Numerical Model for Rocking of Mono-pile in a Porous Seabed

D.-S. Jeng*[1], X.-D. Luo[1] and J.-S. Zhang[1]
[1]Division of Civil Engineering, University of Dundee, Dundee, DD1 4HN, UK
*Corresponding author: d.jeng@dundee.ac.uk

Abstract: Marine energy industries, including offshore wind, wave and tidal energy, have been developed considerably in the last two decades. Most offshore energy structures used in the existing projects are mono-piles in shallow water due to the simplicity of design, installation and control. Numerous researches for the design and modeling of offshore wind energy system have been carried out in the past. However, most previous studies have been limited to a fixed mono-pile, even though rocking of a mono-pile has always been observed and caused damage of offshore wind energy structures. In this paper, a COMSOL model for the rocking of a mono-pile in a porous seabed is presented. In the flow mode, Navier-Stoke equation are solved for flow motion due to the rocking of a mono-pile, Biot's poro-elastic mode is solved for the porous seabed, and structural mechanics theory is solved for the rocking motion of a mono-pile. Such a model has not been available and is not possible to achieve without COMSOL Multiphysics.

Keywords: Periodic rocking; Poro-elastic model; Integrated model; Mono-pile; COMSOL.

1. Introduction

In the last two decades, offshore wind and wave energy industries have been developed considerably to provide the renewable energy. Most offshore energy structures used in the existing projects are mono-piles in shallow water due to the simplicity of design, installation and control. Numerous researches for the design and modeling of offshore wind energy system have been carried out in the past [1-3]. However, most previous research studies have been limited to a fixed mono-pile, even though rocking of a mono-pile has always been observed and caused damage of offshore wind energy structures.

It is well-known that the construction of mono-pile structure in a porous seabed may largely interact with the surrounding flow motion (such as ocean waves) and consequently affect the seabed response around the foundation. The rocking of a mono-pile due to wind, wave, current or a combined loading on the structure may further complicate this phenomenon. When the rocking-induced excess pore pressure is equal to the downward effective soil weight, liquefaction may occur and cause damage to the structure foundation [4-6]. To understand the structure stability of a mono-pile, an integrated model which can accurately predict the rocking-induced seabed response in a porous seabed is desired.

This study, based on COMSOL Multiphysics, is to develop an integrated model for simulating the rocking of a mono-pile and its induced seabed response. The theoretical formulations together with the use of COMSOL Multiphysics are given in Section 2. In Section 3, the model is applied to study the effects of soil and structure parameters on the rocking-induced seabed response. The remarking conclusions and future works are drawn in Section 4.

2. Theoretical Formulations

An integrated model for simulating the flow motion and seabed response induced by rocking of a mono-pile is developed in this study. This model includes three main components: (i) flow mode on the basis of the Navier-Stokes (N-S) equations; (ii) seabed mode on the basis of the Biot's consolidation equations with poro-elastic theory; and (iii) structure mode on the basis of structural mechanics theory.

2.1 Governing equations of flow mode

N-S equations are utilized to describe motion of the water liquid phase. Starting with the momentum balance in terms of stresses, the generalized equations in terms of transport properties and velocity gradients are

$$\rho\frac{\partial\vec{u}}{\partial t} - \nabla\bullet[\eta(\nabla\vec{u} + (\nabla\vec{u})^T)] + \rho(\vec{u}\bullet\nabla)\vec{u} + \nabla p_f = \vec{F} \qquad (1)$$

$$\nabla\bullet\vec{u} = 0 \qquad (2)$$

where η is the dynamic viscosity of fluid, ρ is the fluid density, $\vec{u}$ is the velocity field, p_f is the pressure, t is the time, and $\vec{F}$ is a volume force such as gravity.

