duct
- Micro Heat Exchanger
- Rotating Cone Pump
- Catalytic Wall Microreactor
- T-Micromixer

Example 1: Nonisothermal Flow Over a Heated Cylinder

Problem Statement



COMSOL Geometry



Model Equations

- Equation of Continuity

$$\nabla \cdot (\rho \boldsymbol{u}) = 0$$

- Navier-Stokes Equations

$$\rho(\boldsymbol{u} \cdot \nabla)\boldsymbol{u} = \nabla \cdot [p\boldsymbol{I} + \mu(\nabla \boldsymbol{u} + (\nabla \boldsymbol{u})^T)] + \boldsymbol{F}$$

- Energy Transport Equations

$$\rho C_p \boldsymbol{u} \cdot \nabla T + \nabla \cdot \boldsymbol{q} = Q + Q_{vd}$$
$$\boldsymbol{q} = -k\nabla T$$

Model Parameters

Name	Expression	Value	Description
Vinf	0.0002[m/s]	2E-4 m/s	Far-field velocity
T0	300[K]	300 K	Temperature of the cylinder
T1	293.15[K]	293.15 K	Temperature of the fluid at the inlet
D	0.2[m]	0.2 m	Diameter of the cylinder
H	10[m]	10 m	Height of the fluid domain

Velocity, Temperature and Pressure Profiles at Various Reynolds Numbers

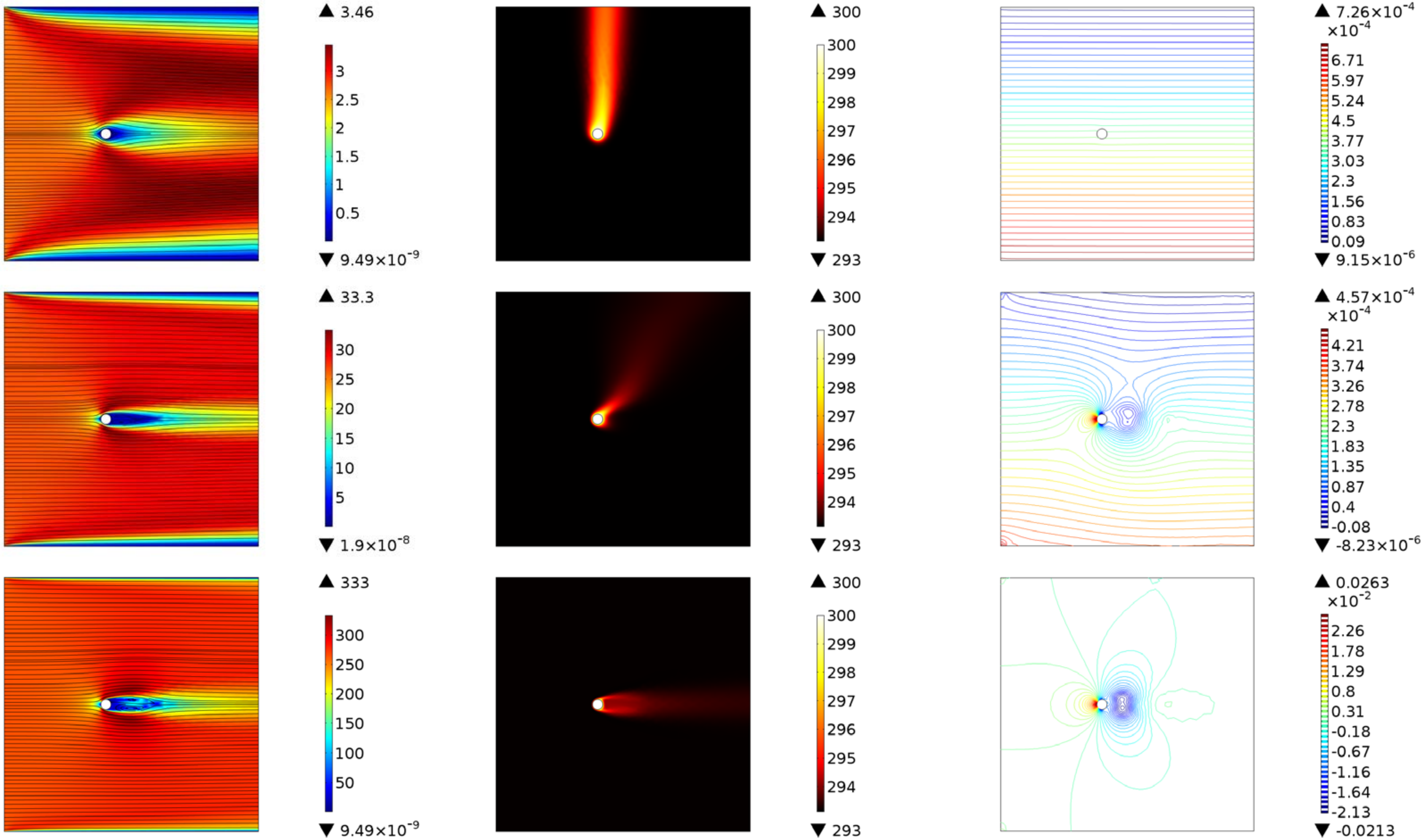
$Gr = 10^8$

Velocity Profiles (Re) | **Temperature Profiles (K)** | **Pressure Profiles (Pa)**

