Modeling And Analysis Of A Direct Expansion Geothermal Heat Pump (Dx): Part II-Modeling Of Water-Refrigerant Exchanger.

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Abstract In this section, we simulate the heat exchanger system in one dimension characterized by two coaxial tubes (Figure 2) with ribbed inner tube using the equations of conservation of mass, conservation of momentum and energy. The COMSOL PDE interface is used to simulate the monophasic and biphasic flow of refrigerant R22 (Chlorodifluoromethane). Heat transfer in water and inner wall of the exchanger are modeled with two Heat Transfer interfaces (solid, fluid).
The developed numerical model was used to simulate the phases of superheating, condensing and subcooling in the condenser. Vapor quality, pressure, enthalpy of the refrigerant and the water temperature are also simulated. To validate this numerical model a comparison between numerical and experimental results was performed.

Keywords: Modeling, heat exchanger, geothermal, direct expansion, heat pump, condenser, evaporator, superheating, subcooling.

1. Introduction

This study is part of a larger project to enhance and develop a new type of direct expansion geothermal heat pump (DX) and to evaluate the performance of such a system with potential fluids (R404A and R407C) for the replacement of R22 refrigerant. A geothermal direct expansion (DX) is a heat pump (Figure 1) with the particularity that one component is buried directly under the ground playing the role of condenser / evaporator according to the mode of operation contrary to the traditional secondary loop. Thus, it has the following advantages among others:
-Reduction of cost by elimination of secondary loop of the ground side,
-Low power consumption for its operation,
-Using a larger energy resource with a relatively constant temperature throughout the year,
-Good even at very low atmospheric temperature,
-Reduced maintenance costs.
As main drawback, the initial investment costs are very high, but because of the savings, return on investment can be done quickly. That is why geothermal systems renewed interest in recent years[1].
The literature review revealed a lack of scientific research and publication concerning direct expansion geothermal heat pump systems. So in terms of modeling and experimental results, the information available does not have sufficient scientific knowledge with respect to this technology. Therefore, the proposed modeling and analysis of this heat pump DX aims to fill this gap.
Modeling and analysis of a direct expansion geothermal heat pump begins with modeling these different components: Ground heat exchanger, compressor, thermostatic expansion valve, reversing valve, pipe, water-refrigerant exchanger, etc. and the coupling of these components to make a closed loop corresponding to the heat pump.
In the first part (pending publication), the numerical model of ground heat exchanger is presented.
In this study, we present a dynamic model of the second heat exchanger system by solving the equations of conservation of mass, momentum and energy conservation. The numerical model was used to simulate the phases of superheating, condensing and sub-cooling. The evolutions of steam as well as the temperature, the pressure of the refrigerant along the exchanger are also presented. The resulting model is validated by comparison with experimental results.

Nomenclature

P	Pitch of the ribs (m)
e	Rib height (m);
α	Helix angle of the ribs (^{o})
N	Number of departures of ribs per section
s	Length of the base of the ribs (m)
De	Maximum internal diameter (m);
t	Wall thickness of the tube twisted (m)
D_b	Minimum inner diameter (m).
D_{1P}	Equivalent inside diameter of the tube means within the perimeter (m).
D_{2A}	Outer diameter of the tube equivalent procedure on the basis of average session (m)
D_{2P}	Outer diameter of the tube equivalent means within the perimeter (m).
Re	Reynolds number;
μ_r	Viscosity of refrigerant (N.s/m^2)
D	Annular diameter (m)
r^*	Ratio of radii for the ring
e^*	Dimensionless rib height
θ^*	Dimensionless helix angle
p^*	Dimensionless pitch of the ribs
ρ	Density of refrigerant (kg/m^3)
$\dot{m}$	Refrigerant mass flow (kg/s)
S_f	Section fluid passage (m^2)
P	Refrigerant pressure (Pa)
h	Refrigerant enthalpy (J/kg)
Q_v	Heat flux per unit exchanged with the inner wall (W/m^3)
F_{vol}	Frictional force per unit volume between the tubes and refrigerant (N/m^3)
x	Vapor quality
ρ_p	Density of the inner wall (kg/m^3)
A_p	Axial wall area (m^2);
C_p	Specific heat of the wall (J/kg.K)
k_p	Thermal conductivity of the wall (W/m.K)
h_{rp}	Coefficient of heat exchange between the refrigerant and the wall (W/m^2.K)
h_{ep}	Coefficient of heat exchange between the inner wall and water (W/m^2.K)
T_r	Temperature of refrigerant (K)
ρ_e	Water density (kg/m^3);
Cp_e	Specific heat of water (J/Kg.K);
T_e	Water temperature (K);
v	Velocity of water (m/s);
T_p	Temperature of the inner wall (K).
Pr	Prandtl number
Nu	Nusselt number
Fr	Froude number
u	Refrigerant velocity (m/s)
We	Weber number
k_r	Thermal conductivity of refrigerant (W/m.K)
k_e	Thermal conductivity of water (W/m.K)
k_f	Thermal conductivity of the liquid phase (W/m.K)
L	heat exchanger length (m)
Cp_f	Specific heat of the liquid phase (J/kg.K)
D_3	Inner diameter of the outer tube (m)

