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Passive and Active Deformation Processes of 3D Fibre-Reinforced Caricatures of Cardiovascular Tissue

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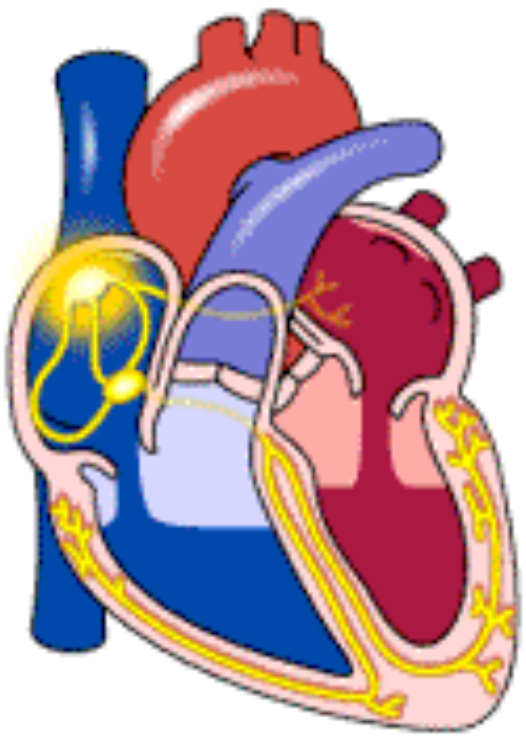
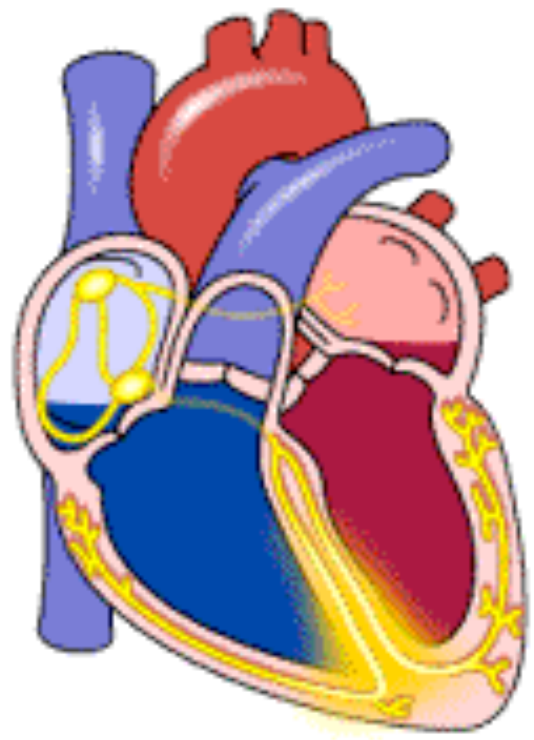
Heart as Muscular Pump