2.2 Governing equations of seabed mode

The consolidation equation for the flow of a compressible pore fluid in a compressible porous medium can be given as [7]

$$\nabla\bullet(K_s\nabla p)-\gamma_w n'\beta\frac{\partial p}{\partial t}=\gamma_w\frac{\partial\varepsilon_s}{\partial t} \tag{3}$$

where p is the pore pressure, K_s is the permeability matrix of the soil, γ_w is the unit weight of pore water, n' is the soil porosity, and $\varepsilon_s=\nabla\bullet\vec{u}_s$ (where $\vec{u}_s$ is the soil displacement) is the volume strain of soil matrix. The compressibility of pore fluid (β) is defined as

$$\beta=\frac{1}{K_w}+\frac{1-S}{P_{w0}} \tag{4}$$

in which K_w is the true modulus of elasticity of water (taken as $2\times10^9\text{N/m}^2$), P_{w0} is the absolute water pressure and S is the degree of saturation of soil.

The relationships between soil displacement and pore pressure are given as

$$G\nabla^2\vec{u}_s+\frac{G}{1-2\mu_s}\nabla\varepsilon_s=\nabla p \tag{5}$$

where G is the shear modulus related to the Young's modulus (E) and the Poisson's ratio (μ_s) in the form of $E/(2(1+\mu_s))$.

2.3 Governing equations of structure mode

Based on the small-displacement assumption, the relationships between strain components and displacement at a point of marine structure are given as follows

$$\varepsilon_x=\frac{\partial u_m}{\partial x}\quad \varepsilon_{xy}=\frac{1}{2}(\frac{\partial u_m}{\partial y}+\frac{\partial v_m}{\partial x}) \tag{6}$$

$$\varepsilon_y=\frac{\partial v_m}{\partial y}\quad \varepsilon_{yz}=\frac{1}{2}(\frac{\partial v_m}{\partial z}+\frac{\partial w_m}{\partial y}) \tag{7}$$

$$\varepsilon_z=\frac{\partial w_m}{\partial z}\quad \varepsilon_{xz}=\frac{1}{2}(\frac{\partial u_m}{\partial z}+\frac{\partial w_m}{\partial x}) \tag{8}$$

The strain tensor ε and stress tensor σ are

$$\varepsilon=\begin{bmatrix}\varepsilon_x & \varepsilon_{xy} & \varepsilon_{xz}\\ \varepsilon_{yx} & \varepsilon_y & \varepsilon_{yz}\\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_z\end{bmatrix}\quad \sigma=\begin{bmatrix}\sigma_x & \tau_{xy} & \tau_{xz}\\ \tau_{yx} & \sigma_y & \tau_{yz}\\ \tau_{zx} & \tau_{zy} & \sigma_z\end{bmatrix} \tag{9}$$

The stress-strain relationship for linear conditions reads

$$\sigma=D_m\varepsilon \tag{10}$$

where D_m is the elasticity matrix.

The structural mechanics theory in this study is based on a weak formulation of the equilibrium equations expressed in the global stress components.

$$-\nabla\bullet\sigma=\vec{F}_m \tag{11}$$

in which $\vec{F}_m$ denotes the volume forces (body forces).

2.4 Boundary conditions

When solving the governing equations, one needs to provide the boundary conditions at external boundaries and internal interfaces for these three modes (see Figure 1).

In the flow mode, a still water with zero velocity is initialized. Wall boundary with no-slip boundary is applied in the left-hand-side boundary Γ_1 and the right-hand-side boundary Γ_2. Zero pressure is applied on the water free surface Γ_3, while no-slip condition is adopted at the solid surface (such as sea floor Γ_4 and surface of mono-pile Γ_5). When the mono-pile is rocking, the impact of the structure movement on flow motion is considered in term of shape deformation and moving velocity of boundary Γ_5.

