ESTIMATION OF VARIABILITIES DUE TO STOCHASTIC VARIATION IN GEOMETRIC PARAMETERS IN MICROWAVE APPLICATIONS

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INTRODUCTION

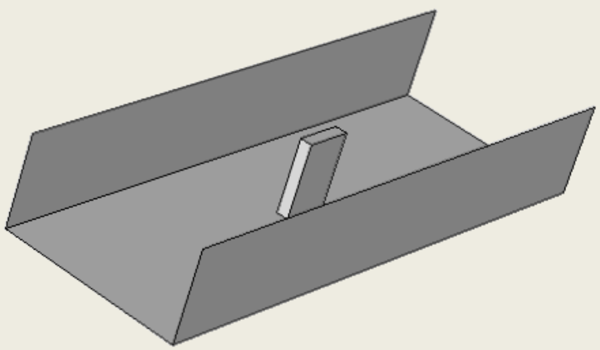
- Predicting the stochastic behaviour of electromagnetic systems is important in microwave circuit, especially
  - Terahertz
  - millimetre wave circuits
  - resonating structures
  - substrate integrated waveguides
- Computational cost using conventional methods high
  - variations due to the stochastic errors in the geometry and material properties
  - number of degrees of freedom is large.
  - Monte Carlo methods can be applied; involve estimating a large number of samples; potentially impractical
- Certain efficient intrusive methods can be applied for material variations
  - for geometric variation, these do not work well; require re-meshing.

INTRODUCTION (CNT.)

- The **stochastic collocation method** is a good candidate for geometrical variation assessment.
  - Involves evaluating a few samples to predict the system stochastic behaviour for general probability distribution of an input parameter.
  - It involves post processing the results from multiple runs of a simulation.
- The collocation uses the Lagrange interpolation scheme to realize the distribution of the transmission coefficient and resonant frequency of the EM system.
- The stochastic collocation method to estimate the geometric tolerance in microwave circuits using the Comsol Live Link™ for MATLAB.

