

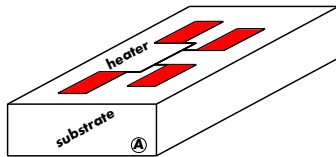
Numerical analysis of the 3ω -method

Comsol Conference, Stuttgart

Manuel Feuchter, Marc Kamlah | October 26, 2011

INSTITUTE FOR APPLIED MATERIALS (IAM)

- 1 Introduction
- 2 Functionality of 3ω -method
- 3 Geometry configurations
- 4 Numerical analysis
- 5 Outlook



Concept of the 3ω -method :

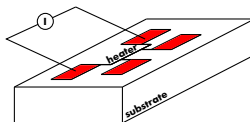
- The 3ω -method is an ac technique to determine the thermal conductivity of an amorphous solid.
- The 3ω -measurement is performed on a thin electrical conducting metal strip (heater) deposited on the sample surface.
- Literature: [Cahill & Pohl 1987], [Cahill 1990], [Lee & Cahill 1997], [Kim & Feldman 1999], [Borca-Tasciuc 2001], [Chen 2004], [Olson 2005]...

Why do we work on the 3ω -method ?

- Involved in DFG SPP 1386 - Understanding size and interface dependent anisotropic thermal conduction in correlated multilayer structures.
- Goal: Minimize thermal conductivity in nanostructured thermoelectric materials.
- Reliable measurement technique is required.

Functionality of 3ω -method

Alternating current

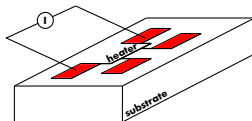


$.. \cos \omega ..$

$I \quad \omega$

Functionality of 3ω -method

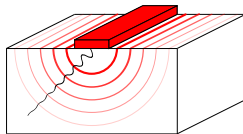
Alternating current



$\dots \cos \omega \dots$

$$I \quad \omega$$

Joule heating
 $P = R \cdot I^2$

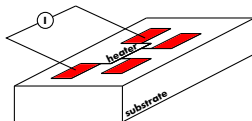


$\cos^2 = \dots \cos 2\omega \dots$

$$P \quad 2\omega$$

Functionality of 3ω -method

Alternating current

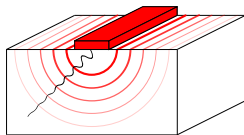


$$I \cos \omega t$$

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Joule heating

$$P = R \cdot I^2$$



$$P \cos^2 \omega t = P \cos 2\omega t$$

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Temperature amplitude ΔT

Analytic solution

Resistance oscillation

FE-Simulation

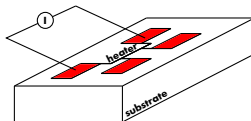


$$R = R_0 + \Delta R \cos 2\omega t$$

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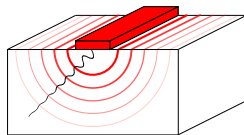


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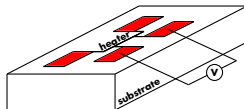


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Modulated voltage

$$U = I \cdot R$$

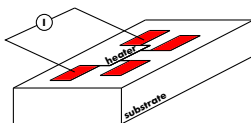


$$U \cos \omega t \cdot \cos 2\omega t = U \cos \omega t \cdot \cos 3\omega t$$

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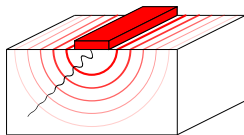


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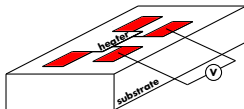
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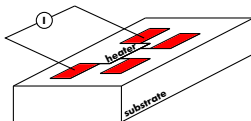


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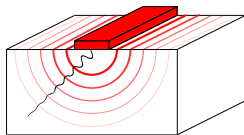