In the seabed mode, it is commonly accepted that vertical effective normal stresses vanish at the seabed surface while the fluid pressure and shear stresses obtained from flow mode are imposed as boundary conditions at the sea floor Γ_4. We assume that the seabed is of a finite thickness and rests on an impermeable rigid bottom, indicating that zero displacement, zero gradient of pore pressure and no vertical flow occur at the horizontal bottom Γ_6. When two vertical side boundaries (Γ_7 and Γ_8) of the seabed are far away from the concerned region (such as the region around a mono-pile foundation), they can be assumed to have zero displacement. At the soil-structure interface Γ_9, the soil has same displacement and velocity as those of mono-pile structure.

In the structure mode, the rocking period T_m and amplitude A_m are imposed at the top of a mono-pile, dominating the rocking motion of structure and providing the external driving force of the integrated model.

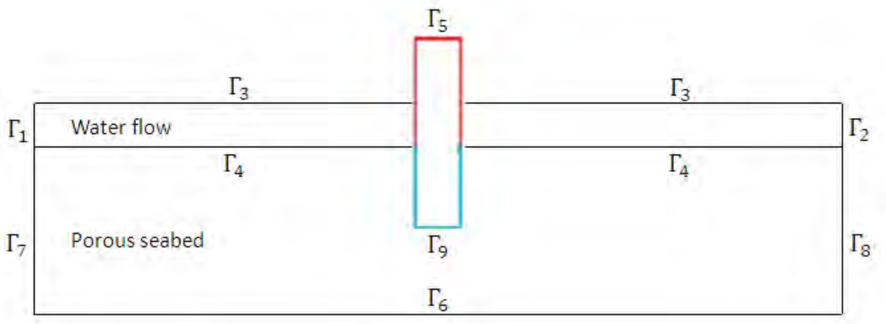
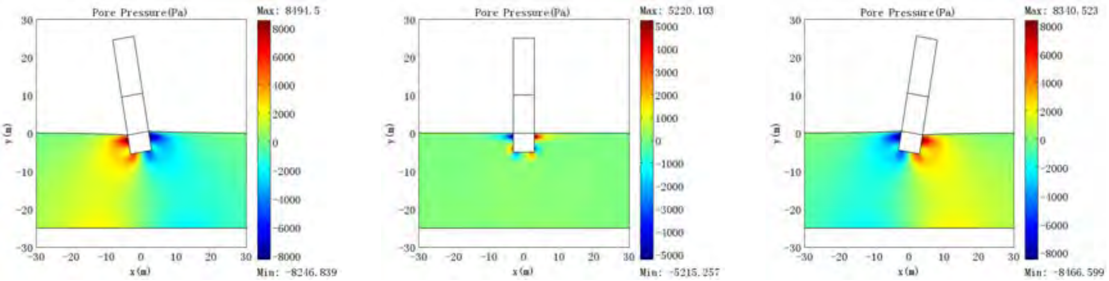


Figure 1. Locations for specification of boundary condition.



(a) (b) (c)

Figure 2. Distribution of rocking-induced pore pressure around mono-pile foundation at different time levles (a) $t = T_m/4$, (b) $t = T_m/2$ and (c) $t = 3T_m/4$. The rocking displacement of the mono-pile is zoomed in 50 times.

2.5 Use of COMSOL Multiphysics

These three numerical modes are integrated by using COMSOL Multiphysics (3.5a version). The main features of COMSOL Multiphysics adopted to set up the integrated model are listed as follows:

(1) 2D space dimension;
(2) Plane strain mode of structural mechanics;
(3) Coefficient form of PDE mode for seabed mode;
(4) Incompressible Navier-Stokes mode of fluid dynamics;
(5) Arbitrary Lagrangian-Eulerian (ALE) method for mesh movement.

3. Results and Discussion

In reality, the rocking mono-pile in a porous seabed may largely interact with the ocean waves, which is an extremely complicated phenomenon. As a starting point, the wave motion is temporarily excluded and only a still water with a water depth $d = 10$ m is considered here. In the example, a computational domain with a length ($L = 200$ m) is used. The original of the Cartesian coordinate system is located at crossing point of the sea floor and the central axis of mono-pile. This integrated model is applied to study the effects of soil characteristics (soil permeability, degree of saturation and two-layer soil type) and structure parameters (structure embedded depth, rocking period and amplitude) on the rocking-induced seabed response.

