

# Simplifying a model of short subsequent sections containing a common detailed cross-section.

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**Abstract:** In Comsol Multiphysics it is possible to implement and simulate several independent models in parallel. This can be meaningful when the different models are coupled to each other by some kind of interaction. If a part of the geometries is identical in all models, this part can be reduced in that it is implemented in only one of the geometries. In the other geometries only its solution towards its surrounding is used. The paper presents the application of a power cable going through different thermal environments. The cable has the same cross-section and current in all submodels. As the surrounding sections are short there is thermal flow in axial direction between the sections. Taking that flow into consideration improves the temperatures calculated for each section.

**Keywords:** Coupled geometries, termination box, umbilical, memory, speed.

## 1. Introduction

Consider a power cable in ground or sea. The termination above ground or sea surface is the most critical part from a thermal point of view. The hot-spot temperature in the termination is the dimensioning factor for the conductor cross-section. Normally the same cross-section is used for the whole cable length. Accurate temperatures calculations are needed for the termination. The termination often consists of different short sections implying thermal interaction in axial direction that cannot be neglected for precise thermal calculations [1]. In Comsol it is possible to create a model for each section in the same overall model and to couple them to take into account their interaction. The required calculation resources increase with the number of modeled cross-sections. This implies a limitation for realistic applications, where the multiphysical behavior of detailed cross-sections is simulated. For instance a power umbilical contains one or more 3-phase power and signal cables, fiber optics, steel rods and fluid pipes in a support structure with holes [2]. Often the

resistive heating due to load current is calculated where the skin effect in the conductors, eddy currents in the cable screens and metallic rods and tubes are taken into account. This makes the model to a complex multiphysical model. For cables with temperatures clearly higher than 20°C the temperature coefficient of current-carrying parts has to be taken into account to obtain realistic losses and thus heating. Often worst-case scenarios are considered with maximum allowed conductor insulation temperature of 90°C. The paper shows that the multiphysical cable model needs to be implemented only once. The different cable surroundings are implemented without the cable geometry as purely thermal models. Apart from dramatically reducing calculation resources it also reduces the time for implementing the model for a given case to a fraction. That in turn reduces the risk for making mistakes during modeling. Another modeling optimization with a few similarities can be found in [3].

## 2. The modeling approach

Modeling consists of the following steps. One of the sections is chosen and modeled as if that section was infinitely long. It contains the power cable and the surrounding of this section. That results in a common 2D model in Cartesian coordinates that can be simulated. The geometry can be very detailed and in addition a multiphysical model.

The model is extended by adding a second geometry for the surrounding of a neighboring section. This section is modeled as a thermal model. Whereas the thermal conditions on the outer boundaries are given for the section, the power cable is reduced to its outer surface boundary conditions. The cable losses per meter due to electric current are calculated by the first cross-section and thus available. These losses are the same in each section as the currents are the same according to Kirchhoff's current law. These losses are injected into the inner boundary of the second geometry. The second model

calculates the resulting temperatures. The temperature on the cable outer surface differs between the two sections whereas the temperature gradient in both cables is the same as the cable losses are independent of the sections. Taking into account the temperature offset between the two cable sections the heat flow  $\Delta Q$  in a copper conductor in axial direction between the two modeled sections is calculated by means of a global equation. The equation uses the conductor cross-sectional area and the lengths of the two modeled sections. These parameters are given and can be entered in Comsol as constants. Often power cables are arranged symmetrically about a centre axis. Then the axial heat flow is the same in all three conductors, which means that only one global equation is required for the three power cores. The axial heat flow in metallic parts not made of copper can often be neglected as their thermal conductivity is about 10 times lower. The axial heat flow from the first to the second section is subtracted from the first cable boundary surface and added on the cable surface in the second section. If the heat flow is positive it will contribute to cooling of the first section and heating of the second section. This gives more realistic temperatures in both sections.

The second step is repeated for the section on the other axial side of the first geometry. If there are further sections the second step is repeated for each new section, where a new global equation is introduced for the heat flow to the adjacent section.

