

Visualization and Exploration of the Dynamics of Phase Slip Centers in Superconducting Wires

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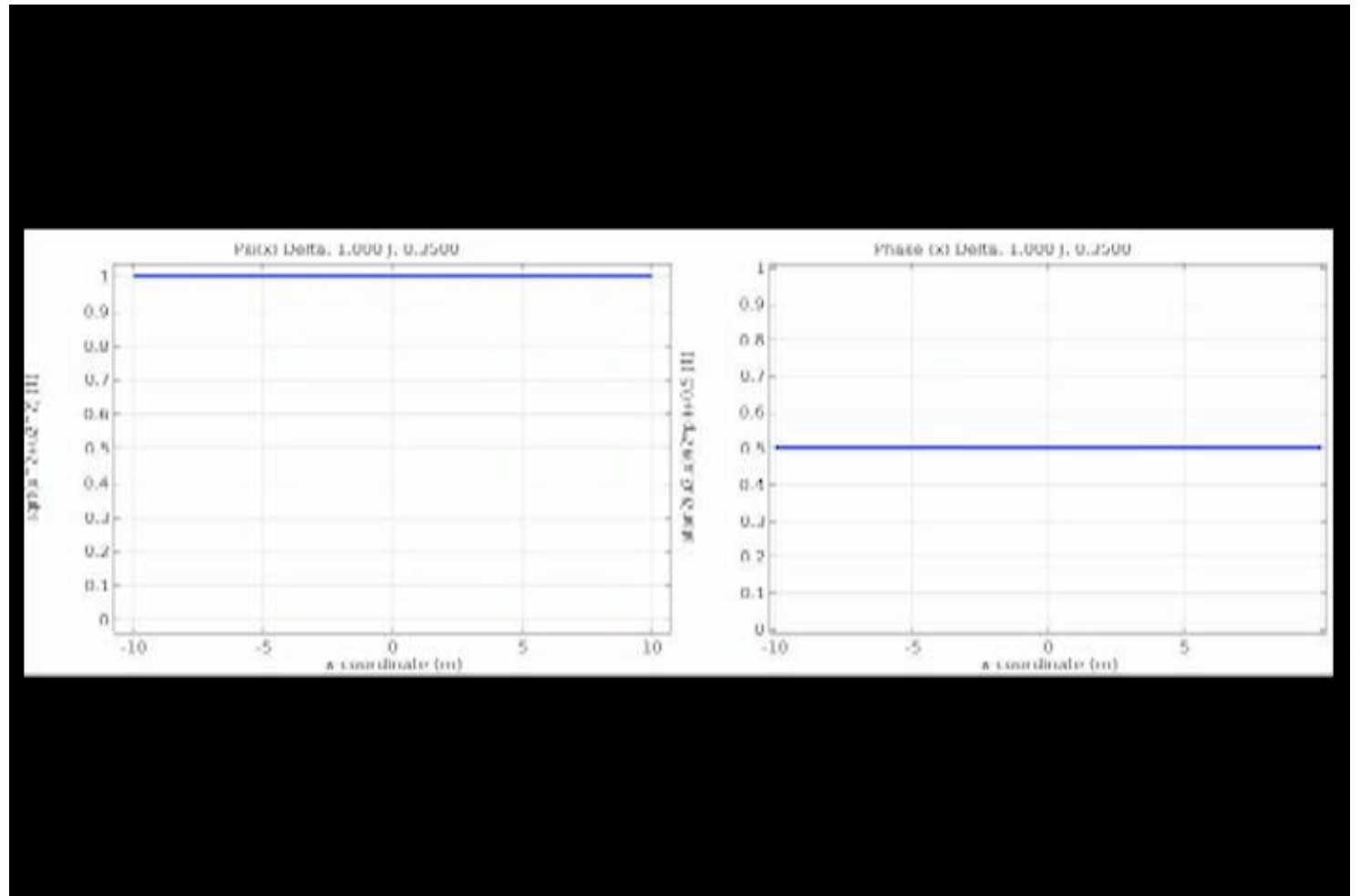
Introduction

- Goal: Visualizing the dynamics of phase slip centers in a 1D model of superconducting wire based on the set of time-dependent Ginzburg-Landau equations (TDGL)
 - COMSOL Multiphysics® General Form PDE interface
 - *Unique to this study*: The set of TDGL equations for superconductors with finite gap was used in full.
 - We took into account interference with normal and superfluid motions.