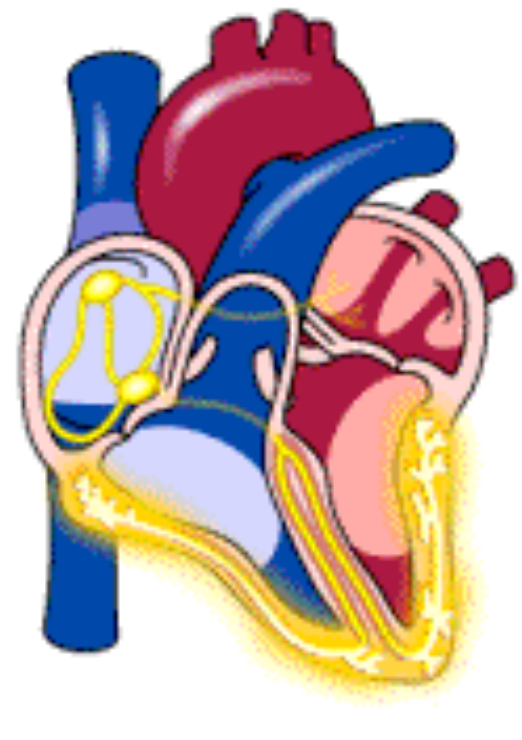
Filling



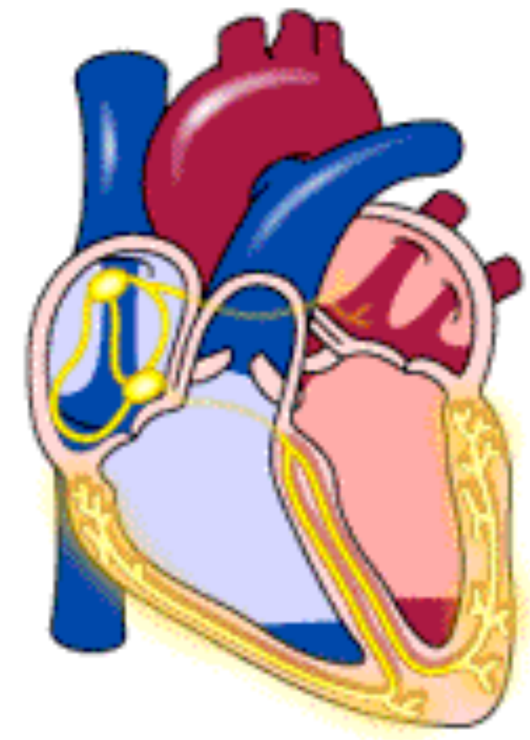
Contraction



Ejection

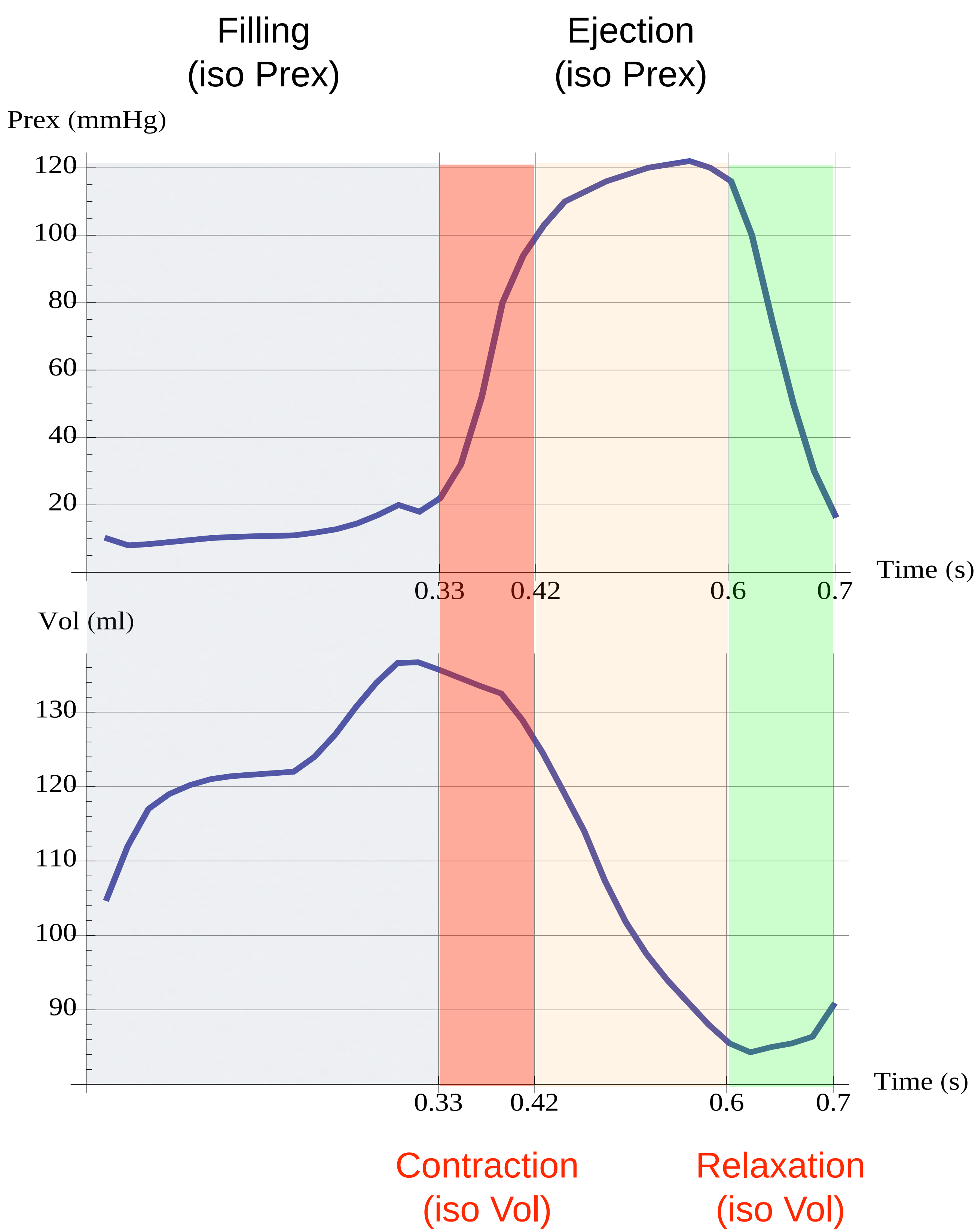


Relaxation