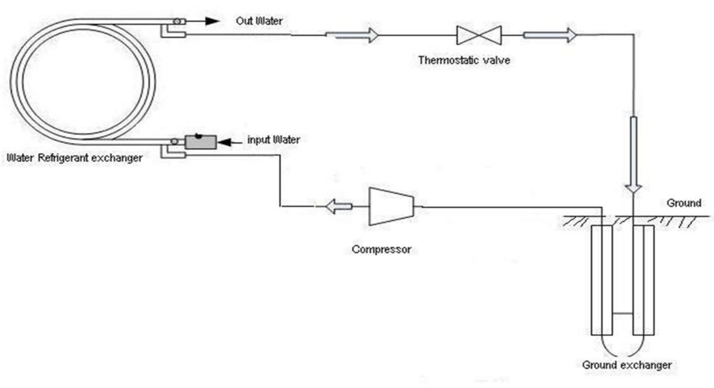

Figure 1. Simplified diagram of a direct expansion geothermal heat pump

2. Modeling Refrigerant-water exchanger

2.1 Geometry description

The heat exchanger of the double type is modeled spirally wound tube whose inner tube is called twisted or ribs (Figure 2). There is characterized the point of view of internal geometry (Figure 3) by:

- Pitch of the ribs, p ;
- Height of the ribs, e;
- Helix angle of the ribs, α (°);
- Number of starts per section of ribs, N;
- Maximum inside diameter, De;
- Wall thickness of twisted tube, t;
- Minimum inside diameter, D_b.

Figure 2. Exchanger diagram

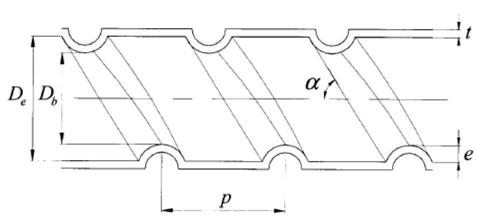

Figure 3. Exchanger geometry

The advantage of such geometry is to increase the exchange surface area to maximize heat transfer between it. Because of this irregular geometry in inner tube, the authors [2], [6] proposed the equivalent of a corresponding smooth tube. The equivalent diameters are calculated using the following formulas:

$$D_{1A} = \sqrt{D_e^{2} - \frac{2Nes}{\pi}} \quad (1)$$

$$D_{1P} = D_e + \frac{N}{\pi}\left(\sqrt{4e^{2} + s^{2}} - s\right) \quad (2)$$

$$D_2 = D_1 - 2t \quad (3)$$

According to Rousseau [6], the ribs helix angle can be calculated by:

$$\alpha = \arctan\left(\frac{\pi D_{2A}}{Np}\right) \quad (4)$$

2.2 Modeling

The heat exchanger is installed on a vapor compression heat pump with reversible cycle power of 10kw. It is used as a condenser when the heat pump is operating in heating mode and as an evaporator in cooling mode. In all cases, the refrigerant circulates in the annular space and the secondary fluid into the inner tube (Figure 4). Refrigerant may flow as monophasic or biphasic. We consider the outer wall of the exchanger to be isolated and that there is therefore no heat transfer between the refrigerant and the outer wall. The temperature of the refrigerant is then equal to the temperature of the outer wall at every point. The modeling presented here, describes three elements that interchange: refrigerant, the secondary fluid and the wall of the inner tube.