3.1 Effects of Seabed Characteristics

Many soil variables affect the seabed response to the rocking a mono-pile and only three of them (soil permeability, degree of saturation and two-layer soil type) are studied here. To study the effects of soil characteristics, the parameters of a mono-pile are fixed as follows: diameter $d_m = 6.0$ m, height above sea floor $h_{m1} = 25.0$ m, embedded depth $h_{m1} = 5.0$ m and rocking period $T_m = 6.0$ sec. Some soil properties are also fixed: seabed thickness $h_s = 25.0$ m, soil shear modulus $G = 10^7$ N/m^2, Poisson's ratio $\mu_s = 1/3$ and unit weight of soil $\gamma_s = 2.65\gamma_w$.

The permeability of a soil (K_s) is a measure of how rapidly fluid is transmitted through the voids between grains. It varies from 9.0×10^{-6} to 1.5×10^{-2} m/sec with a fixed degree of saturation $S = 0.95$ in the example. Figure 2 presents the distribution of pore pressure in seabed with permeability $K_s = 8.4\times10^{-4}$ m/sec at different time levels, indicating that the rocking motion of a mono-pile has a significant impact on the pore pressure around structure foundation. Figure 3 shows the effects of soil permeability on maximal pore pressure around structure foundation at three different values of rocking amplitude A_m . It can be seen from the figure, the maximum of rocking-induced pore pressure decrease shapely when permeability increasing from 9.0×10^{-6} to 9.0×10^{-4} m/sec and then gradually when permeability increasing from 9.0×10^{-4} to 1.5×10^{-2} m/sec. The effect of rocking amplitude on maximal pore pressure will be discussed later.

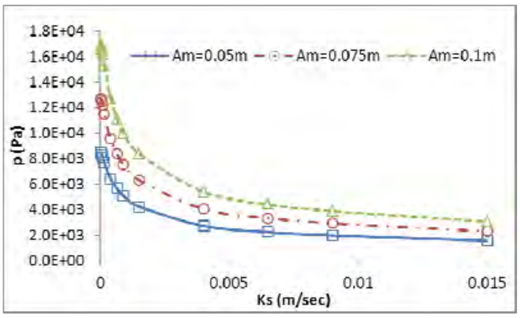
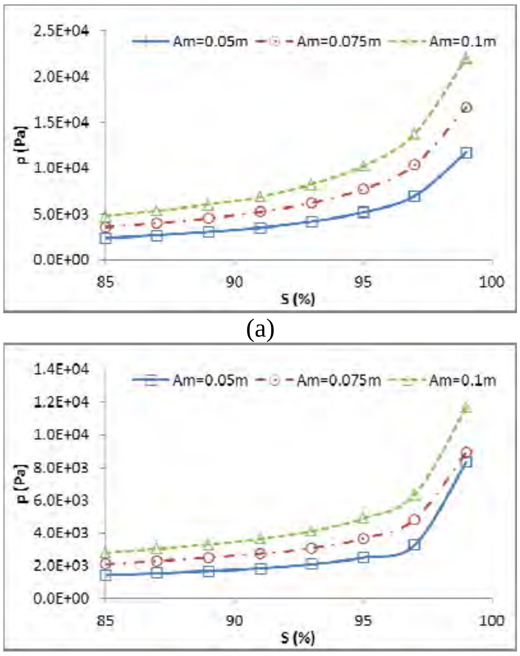


Figure 3. Effect of soil permeability on maximal pore pressure at three different rocking amplitudes.

