Equation-Based Modeling: The Structural Contact Problem Solved by The Velocity Approach

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Abstract:
The contact between infinitely rigid body and deformable part is studied using the velocity as a dependent variable. A simple forging case is evaluated. The velocity approach is realized by mean COMSOL with the Equation-Based Modeling and the Deformed Mesh Module. The Anand's material model is used for the no-linear part of material behavior.

Keywords: contact, continuous, velocity, non-linear, plasticity, Anand

1 Introduction

The contact among mechanical parts is a very important and difficult problem. Its mathematical modeling, theoretical and numerical, is a not concluded challenge. The aim of this work is to evaluate a simple contact model using the velocity as dependent variable [1] . A small amount of penetration is permitted in this contact model and a corresponding system of reactions stress is applied to the surface of deformable part. The contact have not friction.
To handle and solve equations is used COMSOL with its Equation-Based Modeling and Deformed Mesh Module.

2 Governing Equations

The structural equations used here are the same of [1] . Only momentum balance and constitutive equations are considered. The temperature θ and the density ρ are constant and uniform. The momentum balance is expressed by

$$\rho\,\partial_t v_i + \rho\,\mathbf{v}\cdot\nabla v_i = \partial_j \sigma_{ij} + f_i \tag{1}$$

The Anand's plasticity model [2] is expressed by

$$\mathbf{T}^{\nabla} = \mathcal{L}\left(\mathbf{D} - \mathbf{D}^p\right) \tag{2}$$

Where
$\mathbf{v}$ is the velocity,
$\mathbf{f}$ is the body force,
$\mathbf{T}$ is the Cauchy stress tensor ($T_{ij} \equiv \sigma_{ij}$),
$T_{ij}^{\nabla} \equiv \partial_t\sigma_{ij} + v_l\,\partial_l\sigma_{ij} - W_{il}T_{lj} + T_{il}W_{lj}$,
$\mathcal{L} \equiv 2\mu I + [k-(2/3)\mu]\mathbf{1}\otimes\mathbf{1}$ is the fourth order isotropic elasticity tensor,
$\mathbf{D} \equiv \mathtt{sym}(\mathbf{L})$ is the stretching tensor ($L_{ij} \equiv \partial_j v_i$),
$\mathbf{W} \equiv \mathtt{skew}(\mathbf{L})$ is the spin tensor,
$\mathbf{D}^p \equiv \dot{\tilde{\epsilon}}^p(3/2)(\mathbf{T}'/\tilde{\sigma})$ is the flow rule,
$\mathbf{T}'$ is the deviator of the Cauchy stress tensor,
$\tilde{\sigma} = \sqrt{(3/2)\mathbf{T}'\cdot\mathbf{T}'}$ is the equivalent tensile stress,
$\dot{\tilde{\epsilon}}^p \equiv A\exp\left(-Q/(R\theta)\right)\left(\sinh\left(\xi\tilde{\sigma}/s\right)\right)^{1/m}$ is the flow equation,
$\dot{s} = h_0\left|1-s/s^*\right|^a \operatorname{sign}\left(1-s/s^*\right)\dot{\tilde{\epsilon}}^p$ is the evolution equation with
$s^* = \tilde{s}\left[\dot{\tilde{\epsilon}}^p/A\exp\left(Q/(R\theta)\right)\right]$
Where θ is the temperature in $\mathtt{K}$.
The material parameters are: k, μ (Hooke and Anand), A, Q, m, ξ, h_0, a, $\tilde{s}$, s_0 (Anand).
The reaction stress in the point $\mathbf{P}$ is computed by

$$\mathbf{f}_r = \sigma_r\,\mathbf{n} \tag{3}$$

$$\sigma_r = \begin{cases} k_c\,\delta & \delta < 0 \\ 0 & \delta \geq 0 \end{cases} \tag{4}$$

Where $\mathbf{P}$ is a point of the surface of the deformable body, $\mathbf{n}$ is the normal to this surface and δ is the distance from $\mathbf{P}$ to the surface of the rigid body (if $\delta < 0$ there is penetration). The contact stiffness is k_c.

3 Methods

To study this contact model a simple forging case is simulated. The tool (the rigid body) is a sphere, the work-piece (the deformable body) is a hexahedron. Anand's material model is adopted. To execute this task, deformed geometry, displacements, stresses and penetration are evaluated.

4 Numerical Model

The simulations are performed with the software COMSOL 3.5a and on a `2 Quad-Core AMD Opteron(tm) Processor 2356 8GB RAM Linux WS`.