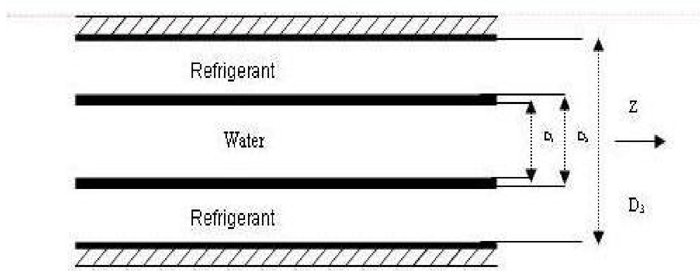

Figure 4. Schematic drawing of the heat exchanger Refrigerant-Water

2.2.1 The Refrigerant

Monophasic flow

Modeling assumptions are as follows:
- The fluid or refrigerant is assumed Newtonian;
- The flow is one dimensional along the axis of the tube;
- The gravitational force is negligible;
- The axial conduction is negligible;
- The viscous dispersion is negligible.

Under these conditions, the governing equations are:

$$S_f \frac{\partial \rho}{\partial t} + \frac{\partial \dot{m}}{\partial z} = 0 \quad (5)$$

$$S_f \frac{\partial \dot{m}}{\partial t} + \frac{2\dot{m}}{\rho}\frac{\partial \dot{m}}{\partial z} - \frac{\dot{m}^2}{\rho^2}\frac{\partial \rho}{\partial z} + S_f \frac{\partial P}{\partial z} = -S_f{}^2 F_{vol} \quad (6)$$

$$S_f \rho \frac{\partial h}{\partial t} + \dot{m}\frac{\partial h}{\partial z} - S_f \frac{\partial P}{\partial t} - \frac{\dot{m}}{\rho}\frac{\partial P}{\partial z} = S_f Q_v, \quad (7)$$

with:

$$F_{vol} = 2C_f \dot{m}^2 / S_f{}^2 \rho D \quad (8)$$

Where C_f represents the friction coefficient. The friction coefficient is given by the Garimela correlation [2]:

For Re < 800,

$$C_f = \frac{24 r^{*0.035}}{\mathrm{Re}}\left(1 + 101{,}7\,\mathrm{Re}^{0.52} e^{*1.65+2\theta^*} r^{*5.77}\right) \quad (9)$$

For Re $\geq$ 800,

$$C_f = \left[1.7372 \ln\left(\frac{\mathrm{Re}}{1.964 \ln \mathrm{Re} - 3.8215}\right)\right]^{-2} * \quad (10)$$

$$\left(1 + 0.0925\, r^*\right) e_f$$

$$e_f = \left(1 + 222\ \mathrm{Re}^{0.09} e^{*2.40} p^{*0.49} \theta^{*-0.38} r^{*2.22}\right) \quad (11)$$

In these equations,

- $\mathrm{Re} = \rho u D / \mu_r$ (12)
- $D = D_3 - D_{2A}$ (13)
- $r^* = D_{2A} / D_3$ (14)
- $e^* = e / D_{2A}$ (15)
- $\theta^* = \alpha / 90$ (16)
- $p^* = p / D_{2A}$ (17)

The heat exchanged with the inner tube is given by the relation:

$$Q_v = \frac{4 h_{rp} D_{2p}}{D_3{}^2 - D_{2A}{}^2}\left(T_p - T_f\right) \quad (18)$$

Heat transfer wall-refrigerant coefficient is calculated by Garimela correlation [3]:

$$h_{rp} = \frac{k_r}{D}\left[\frac{0.5 C_f \,\mathrm{Re}\,\mathrm{Pr}}{1 + 9.77\sqrt{0.5 C_f}\left(\mathrm{Pr}^{2/3} - 1\right)}\right]\left(\mathrm{Re}^{-0.20} e^{*-0.32} p^{*-0.28} r^{*-1.64}\right) \quad (19)$$

Two-phase flow

We assume that the flow is homogeneous and that the two phases are in thermodynamic equilibrium. The governing equations take the same form as in the single-phase flow except that we must take into account the parameters that apply to the liquid-vapor mixture as follows:

- The density as a function of the void fraction:

$$\rho = \left(1 - \alpha_b\right)\rho_f + \alpha_b \rho_g \quad (21)$$

f index refers to the liquid phase and the index g refers to the vapor phase.

- The enthalpy can be written as:

$$h = (1 - x) h_f + x h_g \quad (22)$$

In this model, the void fraction α_b proposed by Zivi [7] is used.