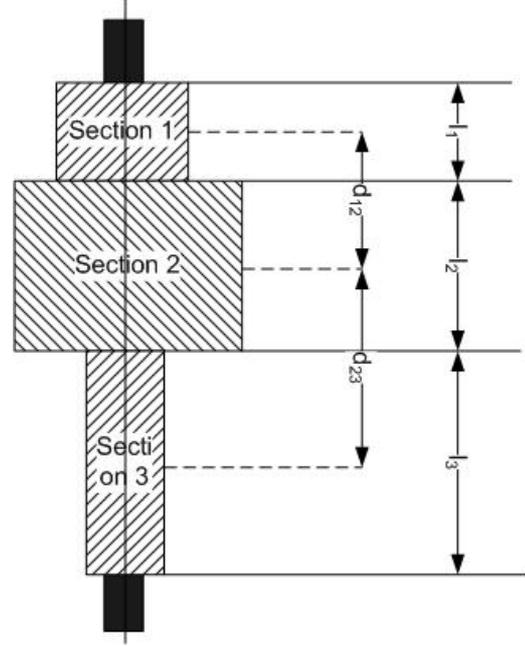
The loss  $q_0$  is the loss per meter of the cable due to the electric current. This value is the similar in all sections. The heat flow in the copper conductor in axial direction between two sections is obtained by the temperature difference  $\Delta T$  between two sections and the thermal resistance of the copper conductor:

$$\Delta Q = \frac{\Delta T}{R_{th}} \quad (1)$$

The thermal resistance is given by the thermal conductivity  $\lambda$  of copper, the length of the thermal path  $d$  and the cross-sectional area of the copper conductor  $A_{Cu}$ .

$$R_{th} = \frac{1}{\lambda} \cdot \frac{d}{A_{Cu}} \quad (2)$$

The distance  $d$  is the distance between the axial centers of the two sections.



**Figure 1.** Definition of section distances used in model.

For two sections with length  $l_1$  and  $l_2$  the distance is:

$$d_{12} = \frac{l_1 + l_2}{2} \quad (3)$$

The temperature difference  $\Delta T$  is the difference of the temperatures in the axial middle of the two sections:

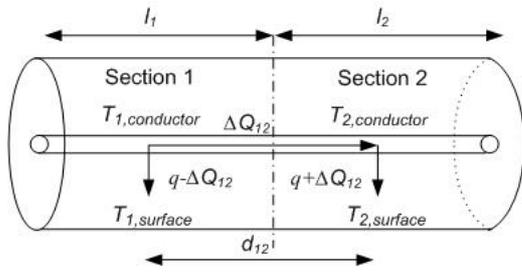
$$\Delta T = T_1 - T_2 \quad (4)$$

As the heat flow through the copper conductor is calculated the temperatures  $T_1$  and  $T_2$  are measured as integration variables in the conductor. But the conductor temperature can only be measured in a geometry where it is modeled. As the cable cross-section is inserted into only of the model geometries, the temperature has to be captured on the cable outer surface as that exists in all modeled geometries. In order to eliminate temperature variations on the cable outer surface the mean surface temperature is obtained by integration coupling

boundary variables divided by the length of the boundary.

In the first section the obtained axial heat flow  $\Delta Q$  is added to  $q_0$  and in the second section it is subtracted from  $q_0$ . The sign of  $\Delta Q$  is obtained by Comsol. For a section  $i$  with length  $l_i$  the total normal heat flux through the cable outer surface into the surrounding is:

$$q_i = q_0 + \frac{\Delta Q}{l_i} \quad [\text{W/m}] \quad (5)$$



**Figure 2.** Heat flow between two sections.

Denoting  $T_1$  as conductor temperature and  $T'_1$  as surface temperature for section with cable geometry extrapolates the unknown conductor temperature  $T_2$  for a neighboring section without cable geometry as:

$$T_2 = T'_2 + (T_1 - T'_1) \frac{q + \frac{\Delta Q_{12}}{l_2}}{q - \frac{\Delta Q_{12}}{l_1}} \quad (6)$$

Inserting (6) in (4) and (5) this gives a quadratic equation  $x^2 + px + q = 0$  in  $\Delta Q_{12}$  with solution

$$p = \frac{T_1 - T'_2 + (T_1 - T'_1) \frac{l_1}{l_2} + q \cdot l_1}{2R_{Cu}} \quad (7)$$

$$q = \frac{(T'_2 - T'_1) \cdot q \cdot l_1}{R_{Cu}} \quad (8)$$

In Comsol this solution is inserted as a global equation for  $\Delta Q_{12}$ . Including the conductor temperature coefficient  $\alpha_{Cu}$  and the ambient temperature of that section a second global expression is required using  $T_2$  as input:

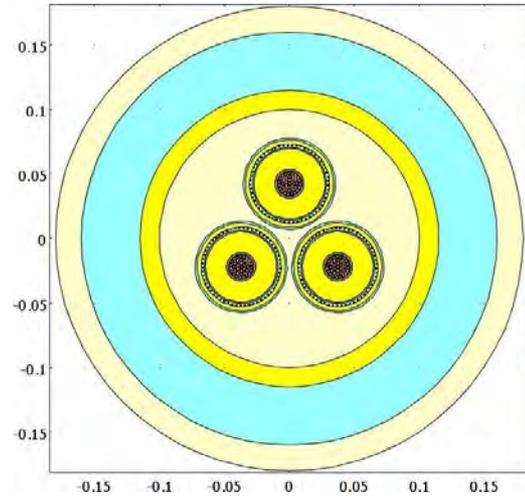
$$T_{2,a} = \frac{T_2 - T_{ambient2}}{1 + (T_1 - T_2)\alpha_{Cu}} + T_{ambient2} \quad (9)$$

### 3. A practical application in Comsol.

#### 3.1 The model parameters.

A standard three-phase power cable goes through three sections as shown in Fig. 1. The cable comes out of cold ground (section 3). It is fixed and supported by means of a resin box (section 1). Between the box and ground the cable is protected by an air filled PVC tube (section 2). On the top of the PVC box the three single cable cores come out of resin box and are cooled by surround air with constant temperature.

Fig. 2 shows the detailed cross-section of the cable in an air-filled tube (section 2). The top and bottom sections, section 1 and section 3, are only modeled as circular and quadratic surroundings, respectively, see Figures 5 to 7.



**Figure 2.** Power cable in air with protection tube.

For this paper the detailed material properties and dimensions are not of interest. Important is that the same cable and surroundings were used for the conventional model and the new model. The dimensions can be seen in Fig. 2 to 7. The electrical and thermal properties of the used materials copper, PE and PVC are commonly known. For the resin box PE was used for simplicity. Concrete and ground was modeled with a thermal conductivity of 1W/K·m. The ambient temperatures are 30°C in air and 10°C in ground.

Instead of a purely thermal model the losses of a 3-phase current with phase shifts of  $120^\circ$  were simulated. Skin effect in the conductors and eddy currents in the cable metallic screens were included. As the cable temperature considerably differs from  $20^\circ\text{C}$  the temperature coefficient was also modeled.

The three cables cores coming out of the top of the termination box are surrounded by air. They do not require any model geometry, merely one common global equation for the axial heat flow. The thermal resistance of one cable conductor to air through the cable insulation can be approximated by an infinite ladder, which in turn results in a known simple equation [4].

### 3.2 Resulting temperatures.

Figures 4 to 7 illustrate the temperatures calculated with the new model for a 20cm long box, a 0.5m long tube and a 5m deep ground section. The cables conductors in the tube are cooled by the neighboring sections to  $67^\circ\text{C}$  from  $77^\circ\text{C}$  without interaction (see later Fig. 9).

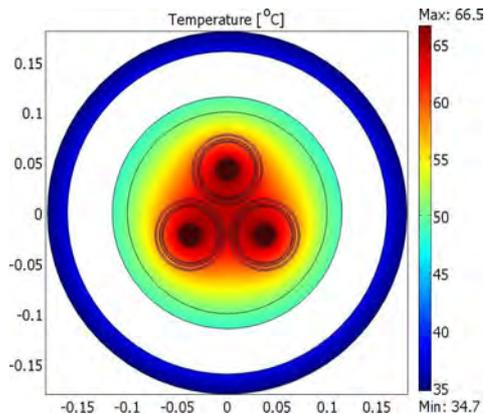


Figure 4. Power cable in protection tube in air  $30^\circ\text{C}$ .

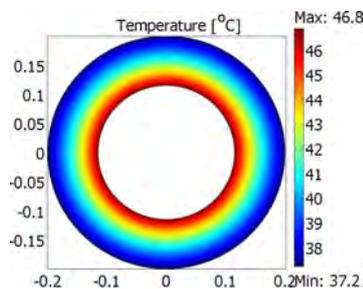


Figure 5. Termination box in air  $30^\circ\text{C}$ .

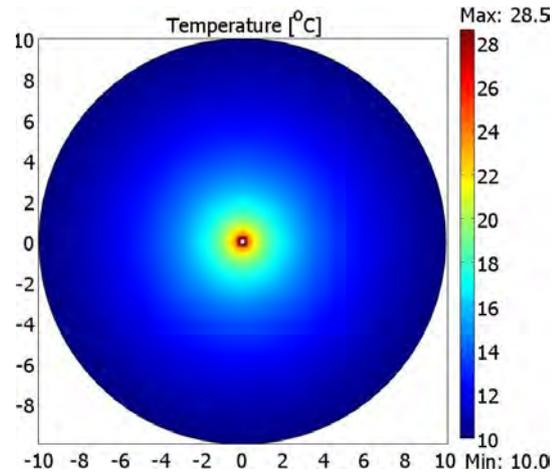


Figure 6. Concrete-filled duct in ground  $10^\circ\text{C}$ .