The elements utilized are Lagrange-linear hexahedron generated with a mapped mesh procedure. All the simulations are transient. Equation-Based Modelling and Deformed Mesh Module are utilized.

The Radius of the sphere is $0.1\,\mathtt{m}$, the sides of the hexahedron are $(0.05, 0.1, 0.1)\,\mathtt{m}$, the velocity of the sphere is $(-0.001, 0, 0)\,\mathtt{m/s}$. The contact stiffness is $1\mathtt{E}{+}12\,\mathtt{N/m^3}$

Only a quarter of the work-piece is modeled utilizing two plane of symmetry ($Y = 0$, $Z = 0$).

The sphere is analytically defined and do not have a geometry to show in COMSOL.

For the coordinate system has been adopted the COMSOL nomenclature: $(x, y, z)_{\mathtt{ale}}$ is the spatial frame, $(X, Y, Z)_{\mathtt{ref}}$ is the reference frame.

The material parameters are reported in the following tables.

Hooke

E	$70\,\mathtt{MPa}$
ν	0.3

Anand

E	$105\,\mathtt{GPa}$
ν	0.41
A	$6.346\,\mathtt{E}{+}11\,\mathtt{s^{-1}}$
Q	$312.35\,\mathtt{kJ/mol}$
m	0.1956
ξ	3.25
h_0	$3093.1\,\mathtt{MPa}$
a	1.5
$\tilde{s}$	$121.1\,\mathtt{MPa}$
R	$8.314472\,\mathtt{J/(mol\,K)}$
s_0	$66.1\,\mathtt{MPa}$

The Anand's parameters are pertinent to a BCC polycrystalline Fe-2% silicon alloy at a temperature $\theta = 1000\,\mathtt{C}$ [2].

5 Experimental Results

The results are reported in the next pictures and graphs. In pictures from 1 to 4 the x component of the displacement $\mathbf{u}$ at various time is reported.

In pictures from 5 to 8 the penetration δ at various time is reported.

In picture 9 the Von Mises's equivalent stress is reported.

In picture 10 the forging force is reported.

6 Discussion

The deformed geometry and displacements obtained from COMSOL are those foreseen. The penetration δ is just for the mesh and element utilized. So, the goodness of the COMSOL simulation is demonstrated.

7 Conclusions

The contact model evaluated in this work is suitable to model the forging process. The simulation have not special numerical difficulties. For a given mesh and element it is possible to choose the optimum value of k_c avoiding numerical not-convergence.

8 Use of COMSOL Multiphysics

COMSOL Equation-Based Modelling and Deformed Mesh Module are tools very suitable for the task to evaluate different and new contact models.