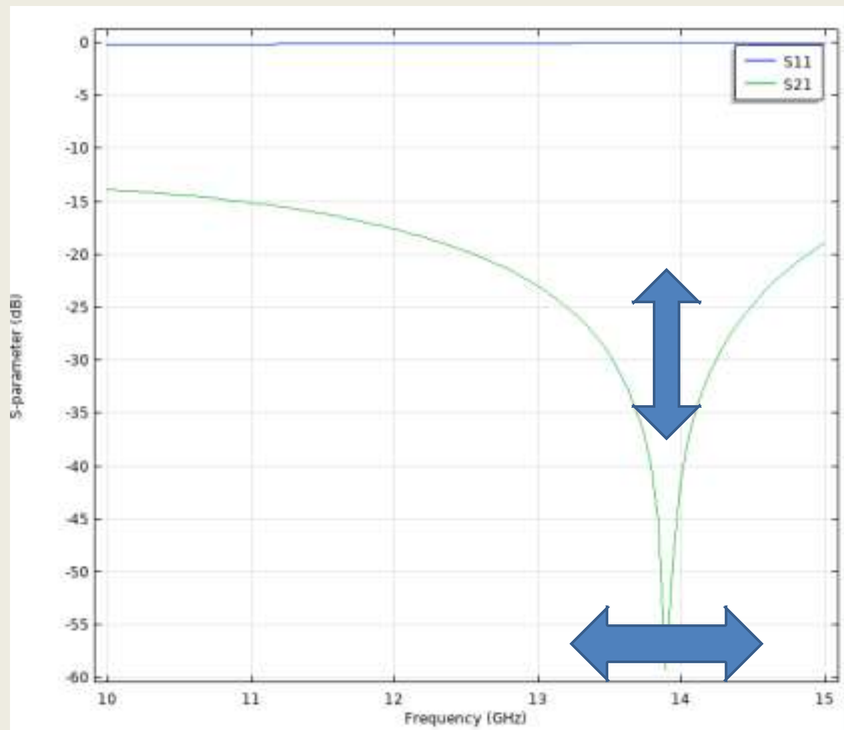
EXAMPLE PROBLEM: WAVEGUIDE WITH A POST

Waveguide Dimensions	
a	19.05 mm
Width	9.254 mm

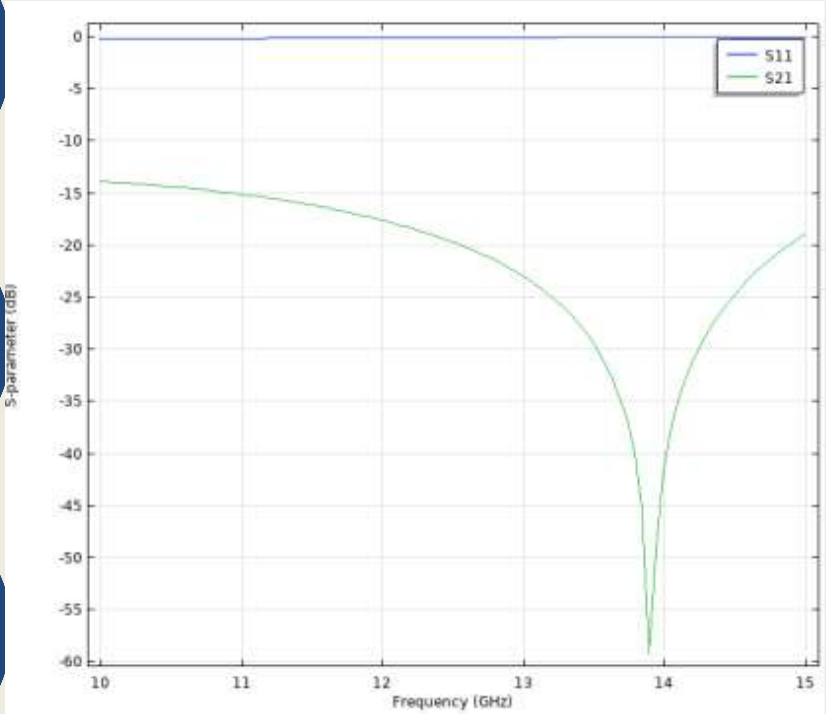


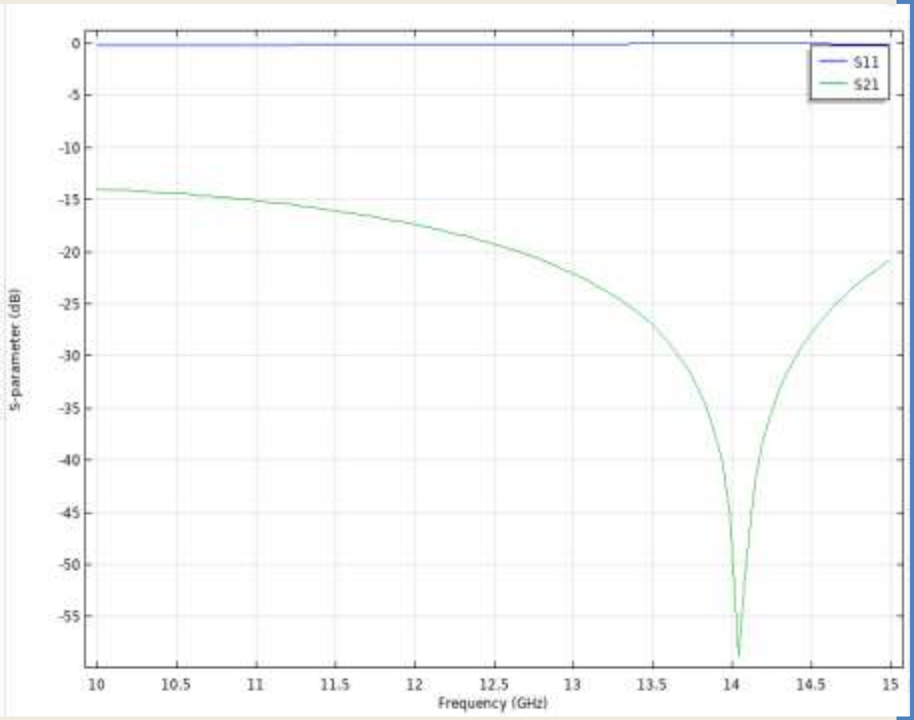
Post Dimensions	
Height	7.619 mm
Thickness	1.016 mm
Width	5.08 mm