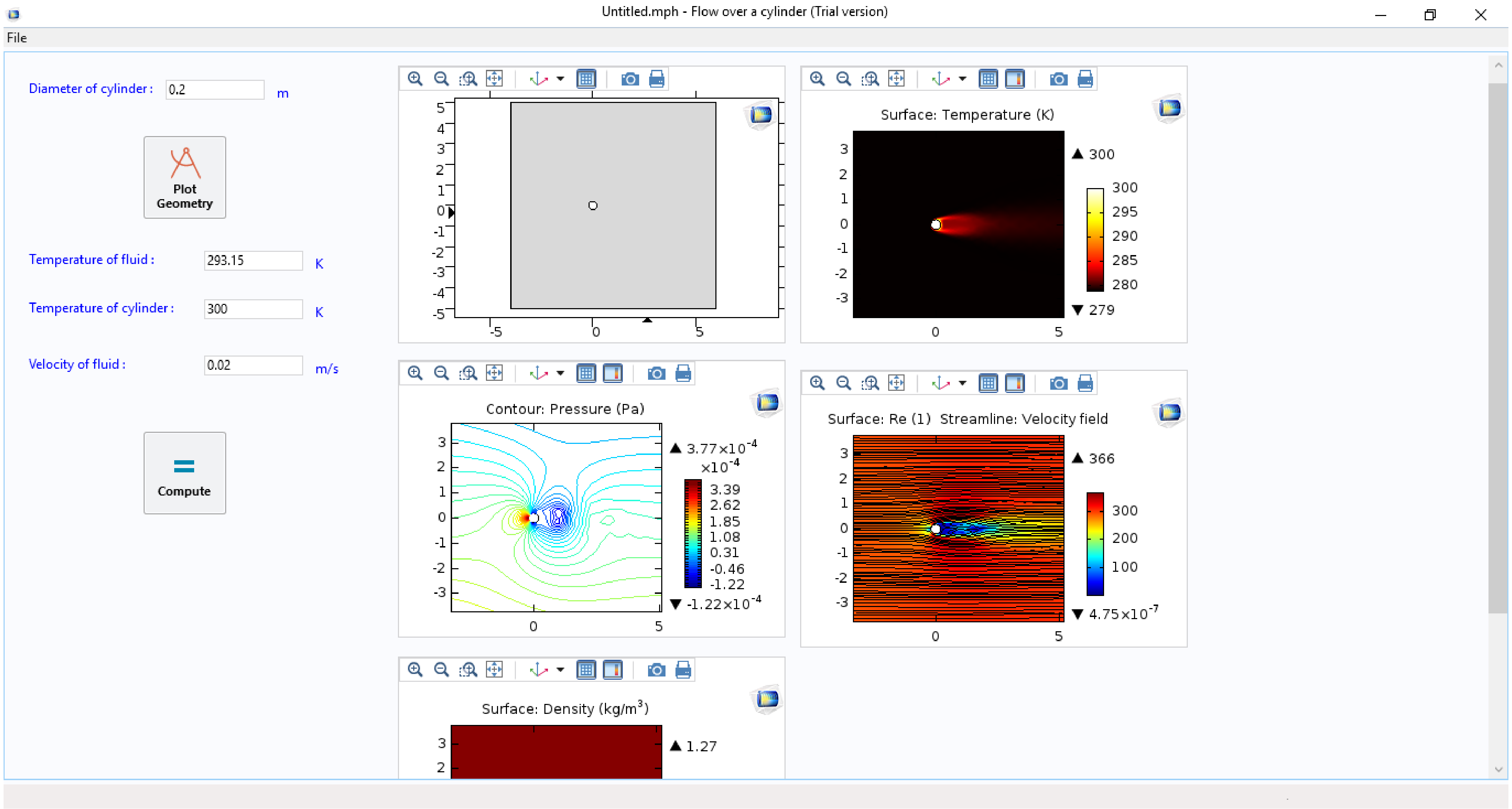
Re = 2.5

Re = 25

Re = 250

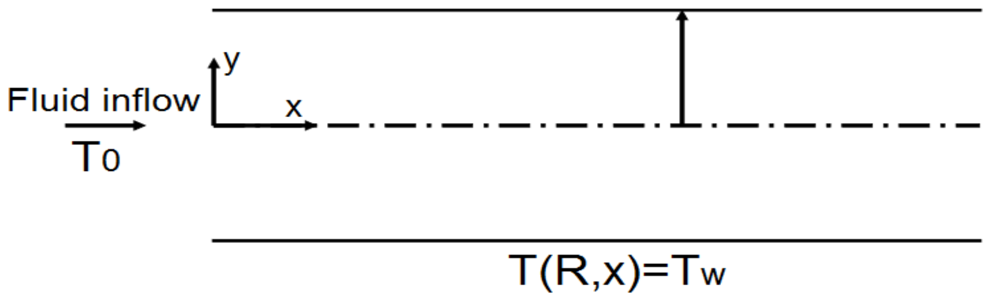


COMSOL Application: Non-isothermal Flow over a Heated Cylinder



Example 2: Graetz-Nusselt Problem (Constant Wall Temperature)