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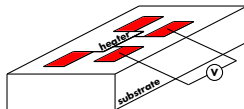
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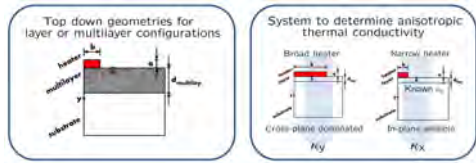
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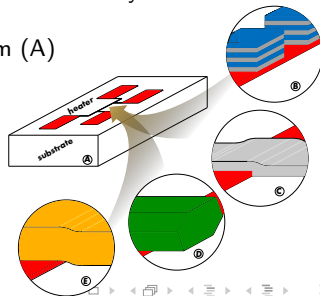


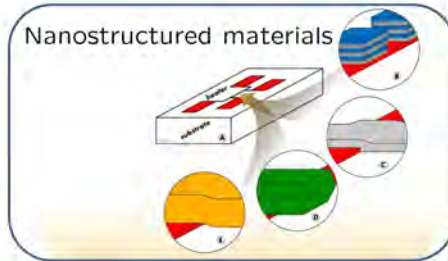
Classic geometry configuration:

- Heater is placed on top of (multi)layer-substrate systems.
- Analytic solutions are available.

Inverse geometry configuration:

- New possibilities to determine the thermal conductivity of nanostructured materials.
- Always the same heater-substrate platform (A) as sample holder.
 - Known system properties.
 - Optimum reproducibility.
- **No** analytic solutions are available.





Measurements on investigated materials

Input:
P/I at certain f_c applied on produced sample

Observed:
 ΔT

Finite Element Simulations

to create a data basis for various influence factors on ΔT

Input parameters:
P/I, f_c , a, b, d_{lay} , $\kappa_{lay \perp}$, $\kappa_{lay \parallel}$, κ_{subs}

Output parameter:
 ΔT

Neural Network

used to solve the inverse problem
to obtain κ from ΔT

Input:
 ΔT from measurement

Output:
emergent κ

- Heat conduction equation has to be solved in the time domain.
- Mathematic interface is used in the general form of the partial differential equation (PDEg).

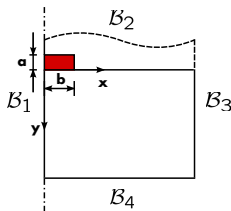
Example for 2d:

$$Q = [\rho_0 \cdot (1 + \alpha \cdot (T - T_0))] / (A) \cdot l_0^2 \cdot \frac{1}{2} \cdot (1 + \cos(2\omega t))$$

$$\frac{\partial^2 T_h}{\partial x^2} + \frac{\partial^2 T_h}{\partial y^2} - \frac{1}{D_h} \frac{\partial T_h}{\partial t} + \frac{Q}{\kappa_h} = 0,$$

$$\frac{\partial^2 T_s}{\partial x^2} + \frac{\partial^2 T_s}{\partial y^2} - \frac{1}{D_s} \frac{\partial T_s}{\partial t} = 0,$$

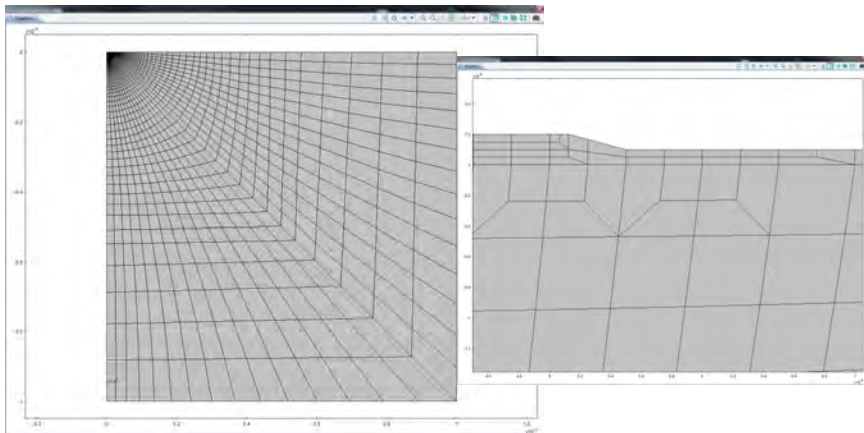
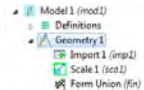
$$\kappa_{m_x} \frac{\partial^2 T_m}{\partial x^2} + \kappa_{m_y} \frac{\partial^2 T_m}{\partial y^2} - \rho_m c_{p_m} \frac{\partial T_m}{\partial t} = 0$$