COMSOL simulation



VARIATION IN POST THICKNESS





Post Dimensions	
Height	7.619 mm
Thickness	1.016 mm
Width	5.08 mm

Post Dimensions	
Height	7.619 mm
Thickness	1.116 mm
Width	5.08 mm

STOCHASTIC COLLOCATION

- Stochastic collocation involves evaluating the problem at a few selected sample points and evaluating the dependence of that random variable using a polynomial interpolation or least square approach.

$$\boldsymbol{P(\xi) = \sum_{j=1}^{n} P_j(\xi)}$$

$$\boldsymbol{P_j(\xi) = y_j \prod_{k=1, k\neq j}^{n} \frac{\xi - \xi_k}{\xi_j - \xi_k}}$$

- where $\xi_1, \xi_2 \ldots\ldots \xi_k$ are the interpolation points, $p(\xi)$ is the polynomial approximation of the dependence of the system on the random variable and y_i is the system response at i^{th} collocation point.
- Generalised polynomial chaos (gPC) based stochastic approximations are used.
- The collocation points are chosen for different distributions based on different quadrature rules as per the Wiener-Askey scheme.
- For higher dimensional problems a nested alternative of these nodes is preferred to generate sparse grids for faster calculations.

LEAST SQUARE APPROACH IN SC

- An alternate approach is to treat this as a least square problem, which is mathematically equivalent and yields the same results.
  - Get data pair for n points, $\{(x\_1,y\_1),(x\_2,y\_2)\ldots.(x\_n,y\_n)\}$.
  - Construct the least square matrix from polynomial chaos expansion.
  - Find the polynomial coefficients from the given data pair.
  - Assuming the system response be the PCE as follows,

$$f(\xi) = \sum_{i=1}^{P} c_i \Phi(\xi)$$

- From the collocation points generate the least square system as below,

$$\begin{bmatrix} \Phi_1(x_1) & \Phi_2(x_1) & \ldots & \Phi_P(x_1) \\ \Phi_1(x_2) & \Phi_2(x_2) & \ldots & \Phi_P(x_2) \\ \vdots & \vdots & \ldots & \vdots \\ \Phi_1(x_n) & \Phi_2(x_n) & \ldots & \Phi_P(x_n) \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_P \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$$

- The solution will yield the values of the PCE coefficients, which can be used for post processing of the data

WORKFLOW

Design and Modelling using COMSOL Multiphysics

Simulation and deterministic Result

Comsol Model and Solver settings as mph file

COMSOL Server with Matlab Connection

Matlab

- Probability Distribution
- Collocation points
- Model parameter altering
- Re-mesh and Solving Instructions
- Sample Collection
- Post Processing
- Plotting Distribution

MATLAB INTERFACE WITH COMSOL

```
mphmodel= mphload([PATHS.comsol_path,'\Center_Post.mph’]);
mphmodel.param.set('W',W);
geom1=mphmodel.geom.get('geom1');
geom1.run;
mesh1 = mphmodel.mesh.get('mesh1');
mesh1.run;
mphmodel.sol('sol1').run;
dat = mpheval(mphmodel,{'abs(emw.S21)'},'selection',1);
s21_sc(:,i)=dat.d1(:,1);
```