It is not uncommon to find air/gas within marine sediment. It is believed that most marine sediments have degree of saturation generally between 75% and 95% [8]. In the example, the degree of saturation (S) varies from 85% to 99% with an equal interval of 2%. Two cases with different values of soil permeability are considered: one is with $K_s = 8.4\times10^{-4}$ m/sec and the other is with $K_s = 6.0\times10^{-3}$ m/sec. Figure 4 gives the relationships between the maximal pore pressure around the mono-pile foundation and degree of saturation. An increasing degree of saturation leads to a significant increase of maximal pore pressure, resulting in more chance to induce the liquefaction around mono-pile foundation.



(a)

(b)

Figure 4. Effect of degree of saturation on maximal pore pressure in soil with different permeabilities: (a) $K_s = 8.4\times10^{-4}$ m/sec and (b) $K_s = 6.0\times10^{-3}$ m/sec.

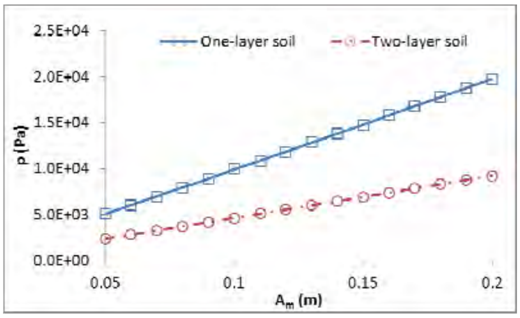


Figure 5. Effect of replacment of upper layer soil on maximal pore pressure with different values of rocking amplitude.

A replacement of upper layer of seabed soil with coarse sand may be usually adopted for the protection of foundation of marine structure. To simulate the effects of rocking of mono-pile in two-layer seabed, the soil in upper layer with 2 m thickness is replaced with coarse sand having a soil permeability $K_s = 9.0\times10^{-3}$ m/sec. At the interface between two consecutive layers in the seabed, the soil displacement, normal effective stresses and pore pressure are concordant. The comparisons of maximal pore pressure within one-layer seabed and that within two-layer seabed are given in Figure 5. As we can see from the figure, a replacement of soil in upper layer can largely reduce the maximal value of pore pressure and consequently increase the stability of mono-pile foundation.

3.2 Effects of Structure Parameters

To examine the effects of embedded depth and rocking strength of a mono-pile on the seabed response, the soil characteristics are fixed as follows: soil permeability $K_s = 8.4 \times 10^{-4}$ m/sec, degree of saturation $S = 95\%$, seabed thickness $h_s = 25.0$ m, soil shear modulus $G = 10^7$ N/m^2, Poisson's ratio $\mu_s = 1/3$ and unit weight of soil $\gamma_s = 2.65\gamma_w$. Some parameters of a mono-pile are also fixed: diameter $d_m = 6.0$ m, height above seabed $h_{m1} = 25.0$ m.

The embedded depth of a mono-pile may largely affect the total construction cost. Eight values of embedded depth of a mono-pile are considered. Figure 6 shows the effect of embedded depth of structure on the maximal pore pressure, indicating that an increasing embedded depth may slightly increase the maximal pore pressure. However, a deeper foundation of mono-pile can significantly prevent the structure from overturning, which is another important factor of structure stability.

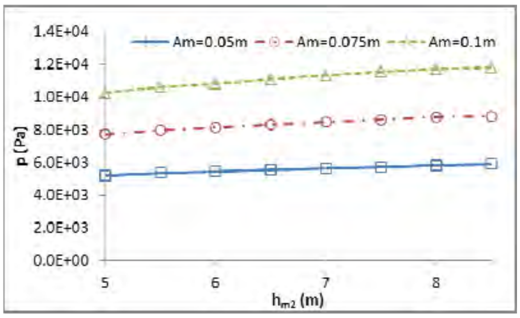


Figure 6: Effect of embedded depth of a mono-pile on maximal pore pressure.