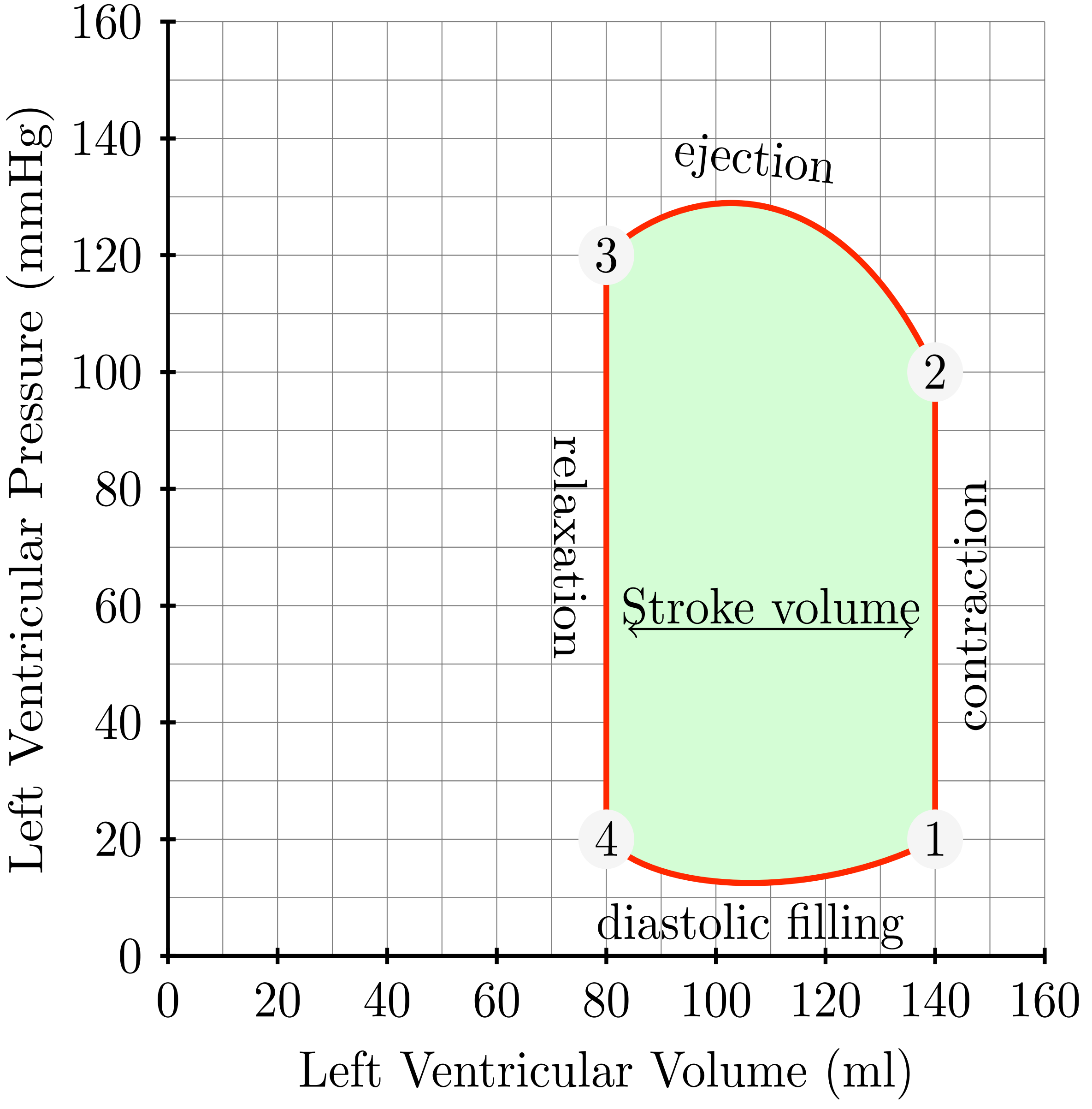


The 4 key phases of cardiac cycle

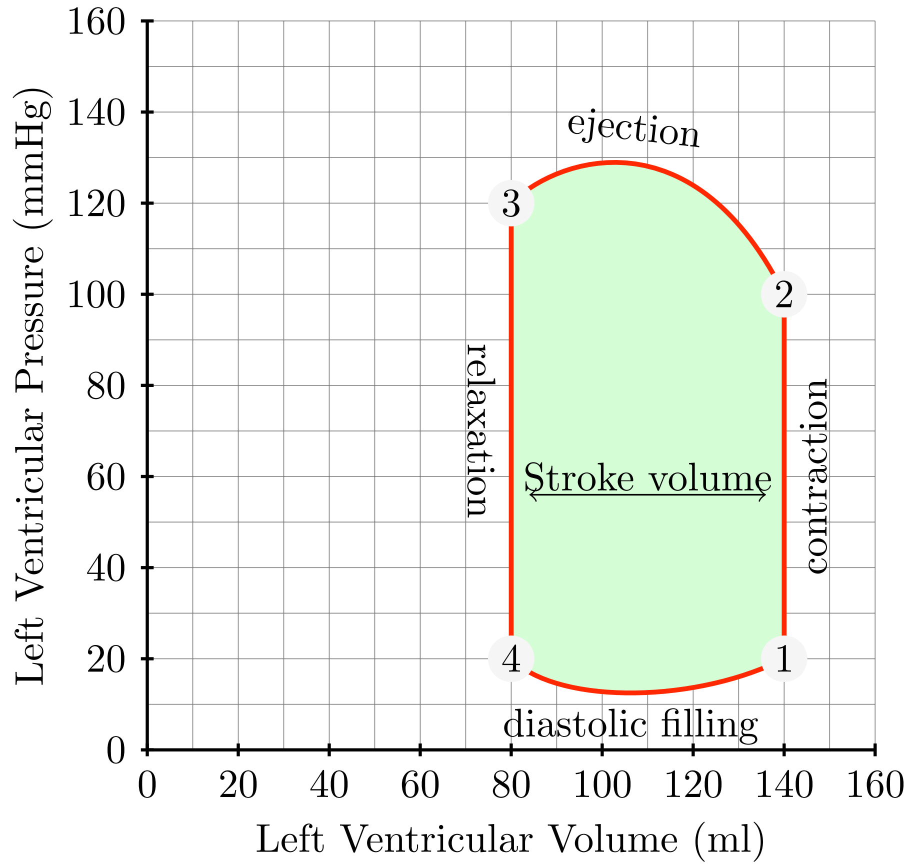
Pressure & Volume Time Course



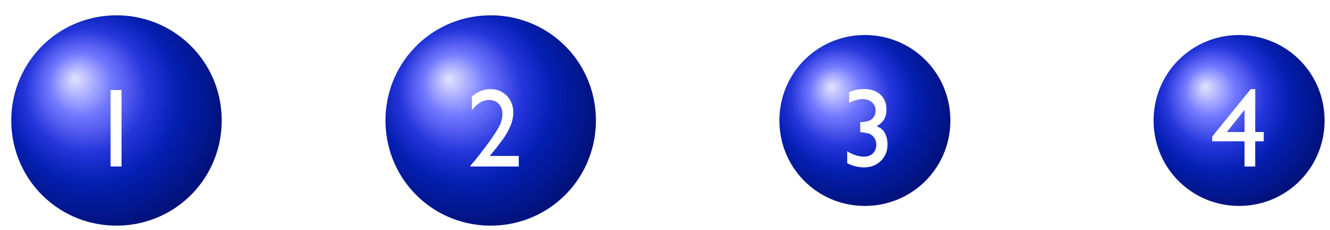
PV - loop