9 Figures

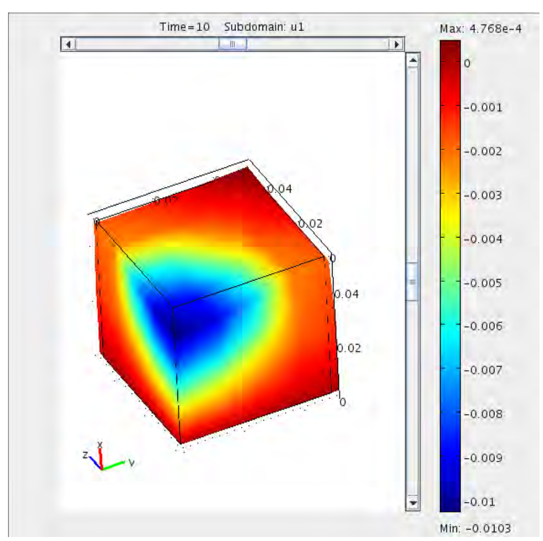

Figure 1: $u_x(t = 10\,\mathtt{s})$

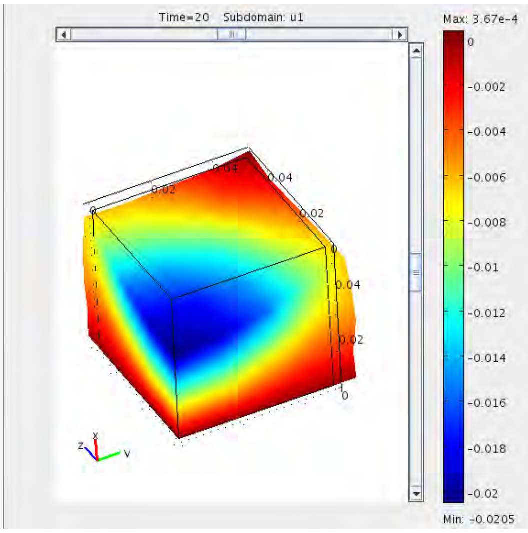

Figure 2: $u_x(t = 20\,\mathrm{s})$

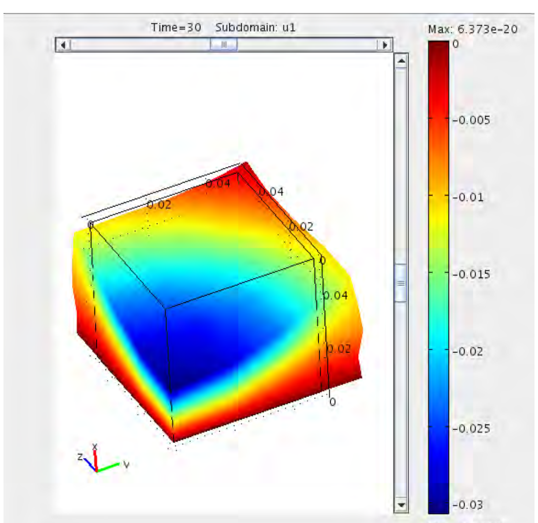

Figure 3: $u_x(t = 30\,\mathrm{s})$

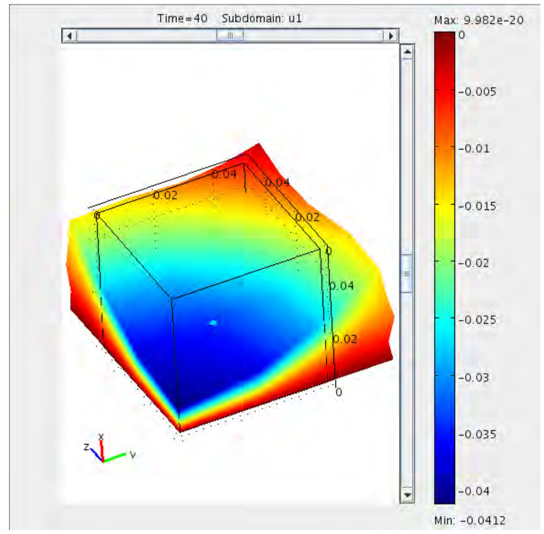

Figure 4: $u_x(t = 40\,\mathrm{s})$

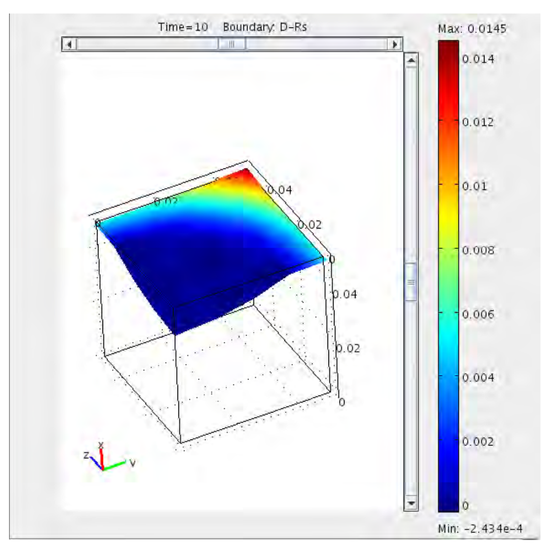

Figure 5: $\delta(t = 10\,\mathrm{s})$

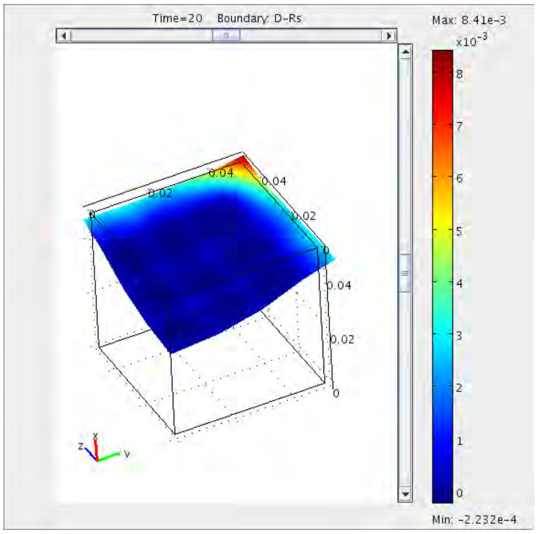

Figure 6: $\delta(t = 20\,\mathbf{s})$

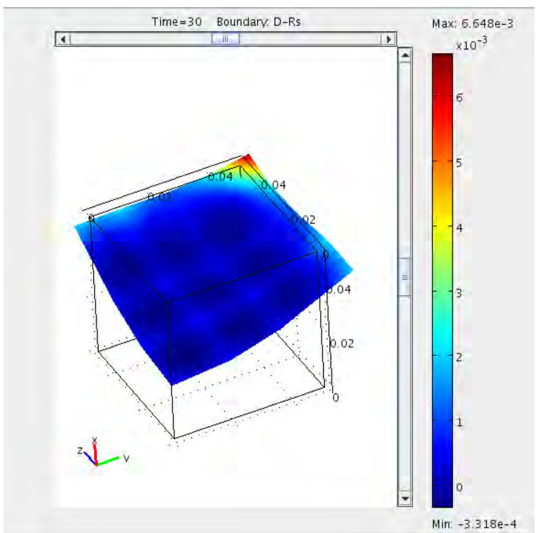

Figure 7: $\delta(t = 30\,\mathbf{s})$

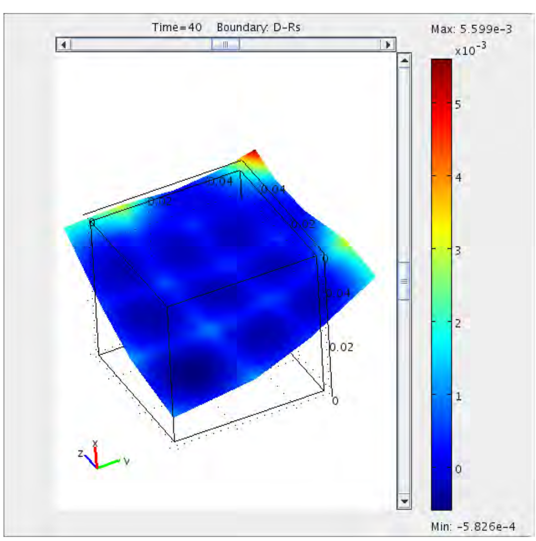