The rocking strength (period and amplitude) mainly dominated by the external force loading is another important factor which plays an important role in evaluation of rocking-induced pore pressure. Figure 7 gives the influence of rocking period on the maximal pore pressure around structure foundation. In general, the maximal pore pressure gradually reduces with an increasing rocking period. This can be due to that the pore pressure in seabed will have more time to dissipate when the rocking period is much longer. The influence of rocking period with larger amplitude is more significant than that with smaller amplitude. All aforementioned results also show that an increase of rocking amplitude always leads to an increase of maximal pore pressure, as the soil in the vicinity of structure foundation suffers from a more violent deformation.

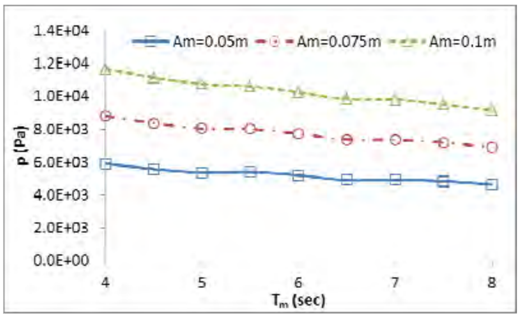


Figure 7: Effect of rocking period of a mono-pile on maximal pore pressure.

4. Conclusions and future work

An integrated model, based on COMSOL Multiphysics, has been developed to study the rocking-induced pore pressure in a porous seabed. The numerical results indicated: (i) COMSOL Multiphysics has a good potential in simulating the rocking of a mono-pile in a porous seabed; (ii) the maximal pore pressure around structure foundation is much larger in a soil with lower permeability and higher degree of saturation; (iii) the replacement of lower permeability soil with higher permeability soil can significantly reduce the maximal pore pressure and consequently increases the foundation stability; (iv) an increasing embedded depth may slightly increase the maximal pore pressure; and (v) a stronger rocking motion with shorter period and larger amplitude always leads to higher maximal pore pressure.

In this paper, only the preliminary results for the rocking-induced seabed response around the foundation of a mono-pile in a porous seabed are presented, providing a general guideline of laboratory experiments. The next step is to validate the integrated model with a series of experimental data.

5. References

1. Mase, H., Kosho, K. and Nagahashi, S., Wave run-up of random waves on a small circular pier on sloping seabed, *Journal of Waterway, Port, Coastal and Ocean Engineering*, **127(4)**, 192-199 (2001)

2. Abdel-Rahman, K. and Achmus, M., Finite element modeling of horizontally loaded monopole foundations for offshore wind energy converters in Germany, *Proceeding of International Symposium on Frontiers in Offshore Geotechnics, Perth, Australia* (2005)

3. Limkatanyu, S., Kuntiyawichai, K. and Spacone, E., Response of reinforced concrete piles including soil-pile interaction effects, *Engineering Structures*, **31(9)**, 1976-1986 (2009)

4. Christian, J.T., Taylor, P.K.,Yen, J.K.C. and Erali, D.R., Large diameter underwater pipeline for nuclear power plant designed against soil liquefaction, *Proceeding of Offshore Technology Conference*, Dallas, 597-606 (1974)

5. Smith, A.W. and Gordon, A.D., Large breakwater toe failures, *Journal of Waterway, Harbor and Coastal Engineering, ASCE*, **109(2)**, 253-255 (1983)

6. Lundgren, H., Lindhardt, J.H.C. and Romhild, C.J., Stability of breakwaters on porous foundation, *Proceedings of 12th International Conference on Soil Mechanics and Foundation Engineering*, 1, 451-454 (1989)

7. Biot, M.A., General theory of three-dimensional consolidation, *Journal of Applied Physics*, **12(2)**, 155-164 (1941)

8. Esrig, M.I. and Kirby, R.C., Implication of gas content for predicting the stability of submarine slopes. , *Marine Geotechnology*, **2(2)**, 81-100 (1977)