The frictional pressure drop is evaluated assuming a single phase flow corrected with two-phase multiplier [7]:

$$(F_{vol})_{di} = \phi^2 (F_{vol})_{mono}, \quad (23)$$

mono index refers to monophasic flow and *di* index refers to biphasic flow. ϕ^2 : The biphasic multiplier factor.

$$\phi^2 = A_1 + \frac{3.24 A_2 A_3}{Fr^{0.045} We^{0.035}} \quad (24)$$

Where:

$$A_1 = (1 - x)^2 + x^2 (\rho_f f_g / \rho_v f_f) \quad (25)$$

$f_g = Cf$: For the gaseous phase flow

$f_f = Cf$: For the liquid phase flow

$$A_2 = x^{0.78} (1 - x)^{0.224} \quad (26)$$

$$A_3 = \left(\rho_f / \rho_g\right)^{0.91}\left(\mu_g / \mu_f\right)^{0.19}\left(1 - \mu_g / \mu_f\right)^{0.7} \quad (27)$$

$$Fr = u^2 / gD \quad (28)$$

$$We = \rho u^2 D / \sigma \quad (39)$$

σ, Stefan's constant.

The coefficient of heat transfer wall-refrigerant is:

- **Condenser Mode:** Koyama's Correlation [4]

$$h_{rp} = 0.53 \frac{k_f}{d}\left(\frac{d}{D}\right)^{-0.4} Ph^{-0.6}\left(\frac{\mathrm{Re}_d \,\mathrm{Pr}_f}{R}\right)^{0.8} \quad (30)$$

with:

$$Ph = Cp_f \left(T_r - T_p\right) / \left(h_g - h_f\right) \quad (31)$$

$$\mathrm{Re}_d = \frac{\rho_f u d}{\mu_f} \quad (32)$$

$$\mathrm{Pr}_f = \frac{\mu_f Cp_f}{k_f} \quad (33)$$

$$R = \sqrt{\rho_f \mu_f / \rho_g \mu_g} \quad (34)$$

- **Evaporator mode**: Takamatsu's Correlation [5]

$$h_{rp} = h_{rpL}\left(16000\ \ Bo + \frac{6.3745}{X_{tt}}\right) \quad (35)$$

Where:

$$h_{rpL} = 0.023\ \frac{k_f}{D} \mathrm{Re}_L^{0.8}\ \mathrm{Pr}_f^{0.4} \tag{36}$$

With:

$$\mathrm{Re}_L = \rho u(1-x)D/\mu_f \tag{37}$$

$$Bo = h_{rp}(T_p - T_r)/\rho u(h_g - h_f) \tag{38}$$

$$X_{tt} = \left(\frac{1-x}{x}\right)^{0.9}\left(\frac{\rho_g}{\rho_f}\right)^{0.5}\left(\frac{\mu_f}{\mu_g}\right)^{0.1} \tag{39}$$

X_{tt} the Martinelli's parameter.

2.2.2 The secondary fluid (water)

We assume that the incompressible fluid flow is unidirectional. In addition, we neglect axial conduction and viscous dissipation. The equation of energy conservation in the secondary fluid can be written:

$$\rho_e Cp_e\left(\frac{\partial T_e}{\partial t} + v\frac{\partial T_e}{\partial z}\right) = \frac{4D_{1P}h_{ep}}{D_{1A}^2}(T_p - T_e) \tag{40}$$

Expression of h_{ep}

The correlation proposed by Ravigurajan and Bergle[7] that take into account the internal geometry of the inner tube is used:

$$h_{ep} = \frac{k_e Nu_a}{D_e} \tag{41}$$

$$Nu_a = Nu_s\left\{1+\left[2.64\mathrm{Re}^{0.036}\left(\frac{e}{D_e}\right)^{0.212}\left(\frac{p}{D_e}\right)^{-0.21}\left(\frac{\alpha}{90}\right)^{0.29}\mathrm{Pr}^{0.024}\right]^7\right\}^{1/7} \tag{42}$$

$$Nu_s = \frac{f\ \mathrm{Re}\ \mathrm{Pr}}{2\left[1+12.7\sqrt{f/2}\left(\mathrm{Pr}^{2/3}-1\right)\right]} \tag{43}$$

$$f = (1.58\ \ln\ \mathrm{Re} - 3.28)^{-2} \tag{44}$$

Where:

The index *a* refers to twisted tube, *s* index for smooth tube, and *f* the friction coefficient.