Figure 8: $\delta(t = 40\,\mathbf{s})$

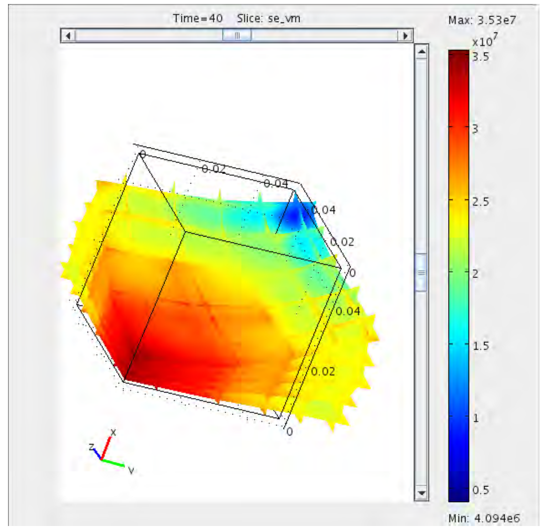

Figure 9: $\sigma_e(t = 40\,\mathbf{s})$

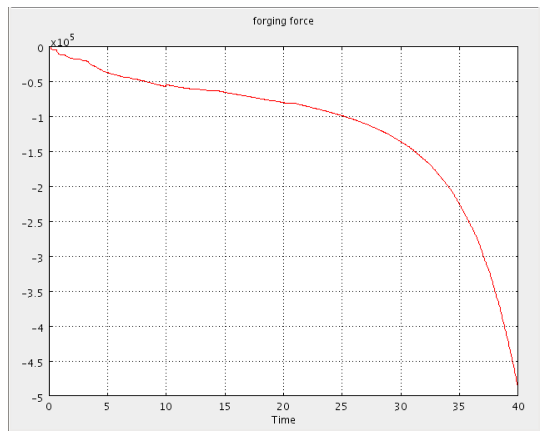

Figure 10: forging force

References

[1] M. Vannucci O. Toscanelli, V. Colla, *Equation-based modelling: True large strain, large displacement and anands plasticity model with the comsol deformed mesh module*, COMSOL CONFERENCE 2009.

[2] Kwon H. Kim Stuart B. Brown and Lallit Anand, *An internal variable constitutive model for hot working of metals*, International Journal of Plasticity **5** (1989), 95–130.