Physics

Phase Slippage:

- Phase difference of the order parameter between the ends of the superconducting wire.
- The density of pair condensate, governed by the order parameter, must vanish at certain times.
- This results in oscillations in the density of pair condensates and the current.



Motivation

- Advantages of Modeling in COMSOL®:
 - Tracing non-linear time dependent solutions and events, which occur in picoseconds.
 - Automation for exploration of various observables:
 - Cooper-pair density, superfluid and normal velocities, etc.
 - High quality animation of solutions.

Theory

Bardeen, Cooper and Schrieffer in the microscopic explanation of superconductivity¹ proposed that the quantum Ψ -function of the Ginzburg-Landau theory² at thermodynamic description of superconducting state may be related to the energy gap in the spectrum of paired electrons.

- Proven by Gor'kov³
- Equation for Ψ -function is not like the Schrödinger equation but rather had a diffusion character proven by Schmid⁴ and by Éliashberg and Gor'kov⁵ on the basis of the Green's function model of superconductivity.

References:

1. "Microscopic Theory of Superconductivity". Physical Review, (1957)
2. Zh. Eksp. Teor.Fiz. (1965)
3. On the energy spectrum of superconductors, Sov. Phys. JETP (1958)
4. A time dependent Ginzburg-Landau equation and its application to the problem of resistivity in the mixed state. Physik der kondensiertenMaterie, (1966)
5. Dynamical properties of gapless superconductors, J. Low Temp. Phys.(1970)



Theory

As shown in Gulian et al.(1989), for superconductors with finite gap the microscopic theory yields additional terms in the current which corresponds to the interference between superconducting and normal motions of electrons. These terms could be essential in many situations. However, they have not been taken into account in a vast amount of research articles. We have included them into our framework of TDGL equations in our studies.

Equations for Simulation: TDGL

- The Order Parameter $\Delta = |\Delta| \exp(i\theta)$

$$-\frac{\pi}{8T_c} \frac{1}{\sqrt{1 + (2\tau_\varepsilon |\Delta|)^2}} \left(\frac{\partial}{\partial t} + 2i\phi + 2\tau_\varepsilon^2 \frac{\partial |\Delta|^2}{\partial t} \right) \Delta + \frac{\pi}{8T_c} [D(\nabla - 2iA)^2] \Delta + \left[\frac{T_c - T}{T_c} - \frac{7\zeta(3)|\Delta|^2}{8(\pi T_c)^2} \right] \Delta = 0$$

- The Current Density with the Interference current terms:

$$j = \frac{\pi\sigma_n}{4T} \mathbf{Q} \left(|\Delta|^2 - \frac{1}{\gamma} \frac{\partial |\Delta|^2}{\partial t} \right) + \sigma_n \mathbf{E} \left\{ 1 + \frac{\sqrt{|\Delta|^2 + \gamma^2}}{2T} \left[K \left(\frac{|\Delta|}{\sqrt{|\Delta|^2 + \gamma^2}} \right) - E \left(\frac{|\Delta|}{\sqrt{|\Delta|^2 + \gamma^2}} \right) \right] \right\}$$