Problem Statement



Model Equations

- **Equation of Continuity**

$$\nabla \cdot \boldsymbol{u} = 0$$

- **Navier-Stokes Equations**

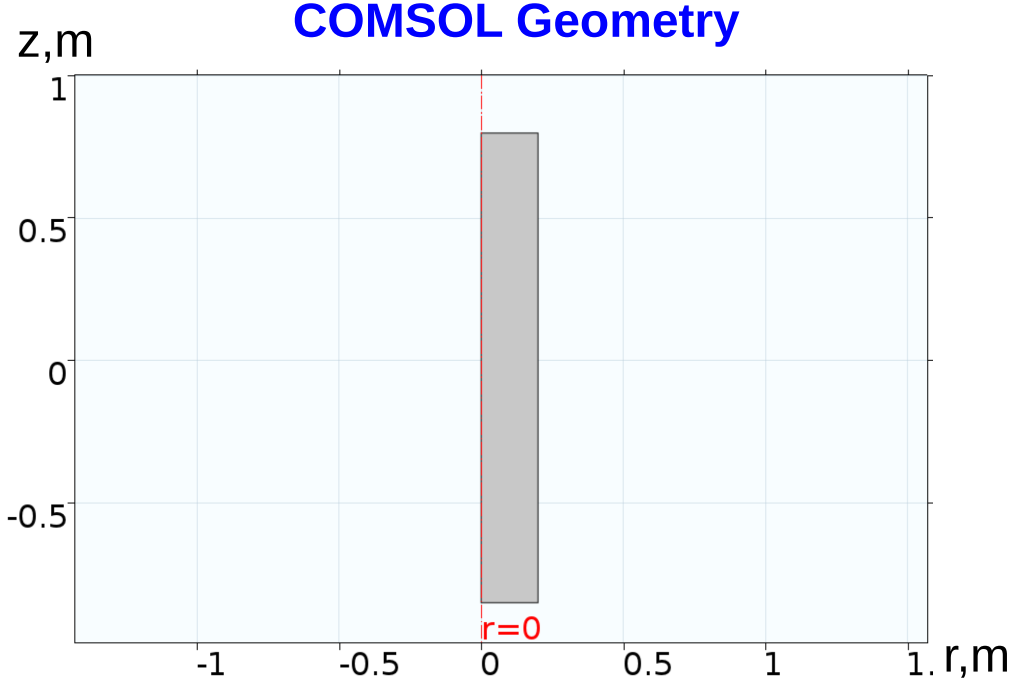
$$\rho(\boldsymbol{u} \cdot \nabla)\boldsymbol{u} = \nabla \cdot [p\boldsymbol{I} + \mu(\nabla\boldsymbol{u} + (\nabla\boldsymbol{u})^T)] + F$$

- **Energy Transport Equations**

$$\rho C_p \boldsymbol{u} \cdot \nabla T + \nabla \cdot \boldsymbol{q} = Q + Q_{vd}$$

$$\boldsymbol{q} = -k\nabla T$$

COMSOL Geometry



Name	Expression	Value	Description
R	0.2[m]	0.2 m	Radius of pipe
L	1.65[m]	1.65 m	Length of pipe
Tin	280[K]	280 K	Temperature of fluid
Tw	350[K]	350 K	Temperature of pipe wall
Vin	0.001[m/s]	0.001 m/s	Inlet fluid velocity