Modeling Muscle Contraction



Visible state

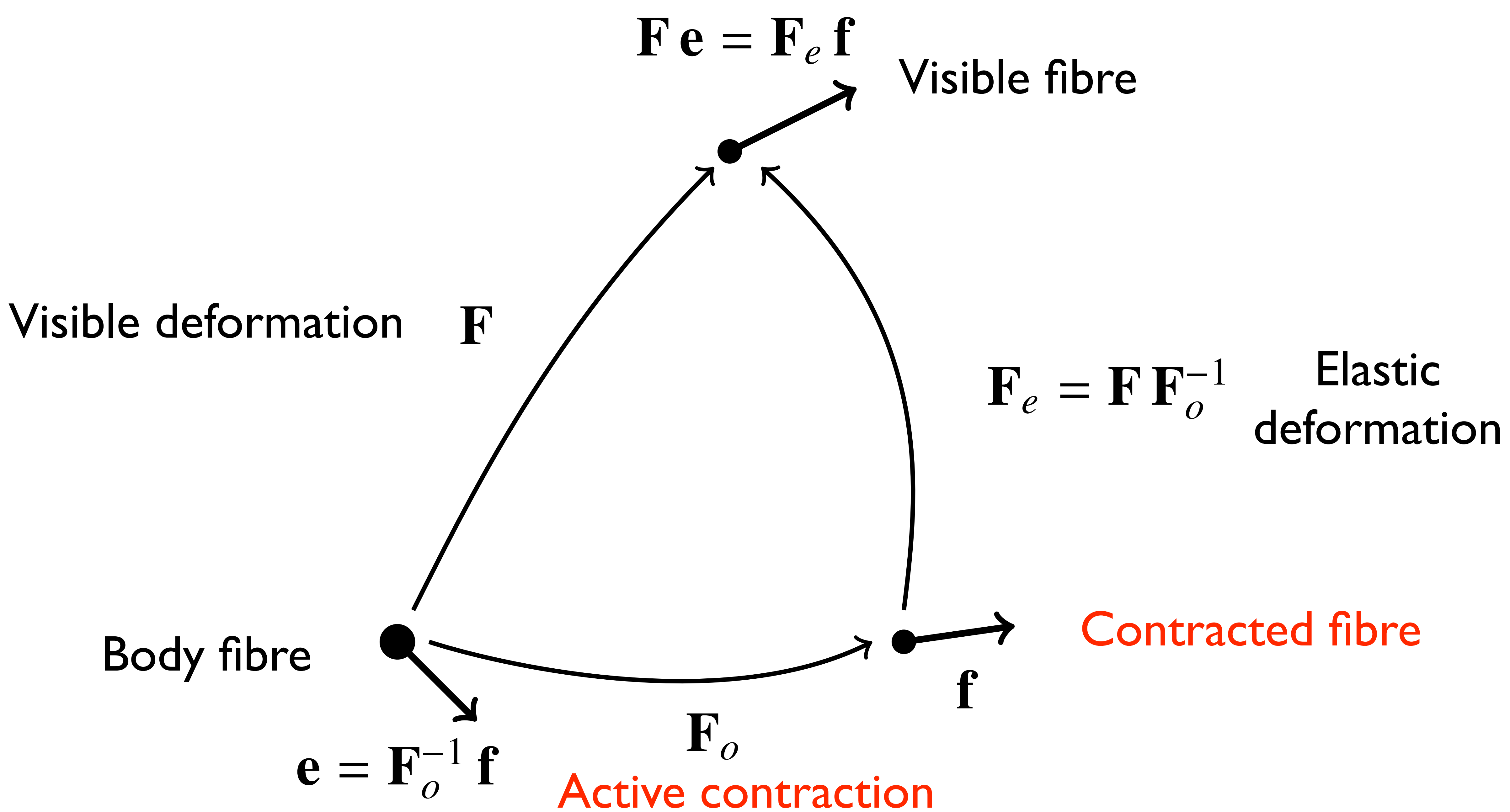


$r_1 = 3.19$ $r_2 = 3.16$ $r_3 = 2.75$ $r_4 = 2.78$

Contracted state

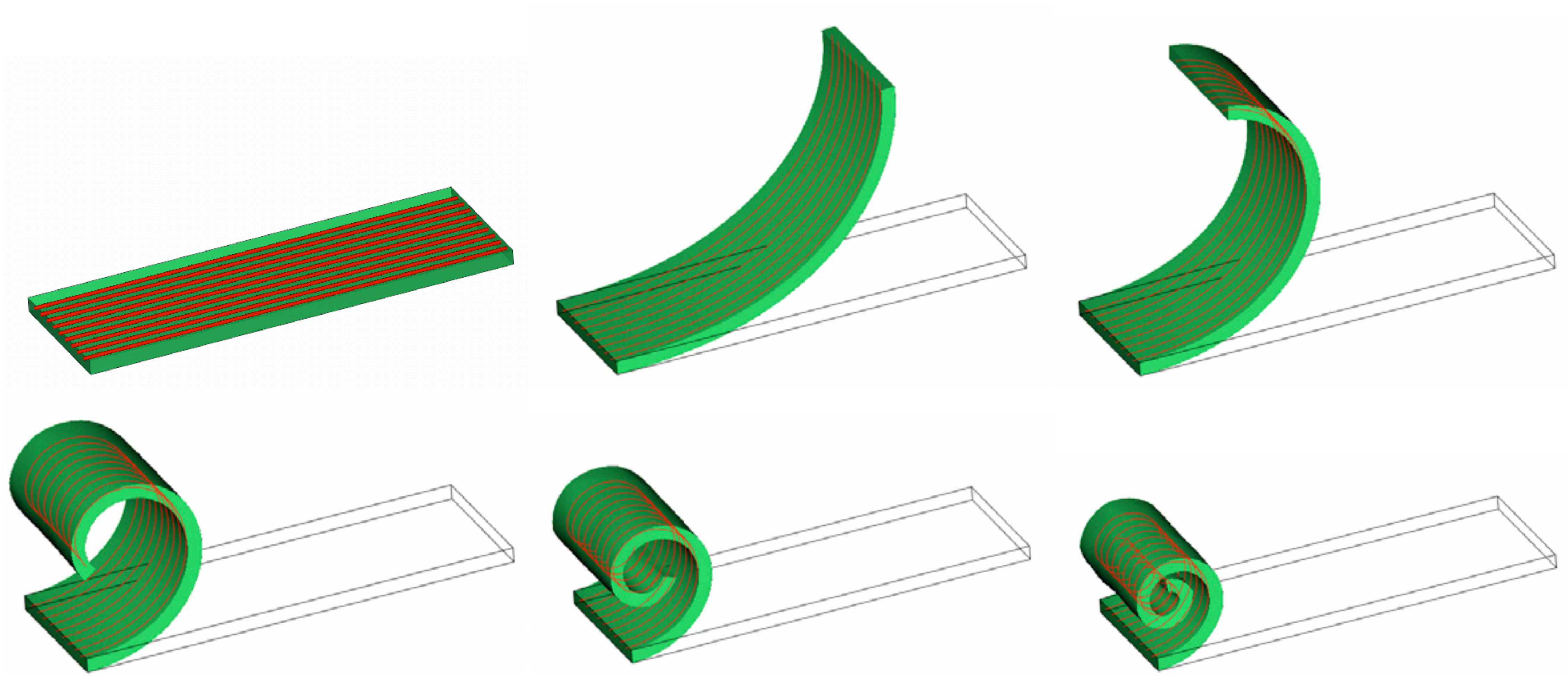
$r_{c,1} = 2.23$ $r_{c,2} = 1.91$ $r_{c,3} = 1.62$ $r_{c,4} = 1.97$

Kröner-Lee Decomposition



Non linear Elasticity with Large Distortions

Extremely Effective!