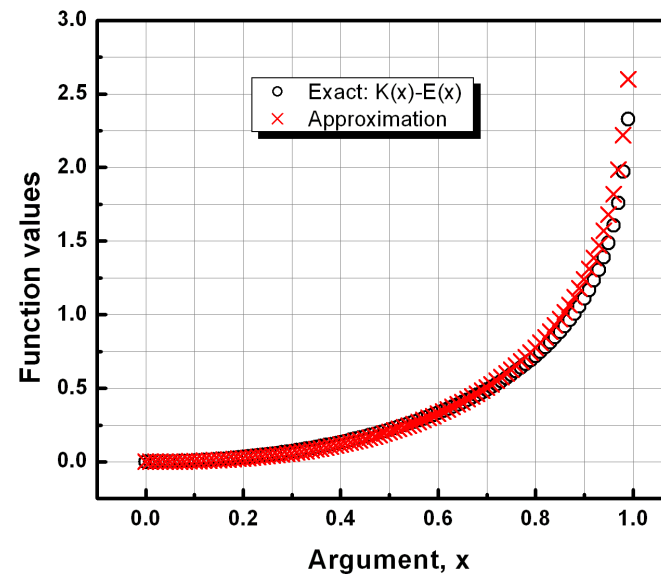
Equations for Simulation

For acceleration of numerical computations, it is convenient to replace the elliptic integrals in the expression the current density by elementary functions. We found a good enough approximation, this could be done by the following relation on the right:

- An Interpolation function using an exact table for elliptic functions was generated, then implemented into the library of COMSOL function.

Replacement of elliptic integrals:

$$K(x) - E(x) \cong \frac{\ln(1+x) - \ln(1-x)}{2} + (1-x) \ln(1-x)$$
$$\equiv \frac{\ln(1-x^2)}{2} - x \ln(1-x) \equiv f(x)/x.$$



Comparison of exact difference of elliptic functions and its approximation by elementary function, as seen above, which was used in the modeling.

Numerical Modeling: Implementation

- Three general form PDE interfaces
 - Real part of ψ , imaginary part of ψ , and the vector potential, A
 - Dirichlet Boundary conditions
- The geometry was built as a simple 1-D wire of half-length L .
- The time-dependent solutions were simulated for given interval of time in seconds with time steps of 0.1.
- The parameters for the results were set to:
 $\sigma = 1$, $\kappa = 0.4$, $A_0 = 0$, $x_0 = 5$, *width* = 0.1 and $\eta = 0.5$.

*Full details can be seen in the appendix of the paper which is in preparation.

Automation Implementation with MATLAB®

- The dynamics of time-dependent solutions were solved for by producing plots of the modulus of Ψ , $\sqrt{Re(\Psi)^2 + Im(\Psi)^2} \equiv \sqrt{u^2 + u_2^2}$, with respect to the x-coordinate.
- The parameter of j_0 , for a fixed value of $\tau_\varepsilon |\Delta|$, was swept in search for the critical current, j_c , in which the first phase slip center occurs. This was accomplished through LiveLink™, which established a connection between COMSOL and MATLAB®, allowing MATLAB script via commands to control the COMSOL model.

Automation Implementation with MATLAB®

The image shows the MATLAB R2019a interface with the following components:

- Editor:** Displays a script named `MinSearch.m` with the following code:

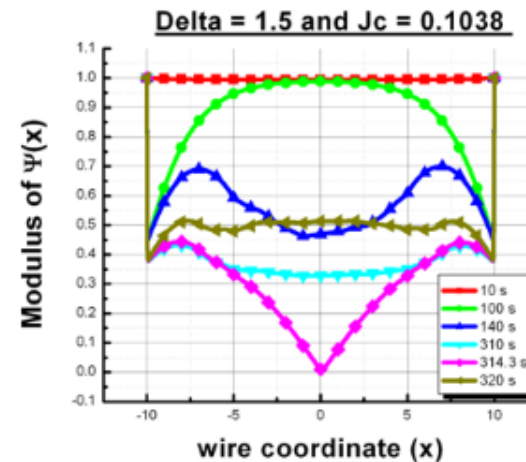
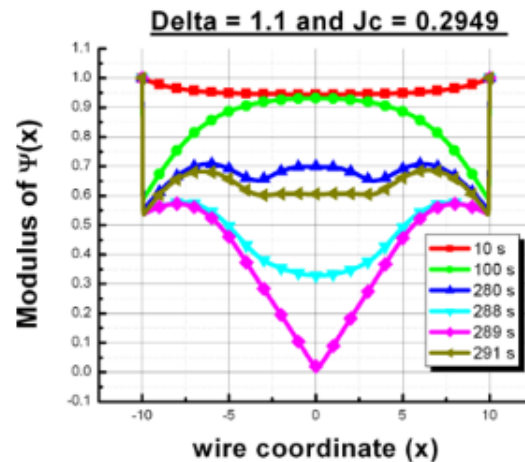
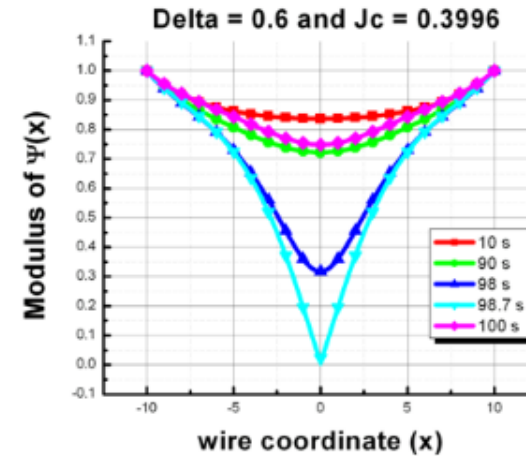
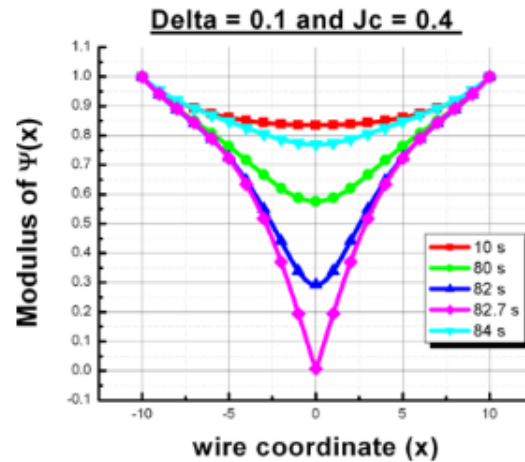
```
1 %jcVsdeltaPsiX = [];  
2 delta = 0.86;  
3 j=0.373;  
4 while (delta < 1.1)  
5     model = mphload('1dWireExactLabelled.mph'); %  
6     model.param.set('j0',j); %set j value  
7     model.param.set('delta',delta); %set delta v  
8     model.study('std1').run; %run study  
9     model.result("pg6").run(); %run plot of Psi(  
10    model.result.export('plot4').run; %export da  
11    B = importdata('NHPsiXFiltered.txt', ' '); %  
12    x = B(:,1); %separate column x  
13    psi = B(:,2); %separate column psi  
14    TF = islocalmin(psi, 'MinProminence', 0.988); %find local min  
15    n = numel(psi(TF)); %count local min  
16    xmin = x(TF); %NEED TO CHECK FOR UNIQUE VALUES , unique(), count how many  
17    progcheck = ['n:', num2str(n), ' j:', num2str(j), ' delta:', num2str(delta)]
```
- Command Window:** Shows the execution of the `MinSearch` script:

```
>> MinSearch  
n:0 j:0.373 delta:0.86  
n:0 j:0.3731 delta:0.86  
n:0 j:0.3732 delta:0.86
```
- Progress Window:** Titled "Solving linear system", it shows the progress of various tasks:

Description	Progress	Convergence	Parameter	Value	Stop
Compute	9%				
Solution 1	9%				
Time-Dependent Solver 1	5%	10	Time	8.600	Stop
Nonlinear solver	100%		Step	1	Stop
Assembling	0%				