Velocity and Temperature Profiles

Dimensionless Velocity for Various Axial Positions

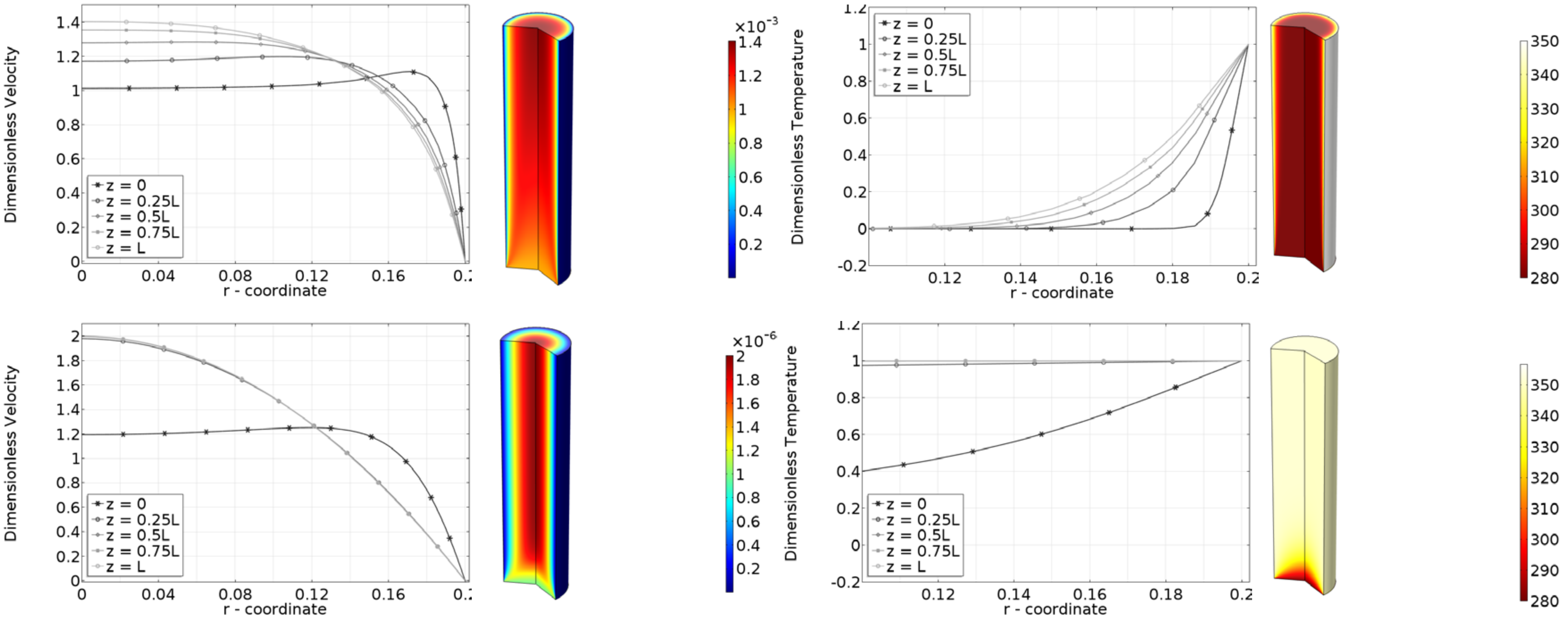
3-D Representation of Velocity Profiles

Dimensionless Temperature for Various Axial Positions

3-D Representation of Temperature Profiles

$V_{in}=10^{-3}$ m/s

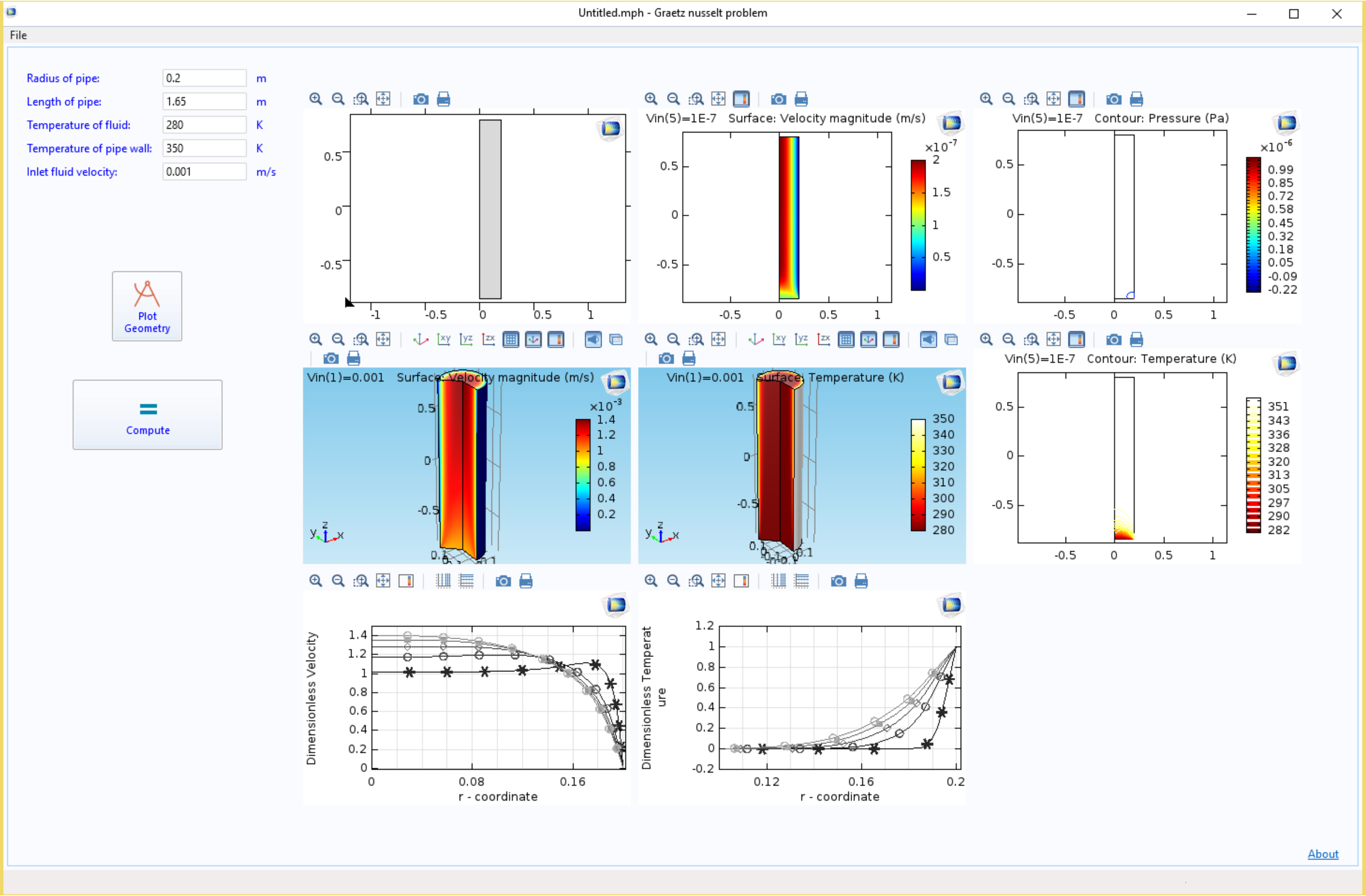
$V_{in}=10^{-6}$ m/s



$$\psi(r,z)=\frac{V(r,z)}{V_{in}}$$

$$\theta(r,z)=\frac{T(r,z)-T_{in}}{T_w-T_{in}}$$

COMSOL Application: Graetz-Nusselt Problem



Example 3: Micro Heat Exchanger

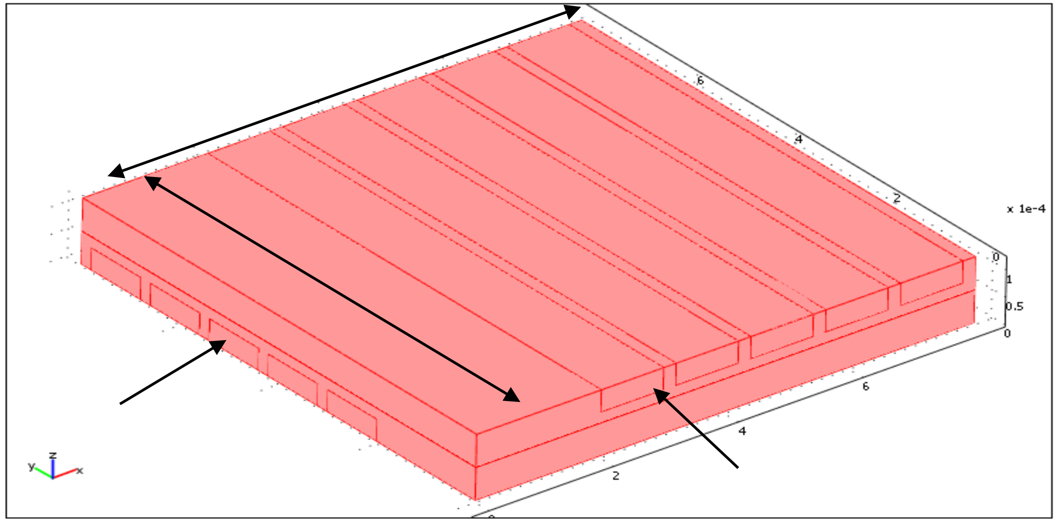
Objectives:

- Compare the heat exchanger effectiveness factor by changing the fluid flow.
- Predict temperature profiles by solving the coupled momentum-energy transport equations for a 3-D geometry.