Reading the model

Setting Parameters

Geometry update

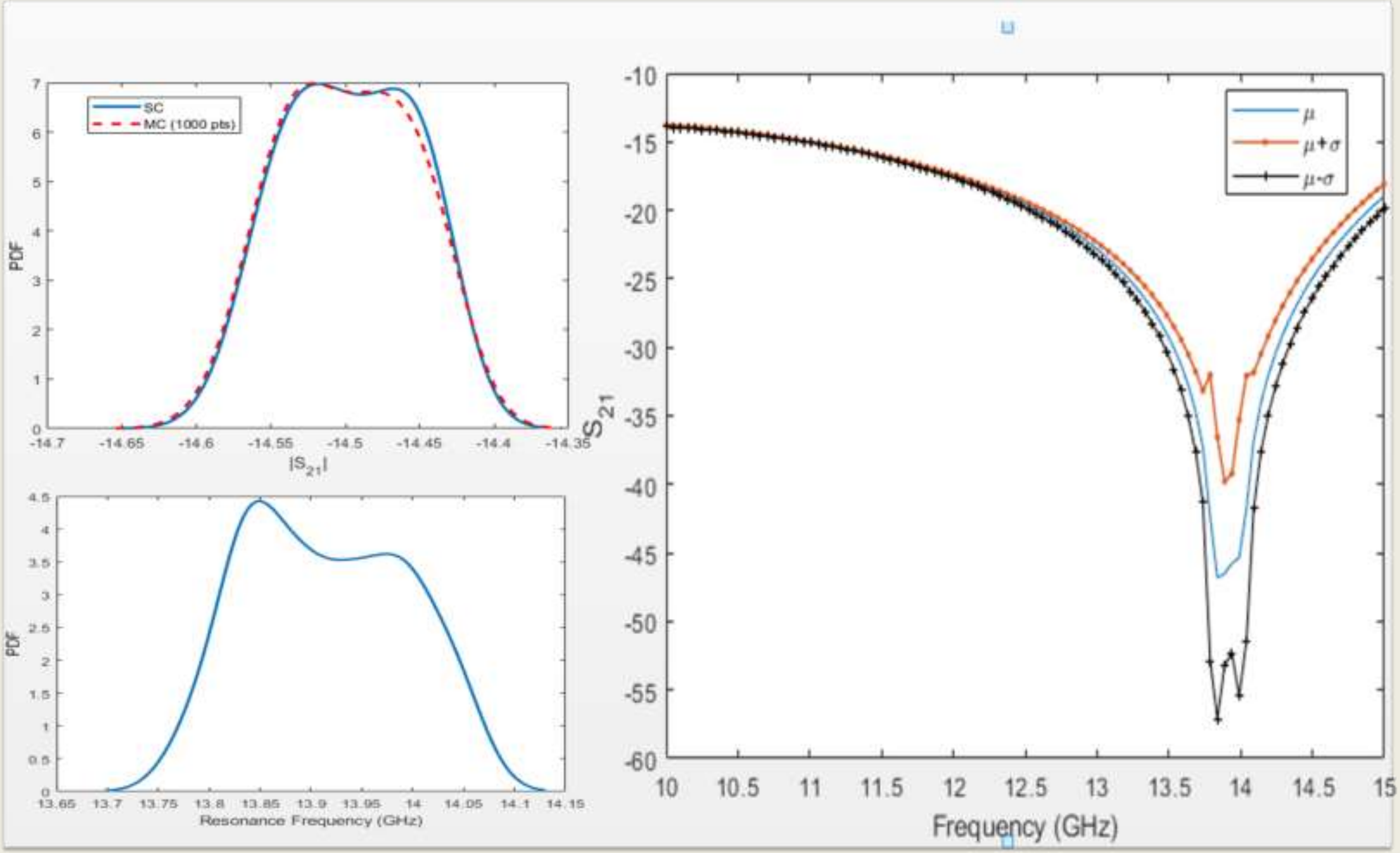
Mesh Update

Simulate

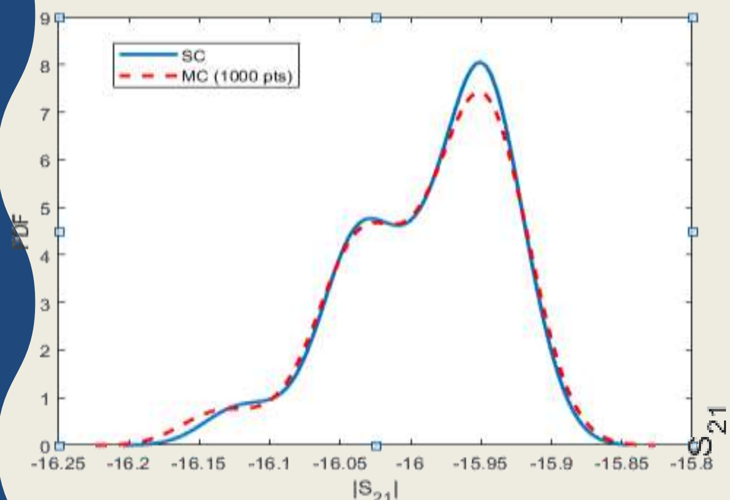
Extract Result

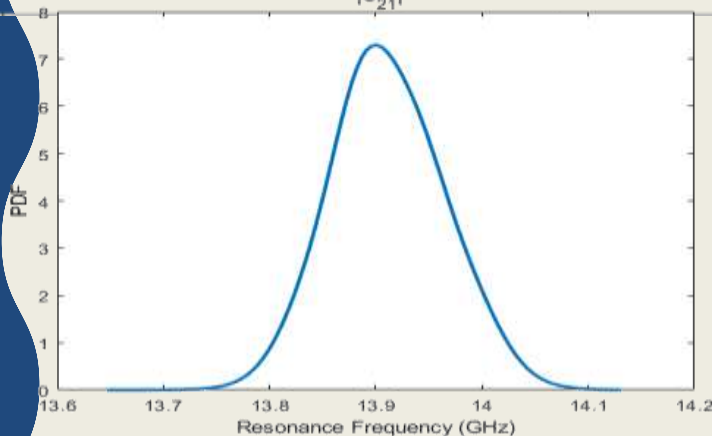