*MATLAB® code available for download at: <https://irisdorn.github.io/automatedcomsol/>

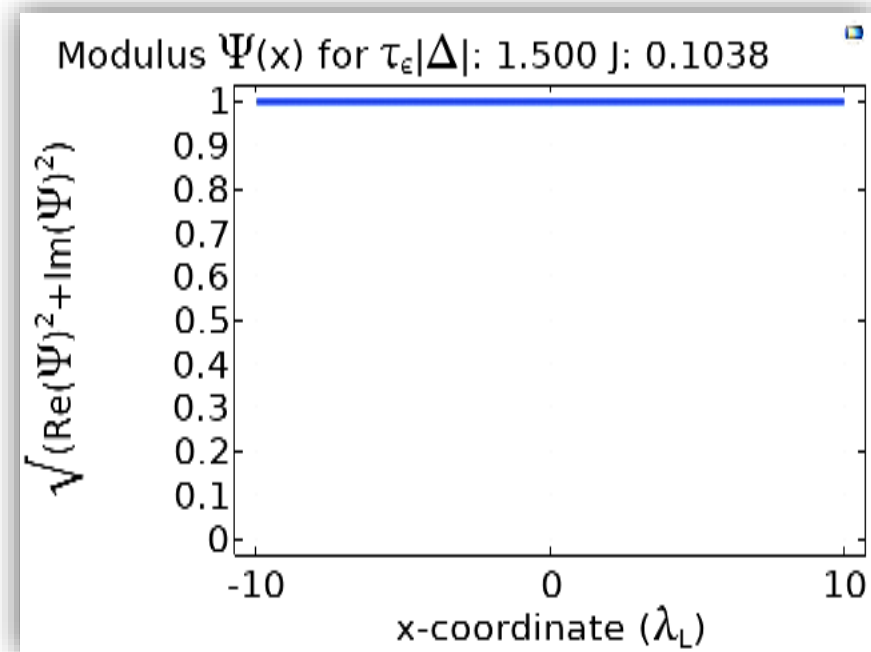
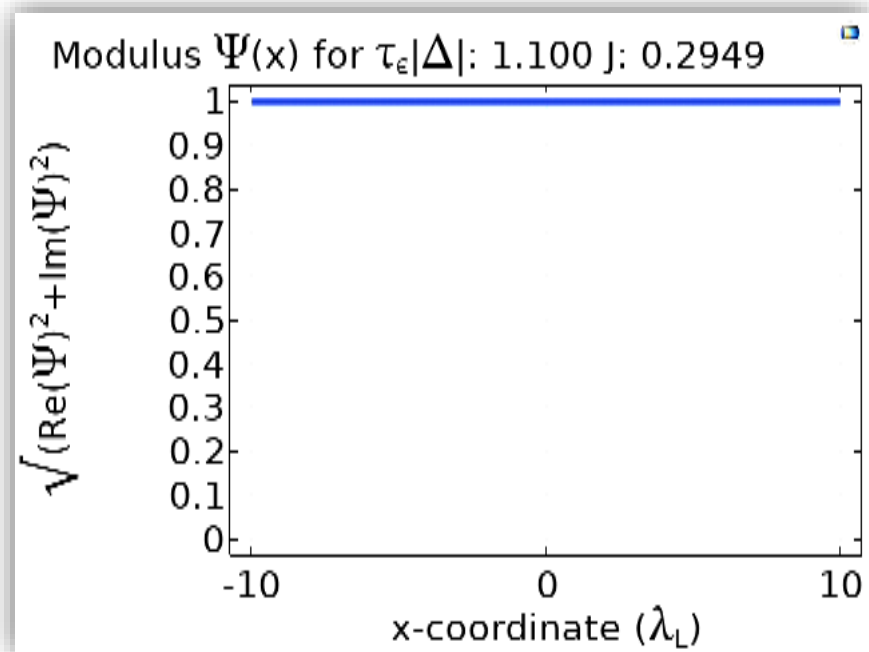
Results of Finite Element Modeling



Set of phase slippage in a 1D wire evolving over time.

