Modeling of 3D Cilium Mechanics using Beam Physics with Extrusion Coupling

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INTRODUCTION: Cilia are slender micro-organelles (200 nm diameter, 10 µm long) that generate propagating waves to propel cells or move fluid. Beating may involve a dynamic instability. The goal of this research is to build a 3D model that can capture modes of instability under different loading and coupling conditions.

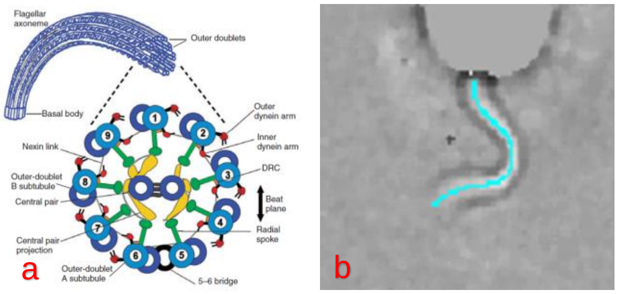

Figure 1. (a) Cytoskeletal structure of cilia. From ref [1].
(b) Cilia under microscope (courtesy M. Bottier)

COMPUTATIONAL METHODS: The cytoskeletal structure is simplified into either a “4-doublet” model or “6+1” model: 6 outer beams and 1 central beam.

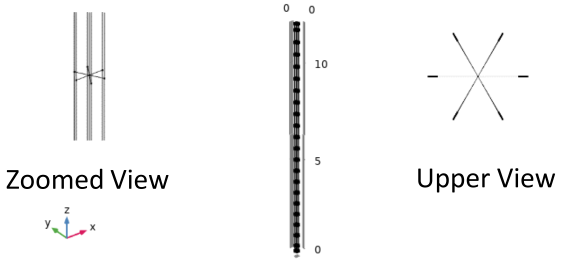

Figure 2. Simplified 6+1 cytoskeletal structure in COMSOL.

Two modeling techniques were used to couple the beams (simulating nexin links and radial spokes): *Beam Structures* were used as links (radial spokes) between the central beam and outer beams. Beams provide radial, tangential, and torsional stiffness. *Linear Extrusion Coupling* was used to couple the outer beams, simulating the effects of nexin links. Extrusion coupling defined a viscoelastic relationship

$$F = kx(1+\alpha\varepsilon^2) + c\frac{dx}{dt}$$

Forces from the motor protein dynein were modeled by follower tangential loads and moments.

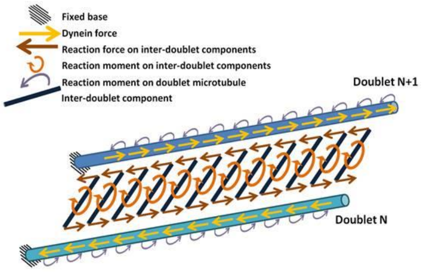

Figure 3. External loading of adjacent beams (doublets) of the model [2].

RESULTS: The COMSOL 3D Beam Physics model with Extrusion Coupling allowed us to compare a full nonlinear simulation with a theoretical prediction based on linear (small-amplitude) approximations.

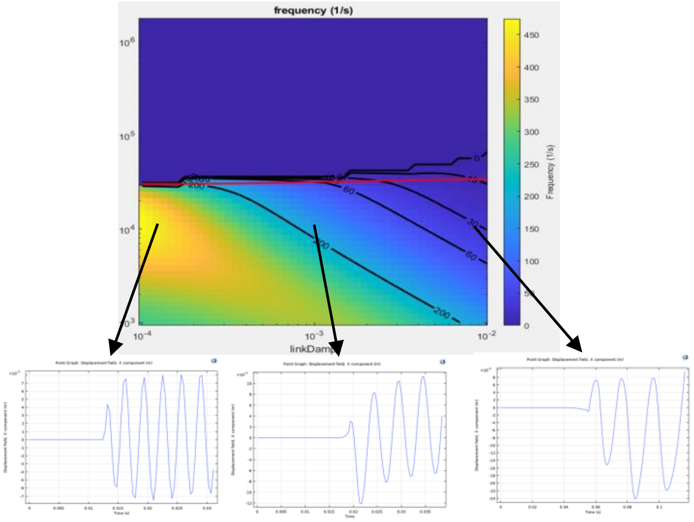

Figure 4. (top) Theoretical (small-amplitude) prediction of the effect of damping coefficient and load amplitude on oscillation frequency . (bottom) Corresponding plots of tip displacement from COMSOL simulations.

Two characteristic dynamic instabilities were observed with the model: *divergence* and *flutter*. Divergence is a static instability in which the beam stabilizes in a new, deformed, equilibrium position.

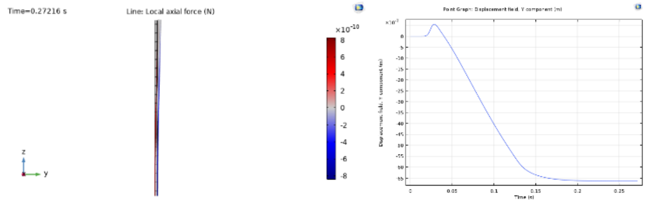

Figure 5. Divergence Mode Shape and Axial Load Plot and Divergence Tip Displacement Plot

In fluttering, the beam loses stability in an oscillatory manner and generates a propagating waveform.

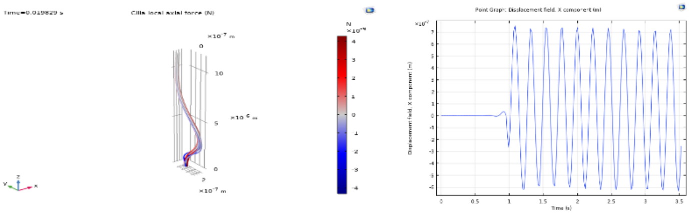

Figure 6. Fluttering Mode Shape and Axial Load Plot and Fluttering Tip Displacement Plot

CONCLUSIONS: The cilia model implemented in COMSOL using 3D beam physics with extrusion coupling leads to oscillatory behavior like that observed in cilia. This model can test the effects of different loading conditions and coupling parameters.

REFERENCES:

1. C.B. Lindemann and K.A. Lesich, Flagellar and ciliary beating: the proven and the possible, Journal of Cell Science, (2010)
2. Tianchen Hu and Philip Bayly, Finite element models of flagella with sliding radial spokes andinterdoublet links exhibit propagating waves under steadydynein loading, Cytoskeleton, (2018)

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