Post processing in Matlab involving weight function and least square matrix creation with polynomial chaos

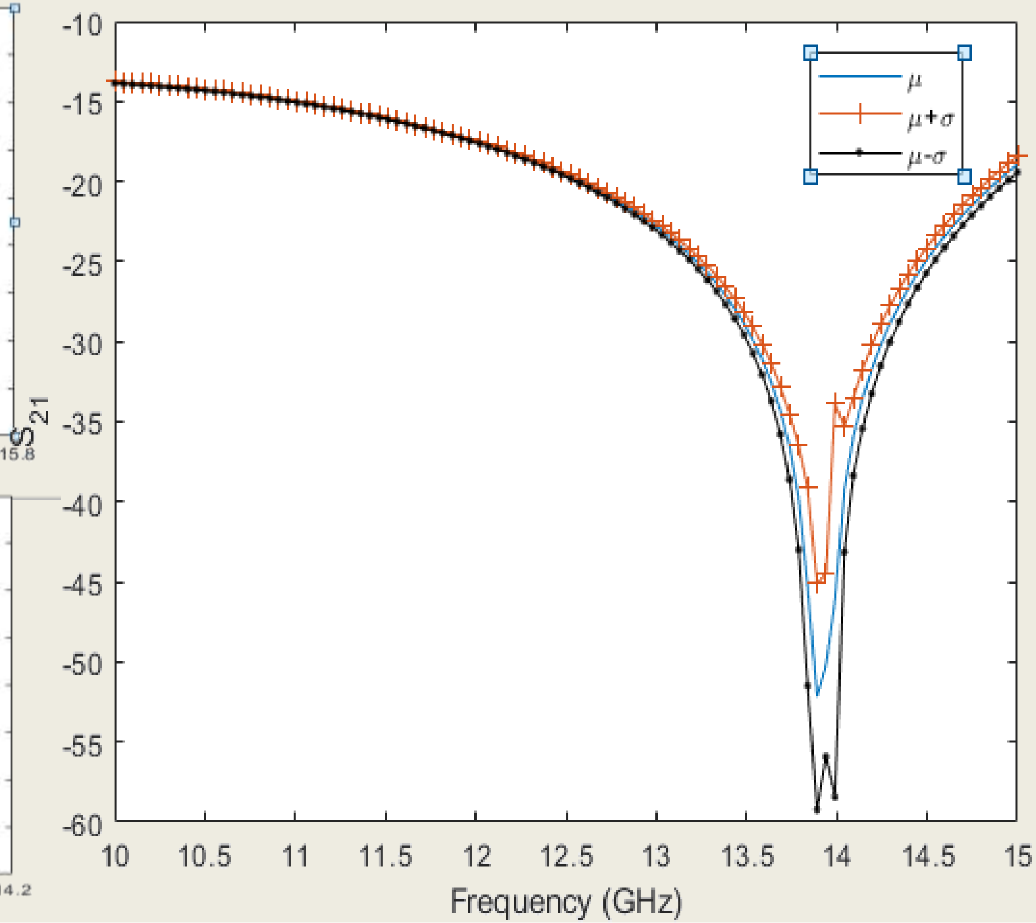
PROBABILITY DISTRIBUTION (UNIFORM DISTRIBUTION)



PROBABILITY DISTRIBUTION (GAUSSIAN DISTRIBUTION)