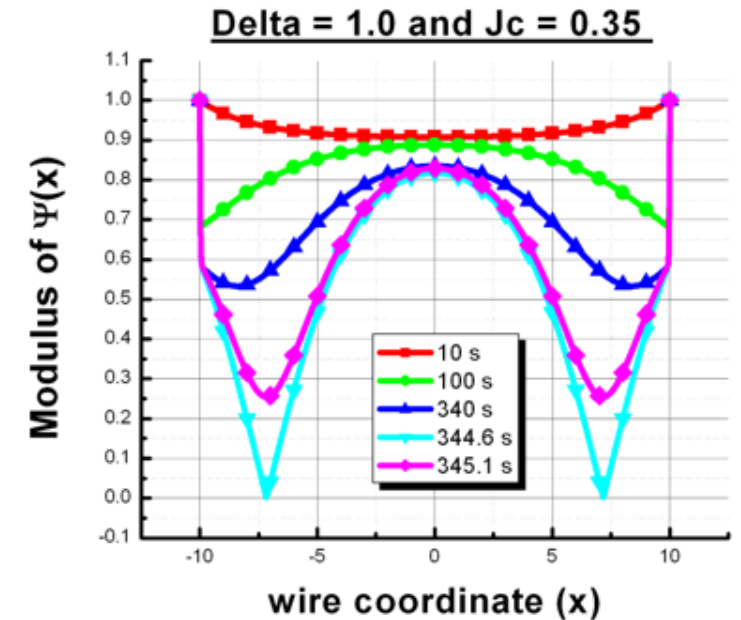
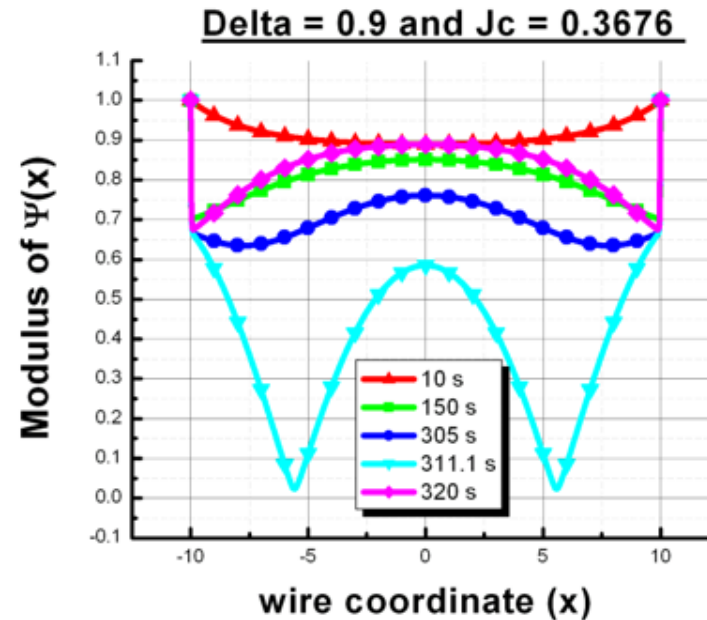