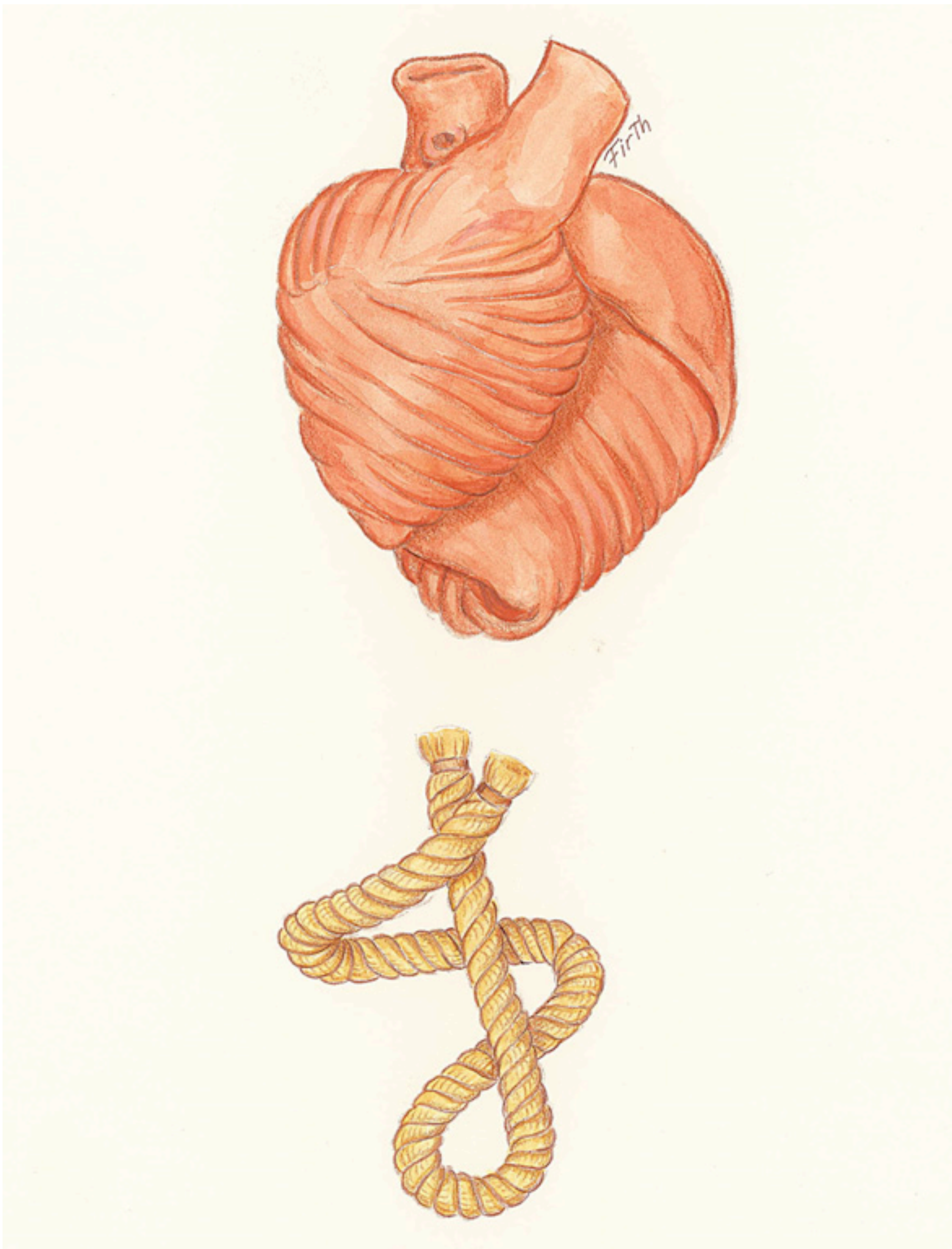
A slender elastic plate reinforced with longitudinal contractile fibres.

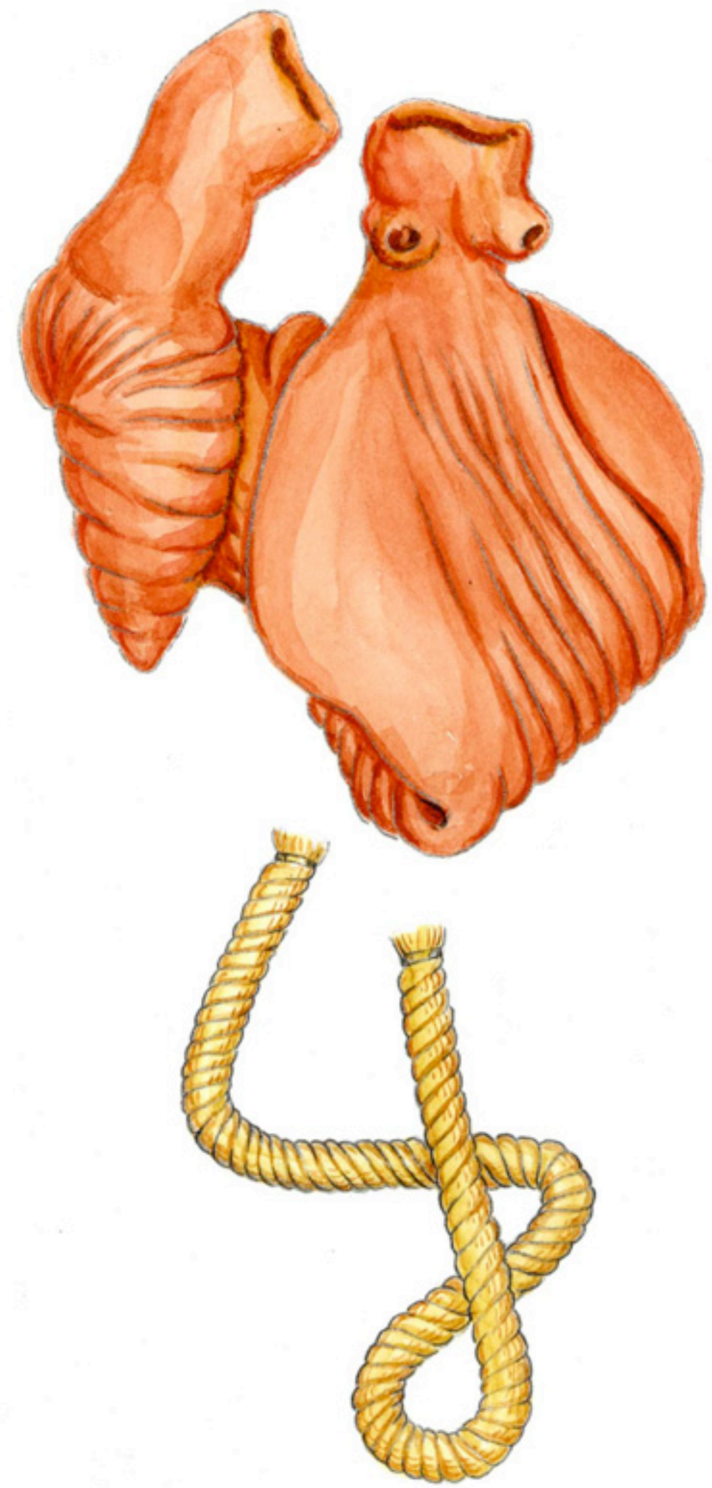
The overall shape of the plate can be controlled to a very large extent by modulating the fibre contraction. The first snapshot shows the plate shape when all fibres are inactive; the others correspond to differential through-the-thickness contractions of increasing intensity.