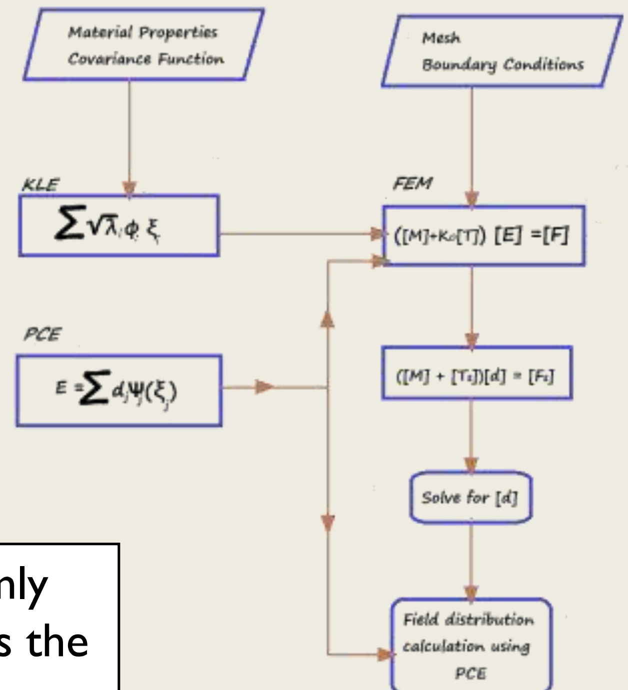
SSFEM: STOCHASTIC ANALYSIS IN A SINGLE MATRIX SOLVE

$$\left[[M^e] + k_0^2 \epsilon_r [T^e]\right][E] = [b^e]$$

- Material property Varying with a probability distribution
- Varies over space as well
- Represented using KL expansion

- As a result electric field varies randomly with a probability distribution which is the unknown in the system.
- Represented as a polynomial chaos expansion

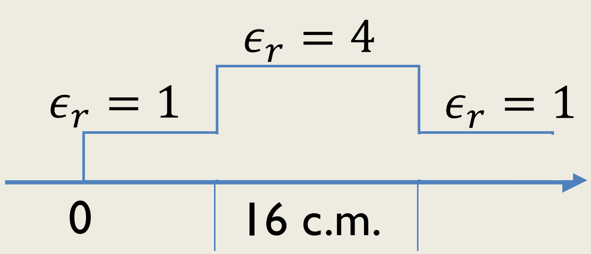
Spectral Stochastic Finite Element Method