2.2.3 The inner wall

The equation of conservation of energy applied to the inner wall can be written assuming a constant thermal conductivity

$$\rho_p Cp_p\frac{\partial T_p}{\partial t} - k_p\frac{\partial^2 T_p}{\partial z^2} = \frac{-\pi D_{1P}h_{ep}}{A_p}(T_p - T_e) + \frac{\pi D_{2P}h_{rp}}{A_p}(T_r - T_p) \tag{45}$$

2.3. Use of COMSOL Multiphysics

PDE module coupled with two heat transfer modules (solid, liquid) are used to solve our equations. PDE module solves the flow through the coolant in the annular space of the heat exchanger, the heat transfer module liquid used to solve the equations of water flow, while the solid Heat Transfer interface is used to solve the heat transfer within the tube inside the exchanger. The properties of the refrigerant are introduced into Comsol sub array and obtained with the software Refprop in the range of pressure and enthalpy used.

Boundary conditions:

Water: Temperature and flow are known to the input (z = 0).

Refrigerant: Temperature, pressure and flow rate are known to the input (z = 0).

Inner tube: Heat flux is zero at the boundary.

The function "flc2hs" is used to couple single phase flow and two-phase flow. The input variables of the model are: The enthalpy, pressure, density and flow rate of the refrigerant, inlet temperature and flow rate of the water. These outputs: enthalpy, pressure and flow rate of the refrigerant, outlet temperature of the water.

3. Results

We present the results of the heat exchanger in the heating mode when it acts as a condenser. Tableau 1 shows the exchanger's parameters. To obtain the experimental results, we run the heat pump for 20 min. This experiment is repeated several times to ensure the representativeness of the results. The experimental apparatus used for this purpose is shown in Figure 10.

Tableau 2. Exchanger's parameters

Characteristic	Unit	Value
L	m	6,30
De	mm	33,75
D_{1A}	mm	25,00
D_{1P}	mm	51,61
D_{2A}	mm	34,33
D_{2P}	mm	53,80
D_3	mm	38,51
e	mm	4,20
p	mm	13,51
θ^*		0,75
ρ_p	Kg/m^3	8300
k_p	W/m.K	419
C_f	J/Kg.K	372