Model Geometry:

5 rectangular ducts

Cross-flow orientation



Dimensions:

Length of each slab	800 μm
Width of each slab	800 μm
Height of each slab	60 μm
No. of Microchannels	5
Microchannel width	100 μm
Microchannel height	30 μm
Mat'l of Construction	Copper

Micro Heat Exchanger Model Equations & Effectiveness Factor

Model Equations:

Momentum Transport Equations

x- direction:

$$\rho\left[\frac{\partial u_x}{\partial t}\right]-\mu\left[\frac{\partial^2 u_x}{\partial x^2}+\frac{\partial^2 u_x}{\partial y^2}+\frac{\partial^2 u_x}{\partial z^2}\right]+\rho\left[u_x\frac{\partial u_x}{\partial x}+u_y\frac{\partial u_y}{\partial y}+u_z\frac{\partial u_z}{\partial z}\right]+\frac{\partial p}{\partial x}=0$$

y-direction:

$$\rho\left[\frac{\partial u_y}{\partial t}\right]-\mu\left[\frac{\partial^2 u_y}{\partial x^2}+\frac{\partial^2 u_y}{\partial y^2}+\frac{\partial^2 u_y}{\partial z^2}\right]+\rho\left[u_x\frac{\partial u_x}{\partial x}+u_y\frac{\partial u_y}{\partial y}+u_z\frac{\partial u_z}{\partial z}\right]+\frac{\partial p}{\partial y}=0$$

z-direction:

$$\rho\left[\frac{\partial u_z}{\partial t}\right]-\mu\left[\frac{\partial^2 u_z}{\partial x^2}+\frac{\partial^2 u_z}{\partial y^2}+\frac{\partial^2 u_z}{\partial z^2}\right]+\rho\left[u_x\frac{\partial u_x}{\partial x}+u_y\frac{\partial u_y}{\partial y}+u_z\frac{\partial u_y}{\partial z}\right]+\frac{\partial p}{\partial z}=0$$

Conduction-Convection Equation:

$$\rho C_p\left(\frac{\partial T}{\partial t}+u_x\frac{\partial T}{\partial x}+u_y\frac{\partial T}{\partial y}+u_z\frac{\partial T}{\partial z}\right)=\left(k_x\frac{\partial^2 T}{\partial x^2}+k_y\frac{\partial^2 T}{\partial y^2}+k_z\frac{\partial^2 T}{\partial z^2}\right)+Q$$

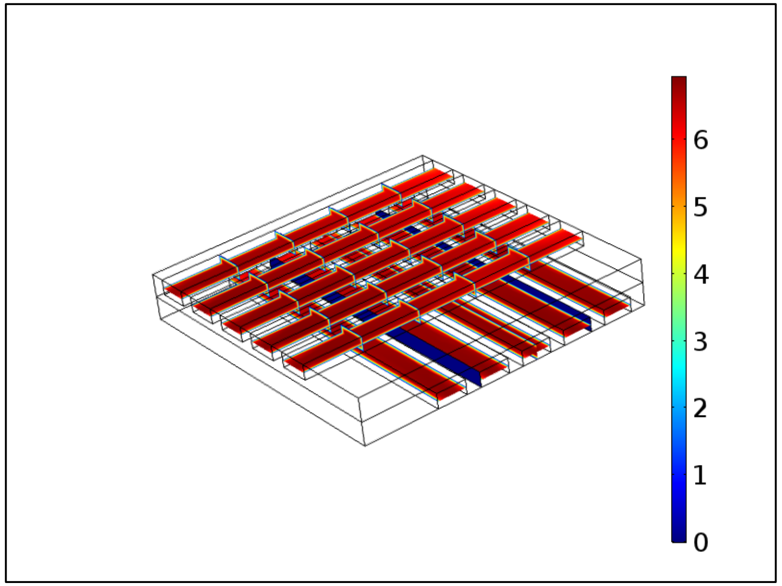
Parameter Estimation:

Effectiveness factor

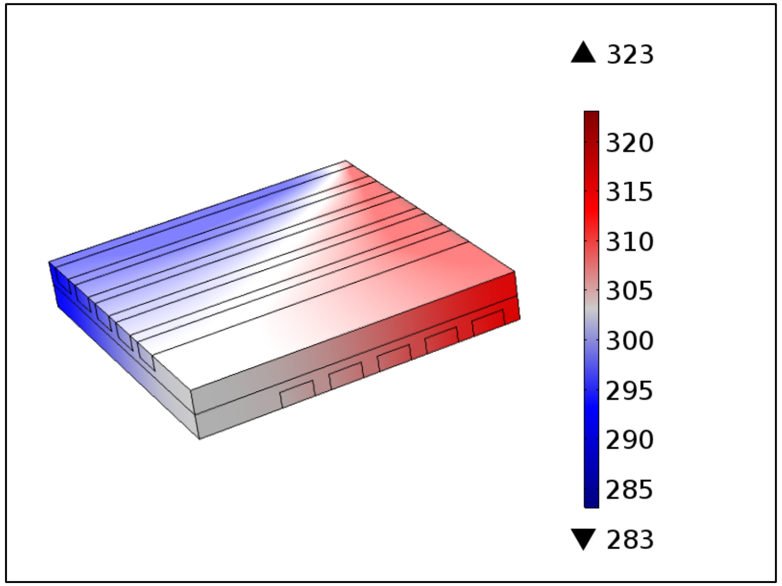
$$\varepsilon=\frac{q}{q_{\max}}=\frac{m^{\circ}C_p(T_{hot-in}-T_{hot-out})}{m^{\circ}C_p(T_{hot-in}-T_{cold-in})}=\frac{(T_{hot-in}-T_{hot-out})}{(T_{hot-in}-T_{cold-in})}$$