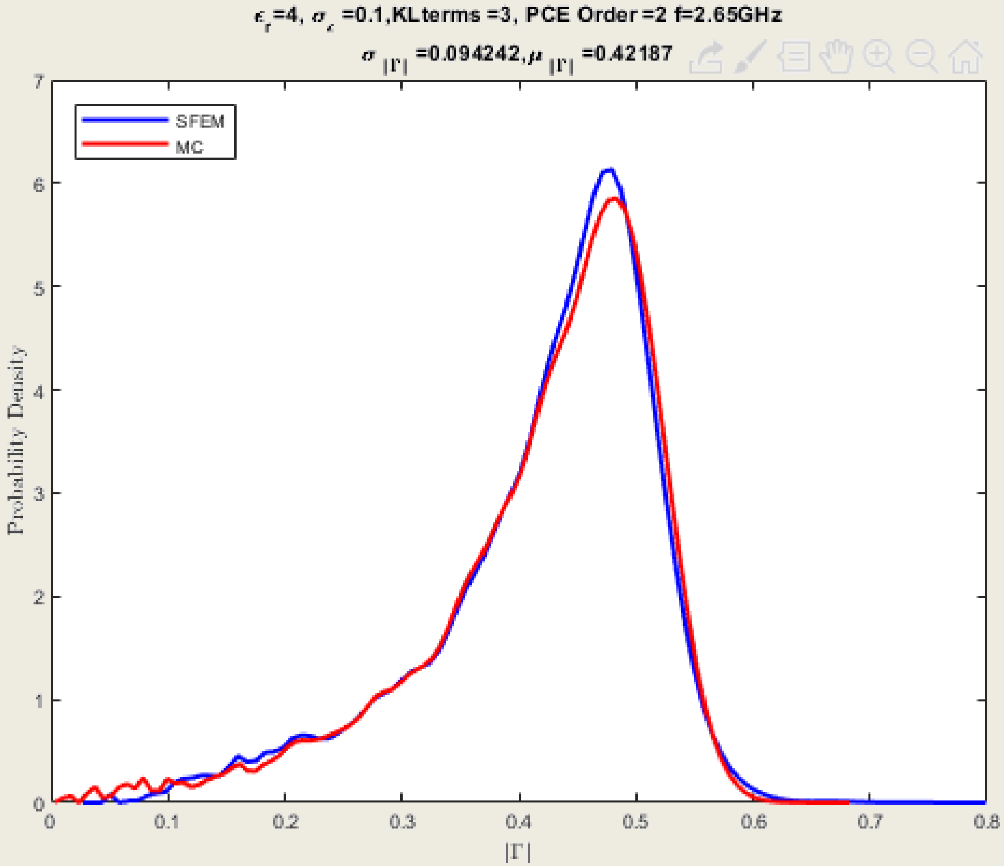


CHALLENGES

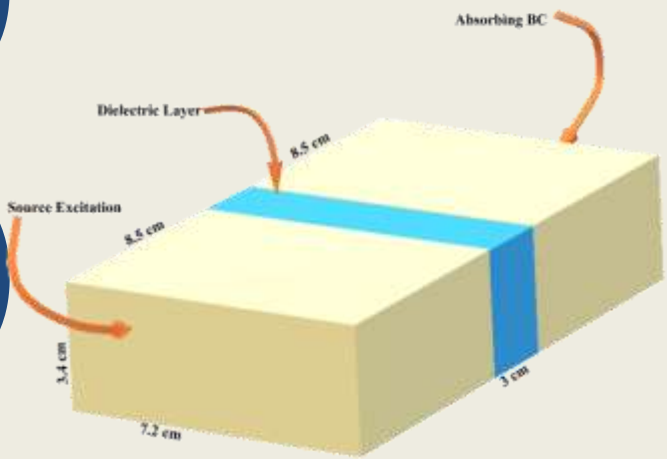
- System matrix need to be altered
- A larger matrix is obtained
- This is still sparse so solving complexity does not increase much if FEM based
- For 1 D Helmholtz system matrix extracted from comsol can be used to do the analysis.
- In 3D models unable to extract and map the matrix to geometry