$$\kappa_n \frac{\partial T_s}{\partial n} - h_{bc} \cdot (T - T_0) = 0$$

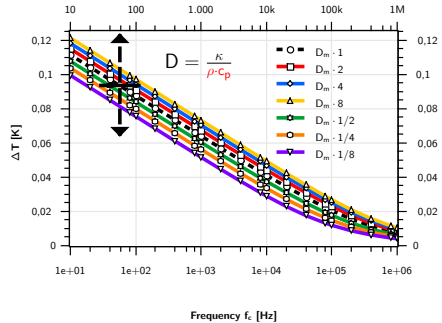
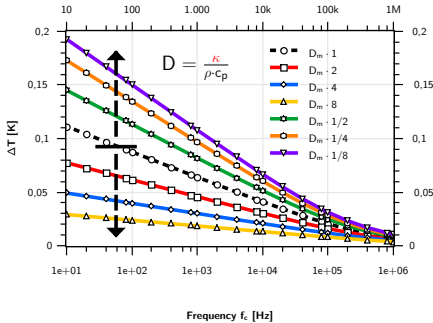
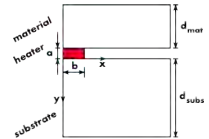
Geometry import to generate mesh

Example for 2d:



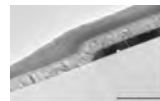
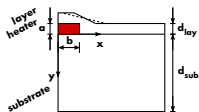
Placed material on top, ideal system:

- Two times substrate, contact just over heater.
 - To study the influence of material parameters on the temperature amplitude.



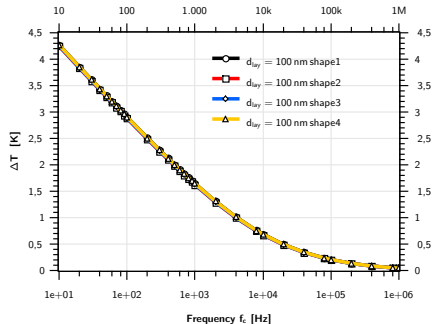
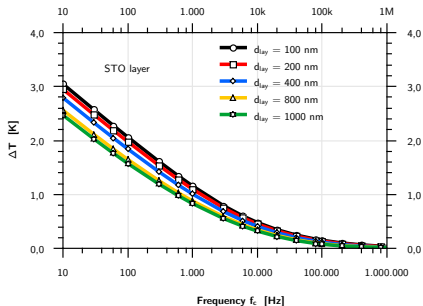
Thin layer on top:

- Bad conducting substrate for a sensitivity in ΔT .
- Contour shape analysis.



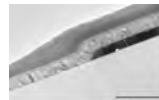
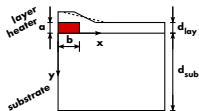
500 nm

By S. Wiedigen, project partner



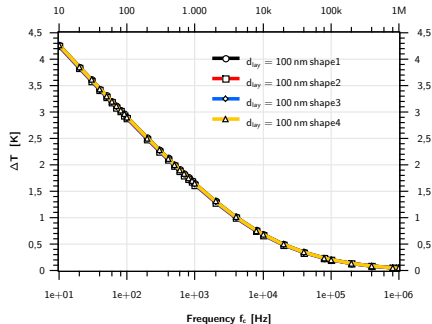
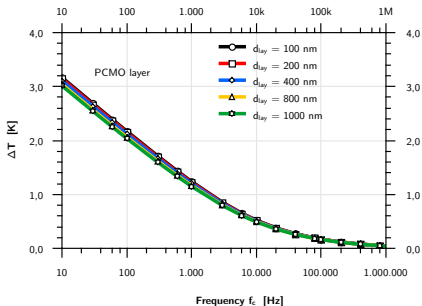
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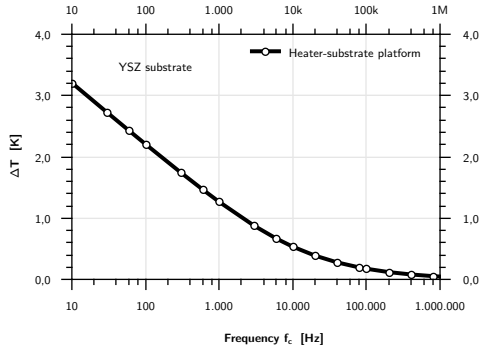
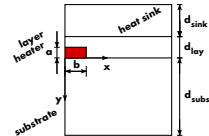