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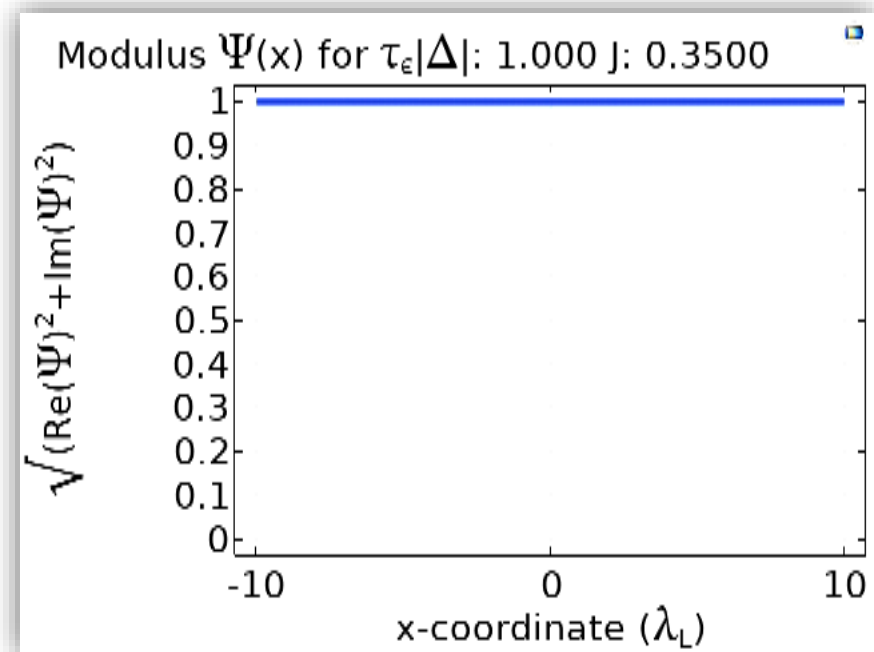
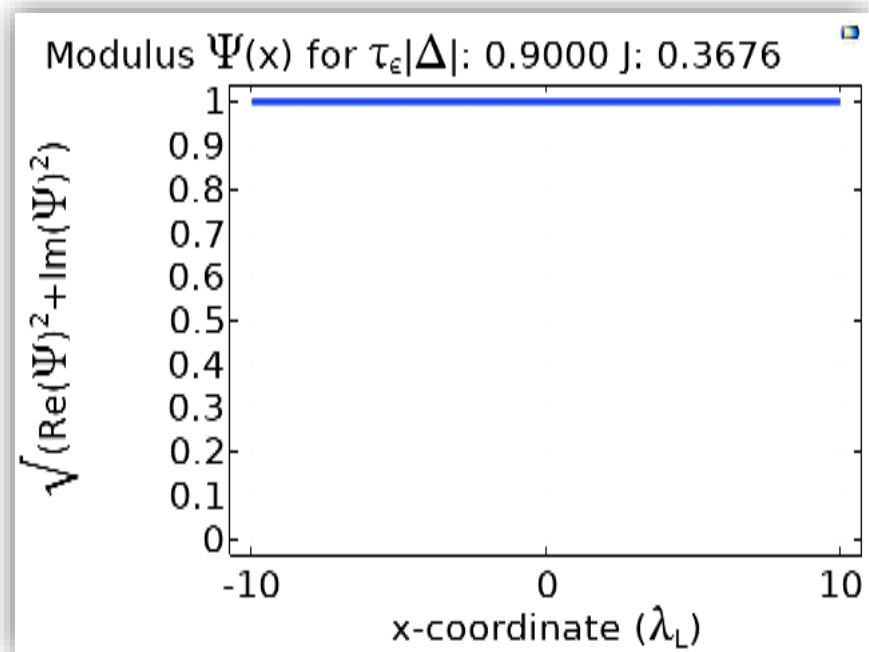


Results of Finite Element Modeling

At and between $\tau_\varepsilon |\Delta| = 0.9$ and 1.0 , we observe the double phase slippage evolving over time, with the low values of $\tau_\varepsilon |\Delta|$ in this region showing the double phase slippages closer to the center of the wire, while increasing $\tau_\varepsilon |\Delta|$ in this region shows the double phase slippages moving away from the center and toward the ends of the wire.