3.1. Vapor quality

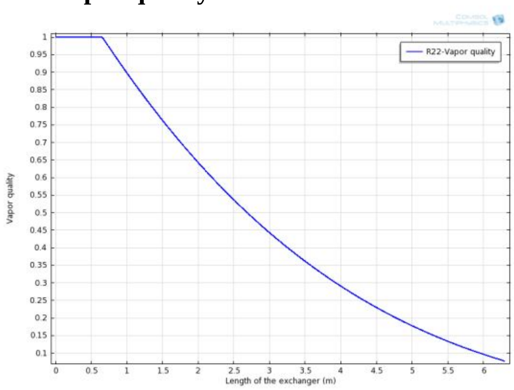

Figure 5. Vapor quality of the refrigerant

3.2. Enthalpy

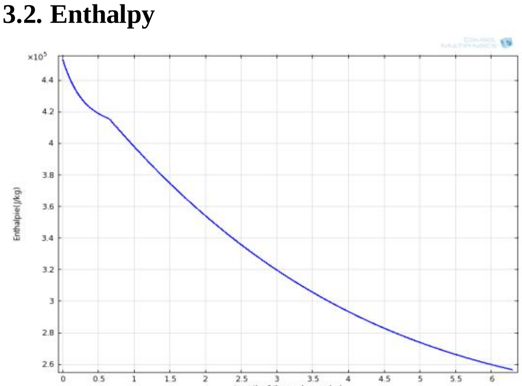

Figure 6. Variation of enthalpy along the exchanger

3.3. Pressure

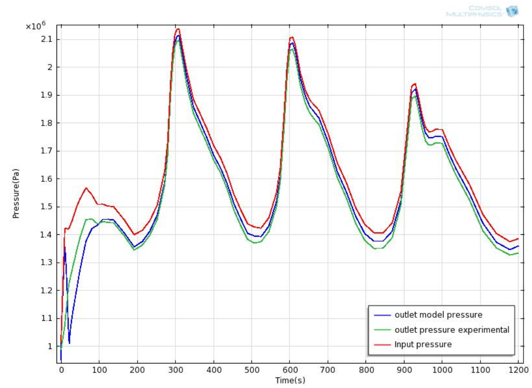

Figure 7. Comparing pressure

3.4. Water temperature

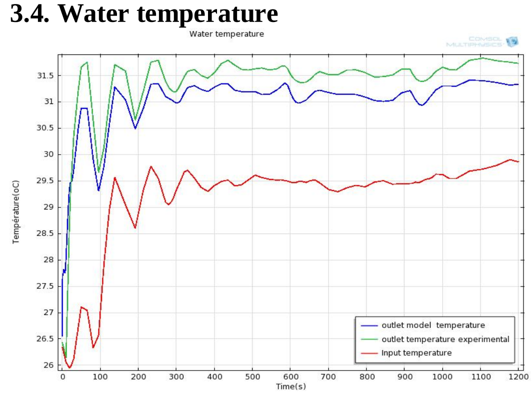

Figure 8. Comparing (water temperature)

3.5. Enthalpy

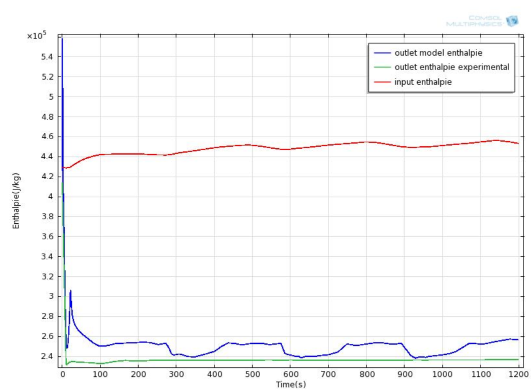

Figure 9 . Comparing enthalpy

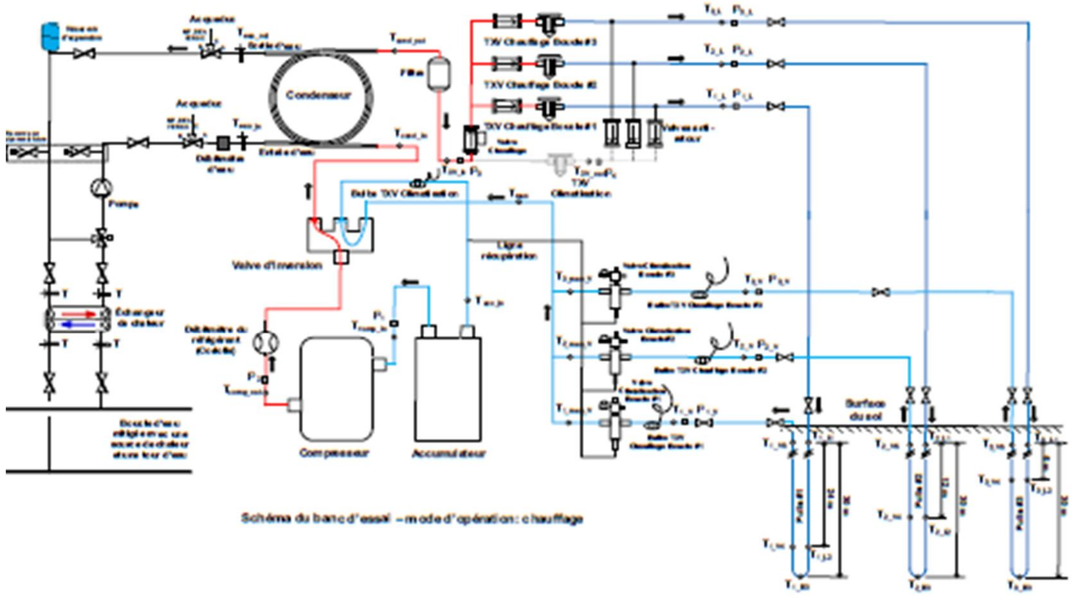

Figure 10. Experimental diagram (heating mode)

4. Conclusions

The numerical model of water-refrigerant heat exchanger with Comsol is developed by solving equations of mass, momentum, and energy conservation in one dimension for the coolant. The validated numeric model obtained will be coupled with the ground heat exchanger for developing the global numerical model of direct expansion geothermal heat pump (Figure 1).

5. References

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