Key Issues

- Active contractions
- Passive response
- Anisotropy
- Fibres architecture
- ...
- Mechano-electro-physiology

Fibres Architecture





The Helical Heart Company

Modeling Muscle Contraction

Deformation measure

$$\mathbf{C}_e = \mathbf{F}_e^\top \, \mathbf{F}_e = \mathbf{F}_o^{-\top} \, \mathbf{C} \, \mathbf{F}_o^{-1} \,, \qquad \mathbf{C} = \mathbf{F}^\top \, \mathbf{F}$$

Contraction

Elastic energy

$$\hat{\psi}(\mathbf{C}_e) = \frac{1}{2}\mu\big((I_1(\mathbf{C}_e) - 3) + \gamma_4 \, (I_4(\mathbf{C}_e) - 1)^2 + \gamma_5 \, (I_5(\mathbf{C}_e) - 1)^2 \big) + \psi(\mathrm{Vol})$$

fibre stiffness moduli

Strain invariants

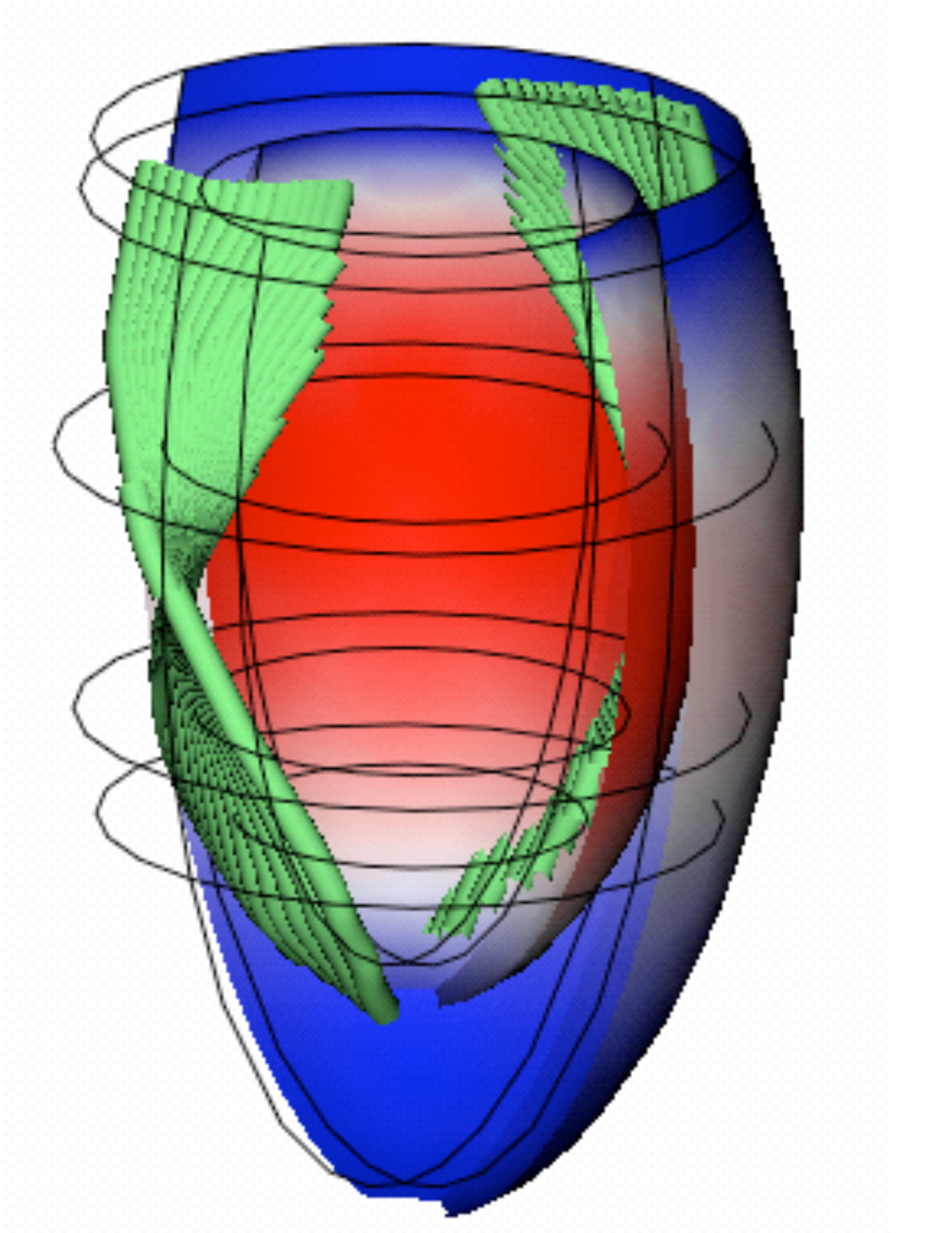
$$I_4(\mathbf{C}_e) = \mathbf{C}_e \cdot \mathbf{e} \otimes \mathbf{e} \,, \qquad I_5(\mathbf{C}_e) = \mathbf{C}_e^2 \cdot \mathbf{e} \otimes \mathbf{e}$$