How to enhance the sensitivity for change in κ_{lay} ?

Examples

Application of heat sink:

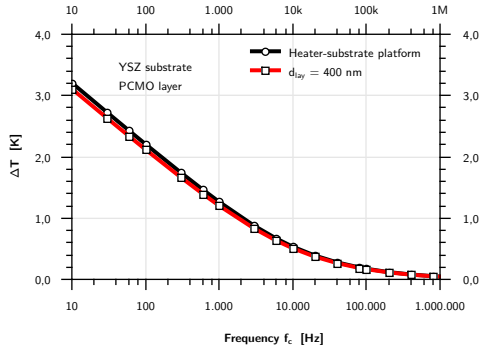
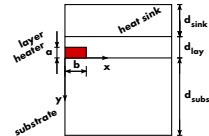
- Heat sink: Good thermal conducting to attract heat.
- Full covering heat sink.



Examples

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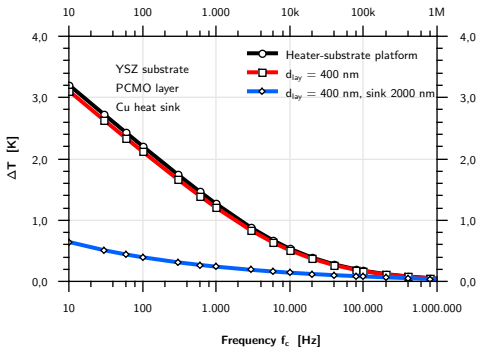
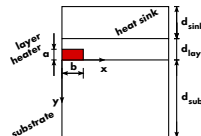
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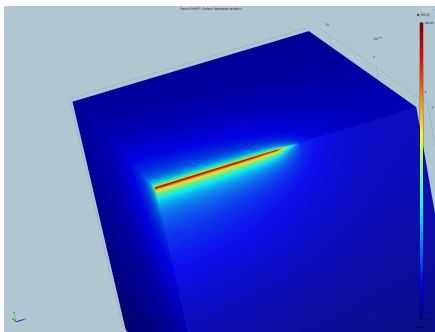
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- Heat flow analysis for 3d structures.
- Multiphysical coupling with respect to mechanical and dielectric properties.
- Bottom electrode configurations with additional heaters to measure Seebeck cross-plane.

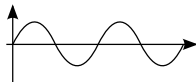


Acknowledgement:

- DFG for funding.
- Project partners from the University of Göttingen,
Institute for material physics
Group of C. Jooss and C. Volkert
- Supervisor M. Kamlah

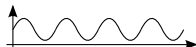
THANK YOU FOR YOUR ATTENTION!

Alternating current



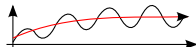
$$I = I_0 \cos(\omega t)$$

Joule heating



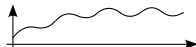
$$P_{JH} = \frac{P_0}{2} (1 + \cos(2\omega t))$$

Temperature oscillation



$$\Delta T(t) = \Delta T \cos(2\omega t + \varphi)$$

Resistance oscillation



$$R = R_0 + \Delta R \cos(2\omega t + \varphi)$$

Modulated voltage



$$U = I \cdot R$$

$$U = I_0 R_0 \cos(\omega t) + \frac{I_0 \Delta R}{2} (\cos(3\omega t + \varphi) + \cos(\omega t + \varphi))$$