Inverse Model for Solving Partial Differential Equations Using a Stochastic Method and COMSOL Multiphysics®

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Abstract

Partial difference equations (PDEs) explain many phenomena in the nature. They can describe fluid flow, thermal behavior, mechanic structures, microwave and many more. These equations can be simplified and solved theoretically for simple problems. But, for complicated structures, which exist in the real world, it is not possible to find the answer simply. In order to overcome this issue, numerical methods are used, and simulation software like COMSOL Multiphysics® has been developed.

In many cases it is needed to solve inverse partial differential equations. Solving these problems involves minimizing the defined error function in a loop. In each step PDE forward problem is solved and desired parameters are adjusted accordingly to meet the criterion. If the number of PDEs needed to be solved in each step is increased, computational time and memory requirement are increased. In order to speed up the solution process, Haber et al. proposed a method to reformat the problem to a stochastic optimization problem [1]. In this method only one PDE is needed to be solved, so the cost of solution can be reduced.

In this paper, LiveLink™ for MATLAB® and COMSOL were used to find the radius of a scatterer in a microwave problem with multiple input and output antennas. COMSOL was used to solve forward problem in our example. The LiveLink™ for MATLAB® allowed us to generate a function which create outputs based on desired inputs in each step of optimization loop. Using this function, not only allows to use finite element method to solve out problem's PDEs, but also lets to include the finite element solution of the problem in the whole process of solving the inverse problem.

Reference

[1] Haber, Eldad, Matthias Chung, and Felix Herrmann. "An effective method for parameter estimation with PDE constraints with multiple right hand side." July 12, 2010.
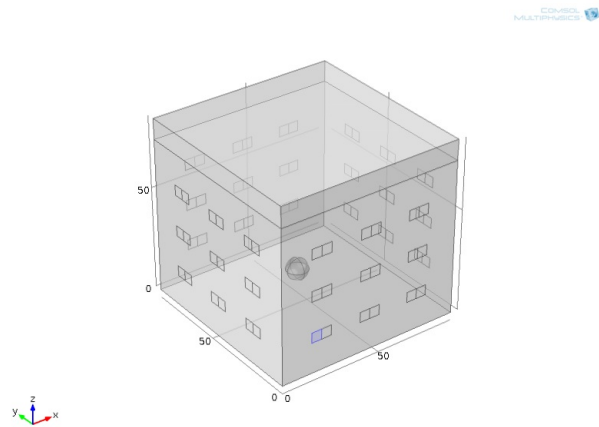
Figures used in the abstract

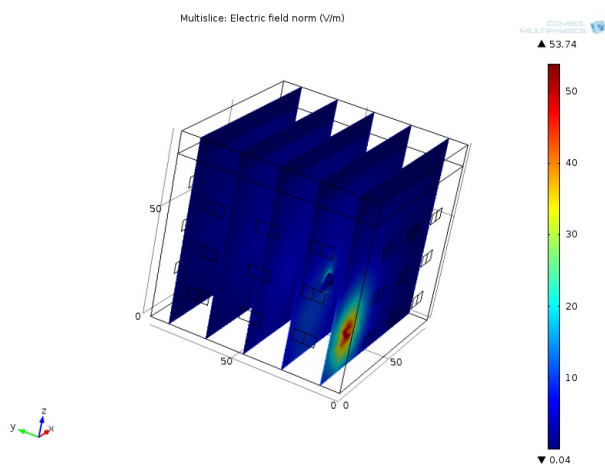**Figure 1**: Microwave imaging with multiple antennas - geometry

Figure 2: Microwave imaging with multiple antennas - results