1-D HELMHOLTZ EQUATION

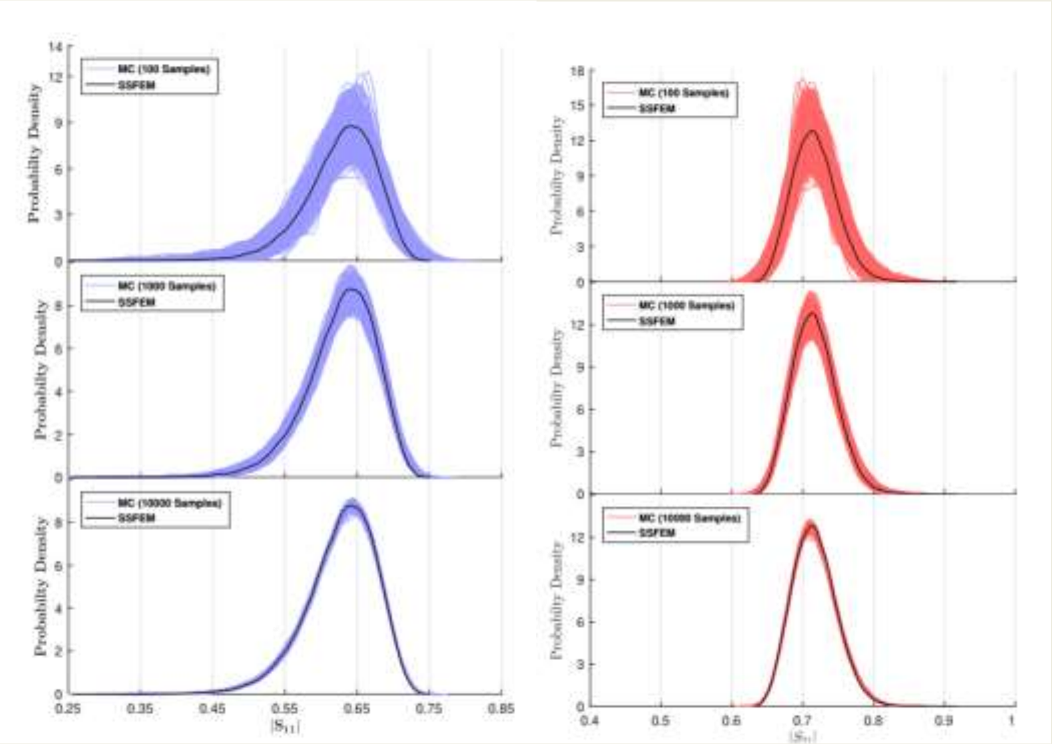




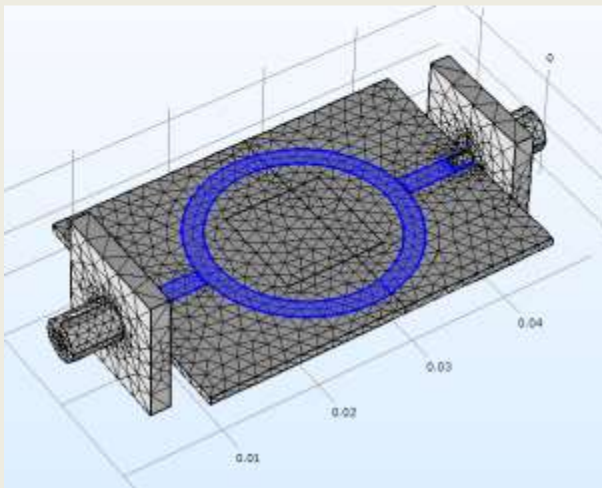
A SAMPLE PROBLEM IN 3D

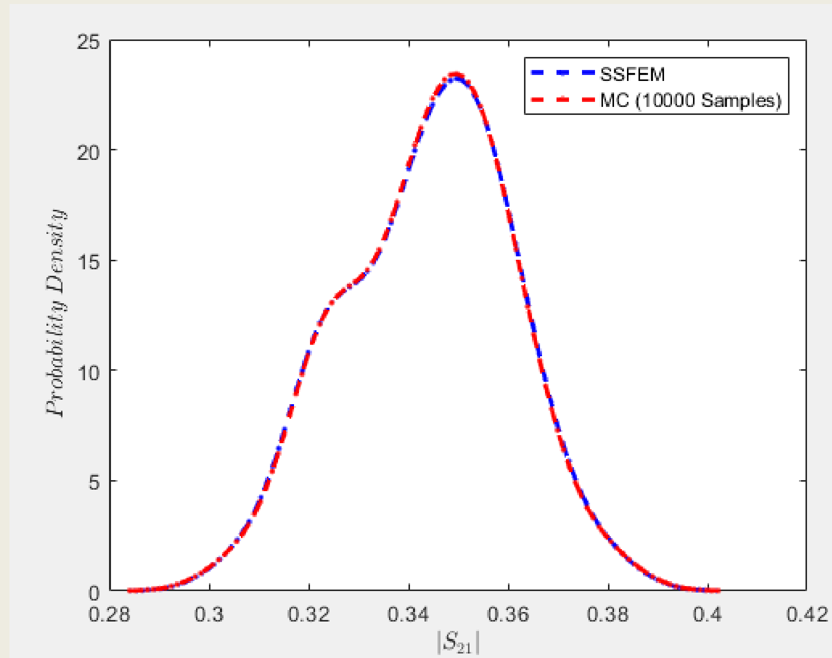


Mean $\epsilon_r' = 4.3$, stdev = 0.3
Mean $\epsilon_r'' = 0.2$, stdev = 0.035



MICROSTRIP LINE





Mean $\epsilon_r' = 4.3$, stdev = 0.25
Mean $\epsilon_r'' = 0.02$, stdev = 0.003

CONCLUSION

- Comsol Multiphysics with Matlab Livelink is used to perform stochastic analysis on electromagnetic model.
- Non-intrusive analysis can be done using the Comsol model as base solver and the intermediate.
- The post processing is handled using Matlab.
- Sensitivity analysis is done over a broadband to obtain the distribution and sensitivity of transmission parameters.