ρ is the fluid density, η is the fluid viscosity, p is the fluid pressure, K is fluid conductivity , T is temperature, m˚ mass flow rate, and Cp is the Specific heat capacity

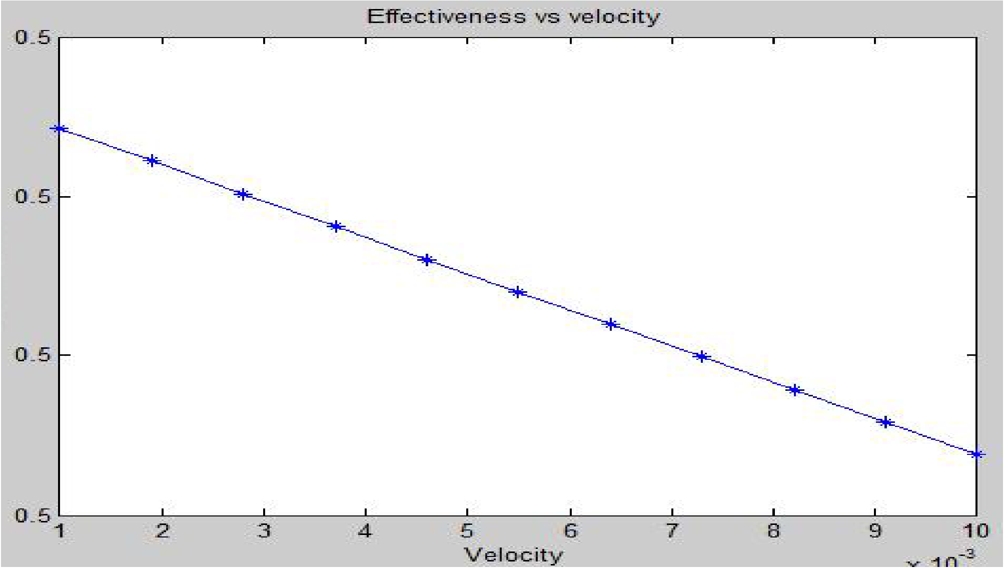
Micro Heat Exchanger Velocity & Temperature Profiles