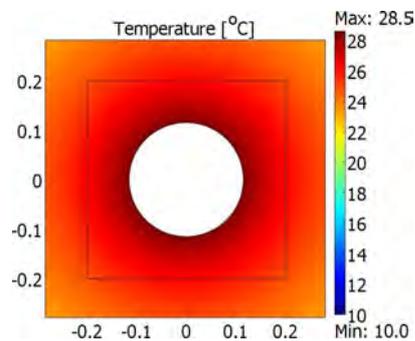


Figure 7. Zoom of concrete-filled duct in Fig. 6.

### 3.3 Accuracy of Results

The large ground surrounding in Fig. 6 without the detailed cable geometry overcomes Comsol's limitation for meshing. The overall size should not exceed 1000 times the smallest detail.

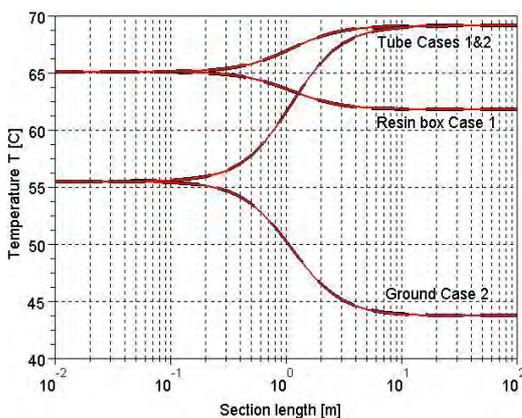
Figures 8 to 11 show that the cable temperature depends on the section length, which justifies taking the axial thermal coupling into account. For simplicity all sections have the same length. For very short sections the conductor temperature is the same in all sections and for very long sections the thermal coupling between the sections vanishes and the temperatures approach asymptotic values.

Fig. 8 shows temperatures for two simulated sections. In Case 1 the heat flows from the tube to the resin box, in Case 2 to the ground section.

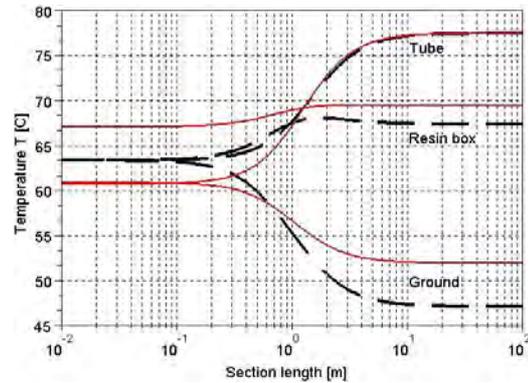
As the model is linear the new model fully agrees with the conventional model.

Fig. 9 shows the result for three sections with same length including the temperature coefficient of the current carrying copper conductor. As a result all temperatures are higher than in Fig. 8. In Fig. 9 the agreement with the conventional model is not high. Taking into account the temperature coefficient according to (9) still does not give perfect agreement, see Fig. 10. The reason is that  $\Delta Q_{12}$  and  $\Delta Q_{23}$  were derived above for thermal coupling between two neighboring sections but implemented in a model with three sections. In order to obtain the correct solution for three sections two unknown  $\Delta Q_{12}$  and  $\Delta Q_{23}$  have to be solved analytically before inserting as global equations into Comsol. It is commonly known how to solve the resulting third order polynomial. The approach is similar to the quadratic equation above. Similarly, thermal interaction between four sections would yield a polynomial of fourth order with three solutions to be entered as global equations into Comsol.

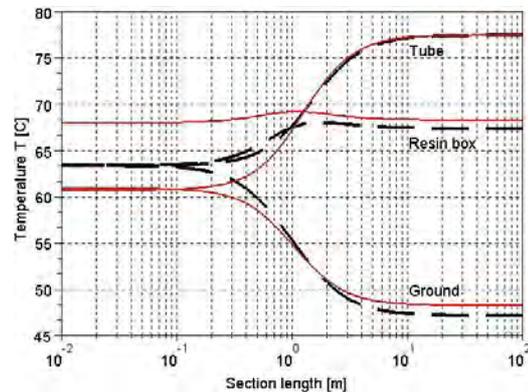
A simple manipulation is illustrated in Fig. 11. For very short sections the temperature has to be the same in all conductors. The inverse of the section length was used as weight for the mean value of simulated temperatures. However, this approach is somewhat manual as it has some degrees of freedom.