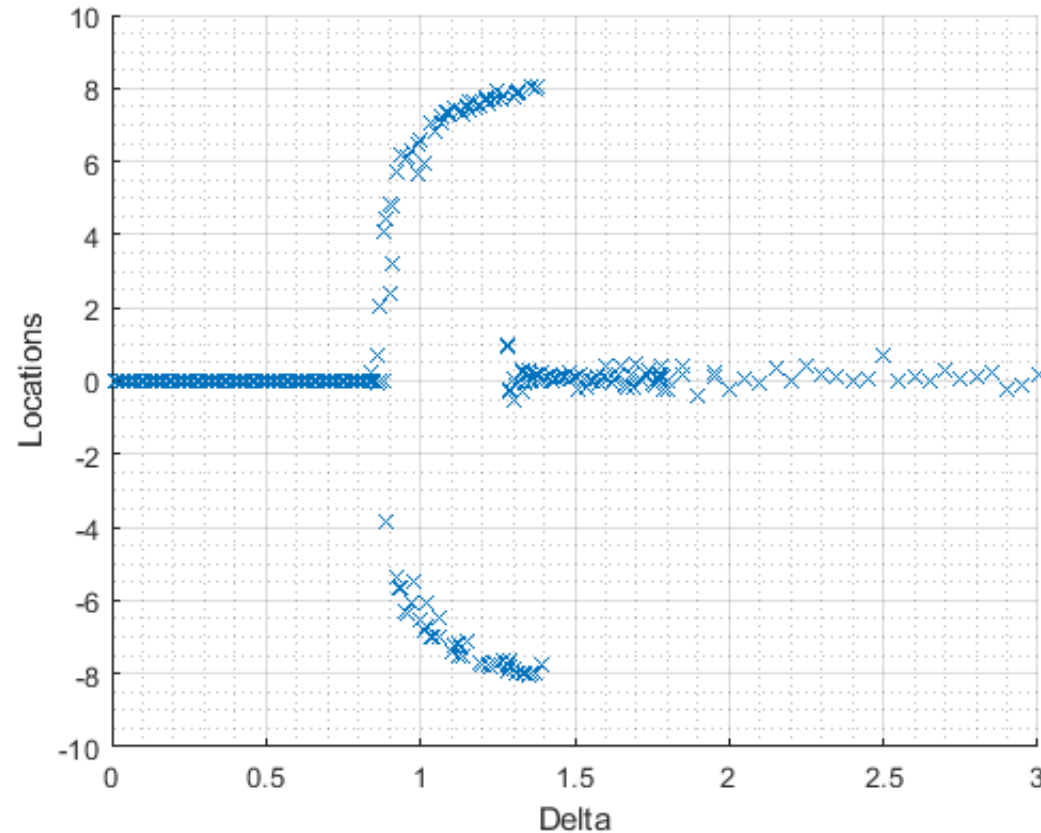


Set of double phase slippage evolving over time for increasing $\tau_\varepsilon |\Delta|$.



Results of Finite Element Modeling

From the automation search of locations of phase slip centers with increasing values of $\tau_\varepsilon |\Delta|$, we obtained "branching" and "anti-branching" of $\tau_\varepsilon |\Delta|$ at their critical currents.



Location of PSCs with increased $\tau_\varepsilon |\Delta|$ with branching and anti-branching for different parameters than previous.

Summary

- Goal: Exploration of the parameters governing the dynamics of phase slip centers in a 1D model of superconducting wire
 - Automation of COMSOL® utilizing MATLAB®
 - “Branching” & “Anti-Branching” Solutions discovered using Automation method.
- To help experimentalist understand better properties of resistive states in superconducting filaments.

Conclusion

Thanks to the power of COMSOL paired with MATLAB® we obtained many results with only a minor part of our findings shown here. These two software combined have provided our research the exceptional ability to go into great details in regards to visualizing and exploring the microscopic phenomena fine enough to be close to experimental findings. With this ability, we are directing our future work towards gaining deeper understanding of the role of interference current, exploring the dynamics of phonon feedback and expanding our model to 2D and most realistic 3D samples.

Acknowledgements

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Thank you for your attention!

Questions?



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