Velocity Profile
Slice Plot

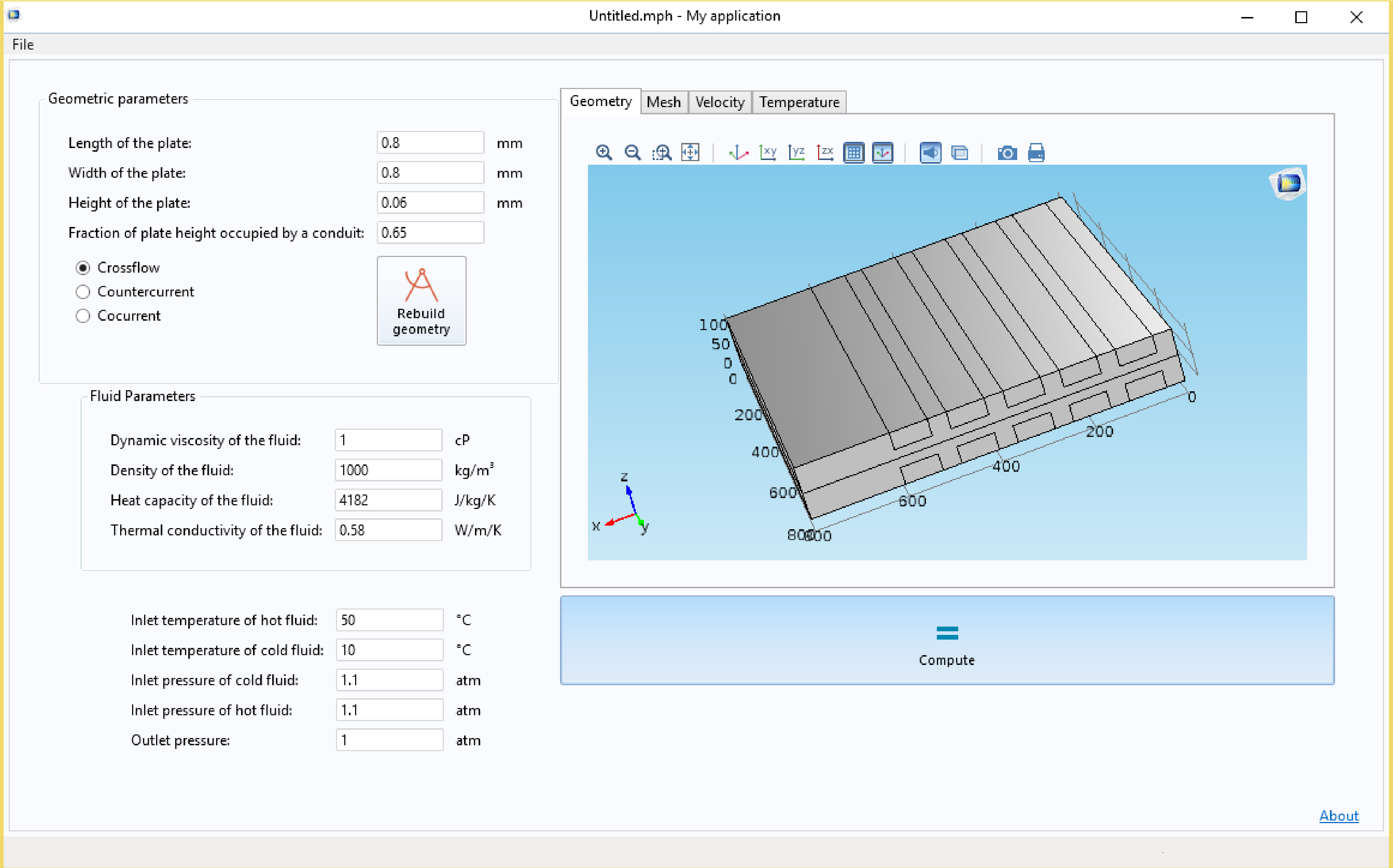


Temperature Profile
Isosurface Plot

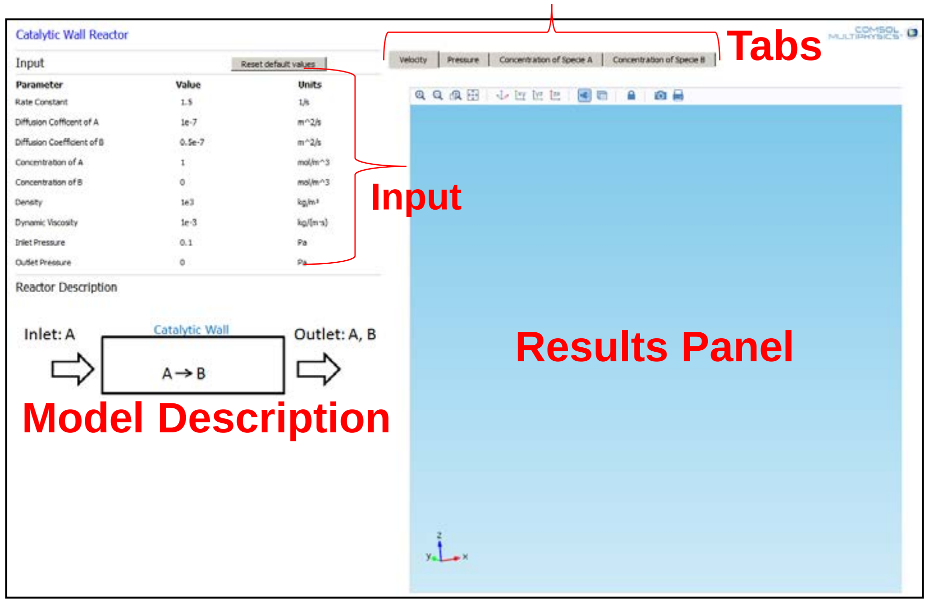


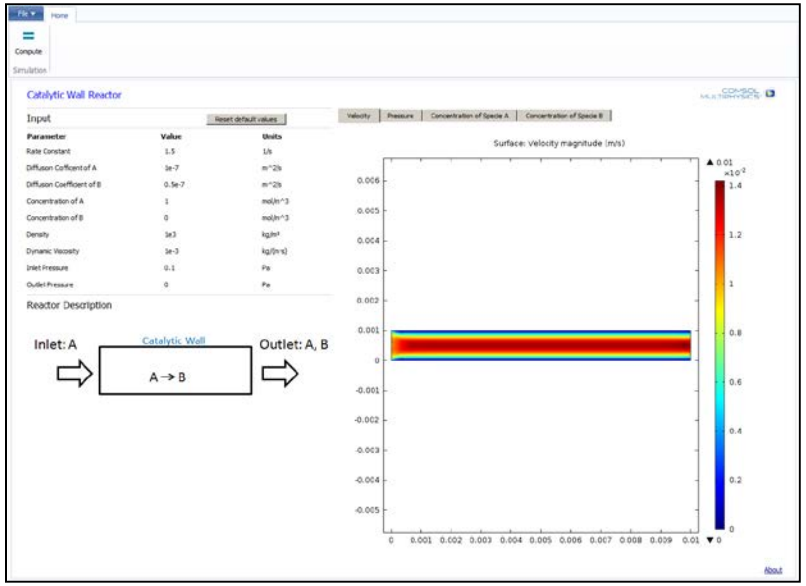
Effectiveness
Factor vs Fluid Velocity

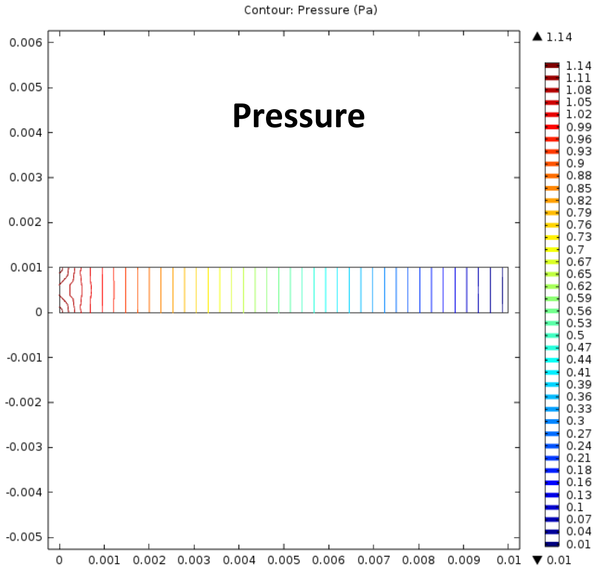
COMSOL Application: Micro Heat Exchanger