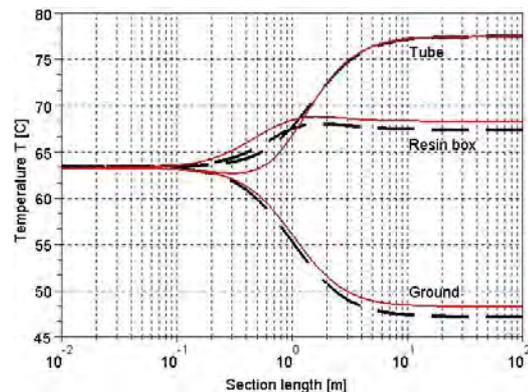
**Figure 8.** Two sections. Cable conductor temperature in tube and resin box. Conventional model (black dashed) and new model (red).



**Figure 9.** Three sections. Cable conductor temperature. Conventional model (black dashed) and new model (red).



**Figure 10.** Fig. 9 with temperature coefficient also in sections without cable geometry.

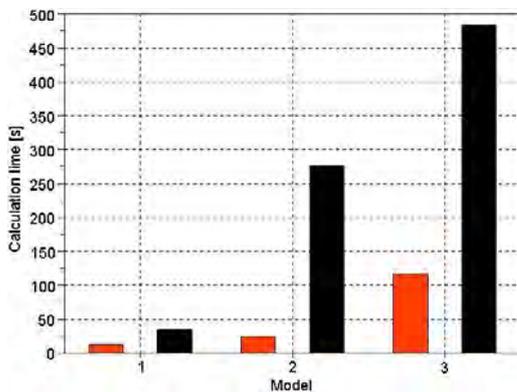


**Figure 11.** Fig. 10 manipulated by use of mean temperature for very short section lengths.

### 3.4 Convergence, Required Memory and Calculation Time

For the length sweep in Figures 8 to 11 there were no convergence issues with the new model but with the conventional model. That was improved by starting with high lengths where the axial flow is low compared to the cable loss.

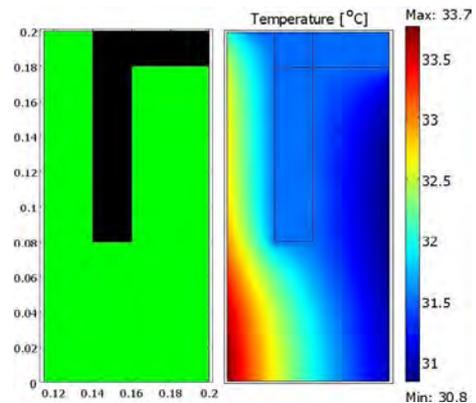
The conventional model required the solver Paradiso Out of Core whereas the new model worked well with Spooles. Fig. 12 shows that the new model calculates about five times faster than the conventional model. The purely thermal model (left) needs little time and the electro-thermal model including temperature coefficient in the conductor is the slowest model (right).



**Figure 12.** Required simulation time by new model (red) and conventional model (black). Thermal model, electro-thermal model and electro-thermal model with temperature coefficient.

### 4. Further Improvements

If the material thermal properties in the surrounding of one section changes in axial direction, the surrounding cross-section can be replaced by an axial symmetric profile. If for instance the termination box in Fig. 5 is mechanically supported by an integrated steel plate the profile may look as in Fig. 13.a). The thermal result in Fig. 13.b) confirms that such a steel plate would imply considerable cooling in the upper part of the section. This model improvement did not increase the simulation time nor memory use.



**Figure 13.** Profile of termination box including a steel plate for mechanical support. a.) Materials b.) Calculated temperatures.

### 5. Conclusions

The paper showed that the model simplification reduces calculation time to a fraction. Also the memory requirement is considerably reduced, which is important for real detailed cross-sections that would require too much memory if implemented several times.

For linear materials the results are the same as for the conventional model. That requires, however, implementing the zeros of a polynomial. The degree of the polynomial is the same as the number of sections. That involves larger expressions for more sections. If the model is non-linear, e.g. due to the temperature coefficient of the current carrying conductor, the results are still useful but the accuracy is not perfect any more. It can be increased by defining dependent global expressions plotted at post-processing, which can be done conveniently.

### 6. References

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