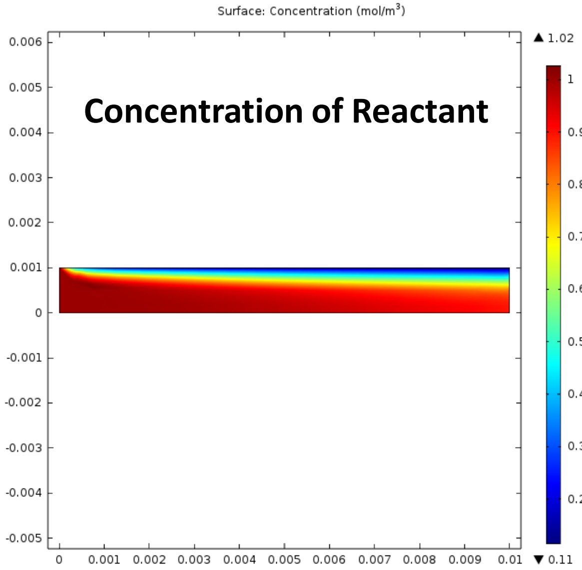


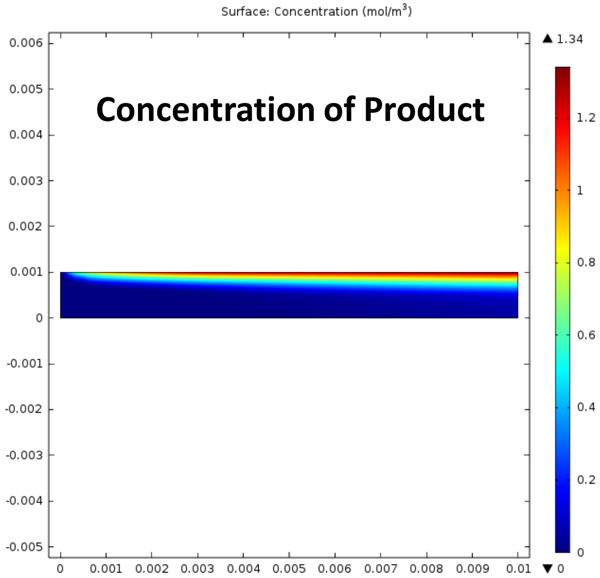
COMSOL Application: Catalytic Wall Microreactor



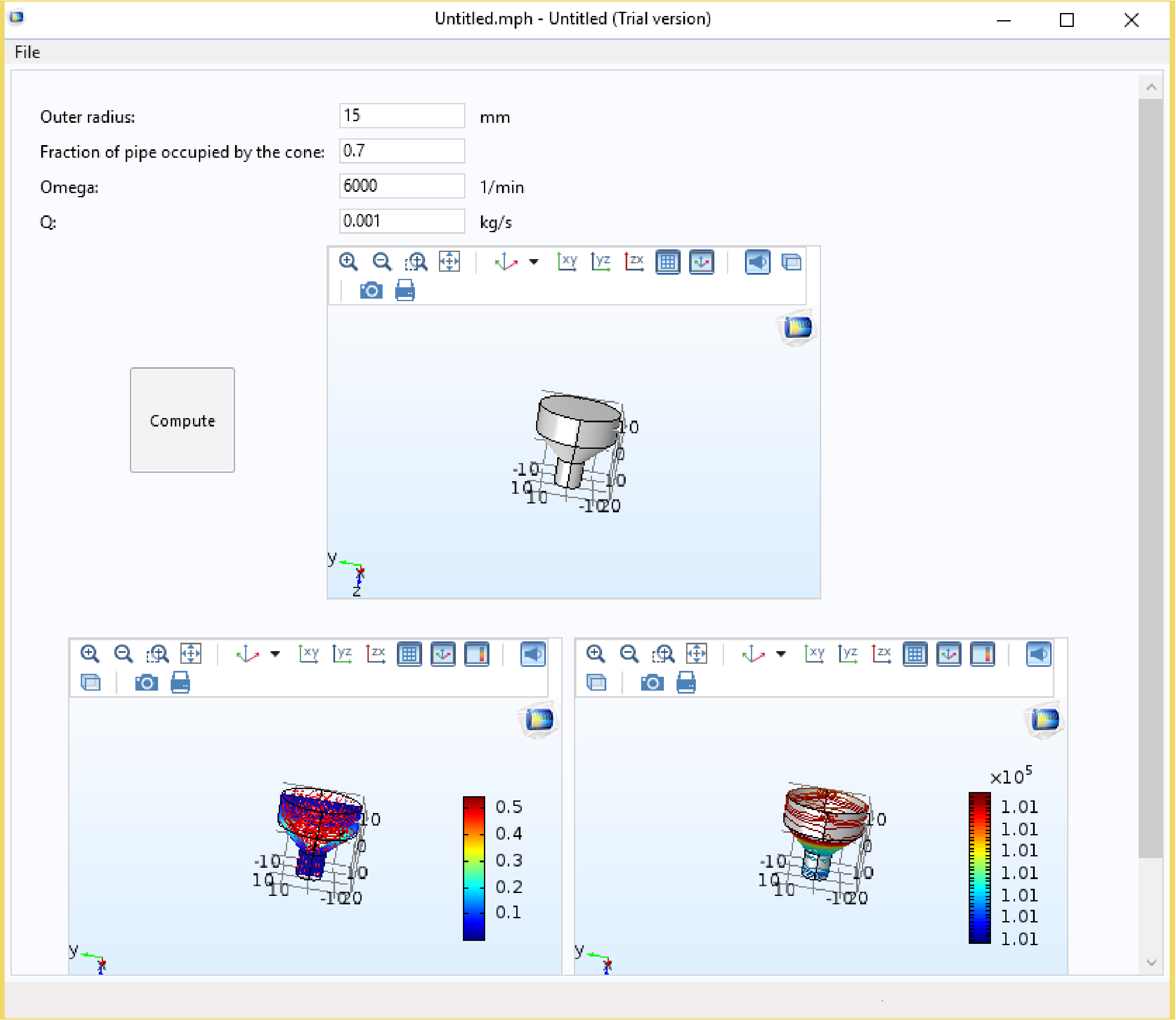








COMSOL Application: Rotating Cone Pump



Conclusions

COMSOL Multiphysics™ :

- Provides robust representation of different multiphysics problems that can be used as a tool for teaching transport phenomena principles.

- Can be used as an intermediate learning instrument between university education and real-world applications.

- Allows different cases of the same problem to be combined into a single application for parametric studies.

- Allows more realistic simulation of real world problems for improved insight.

The student can observe the effect of input parameters on any output variable.

Thank you for your attention.