fibre direction

Fibred Caricature of the Left Ventricle

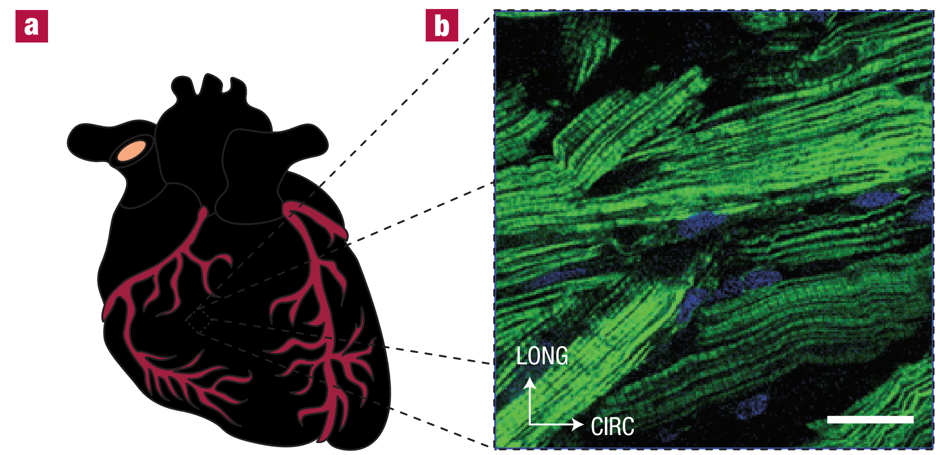
Goals

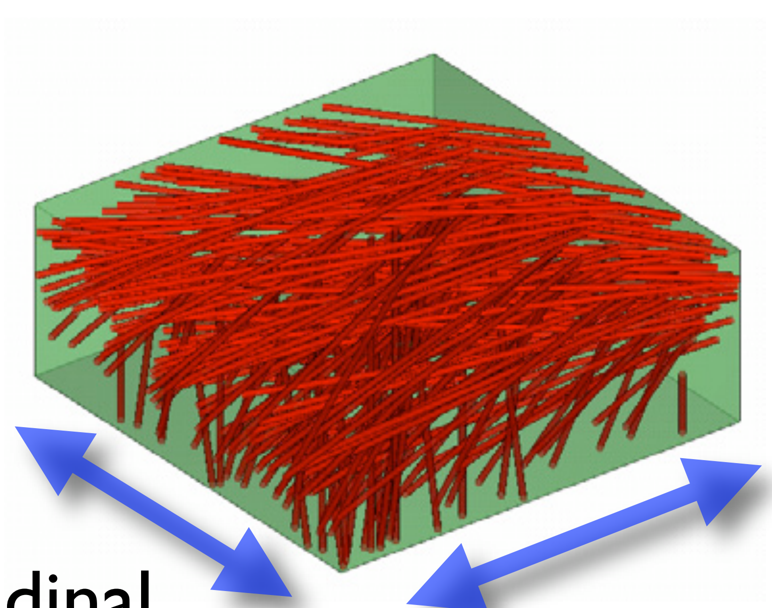
- Capture passive response
- Reproduce End Diastolic & End Systolic PV relationships
- Reproduce an actual PV loop
- Capture contraction-volume loop
- ...
- Coupling contraction with electrophysiology



Tuning the Material Response

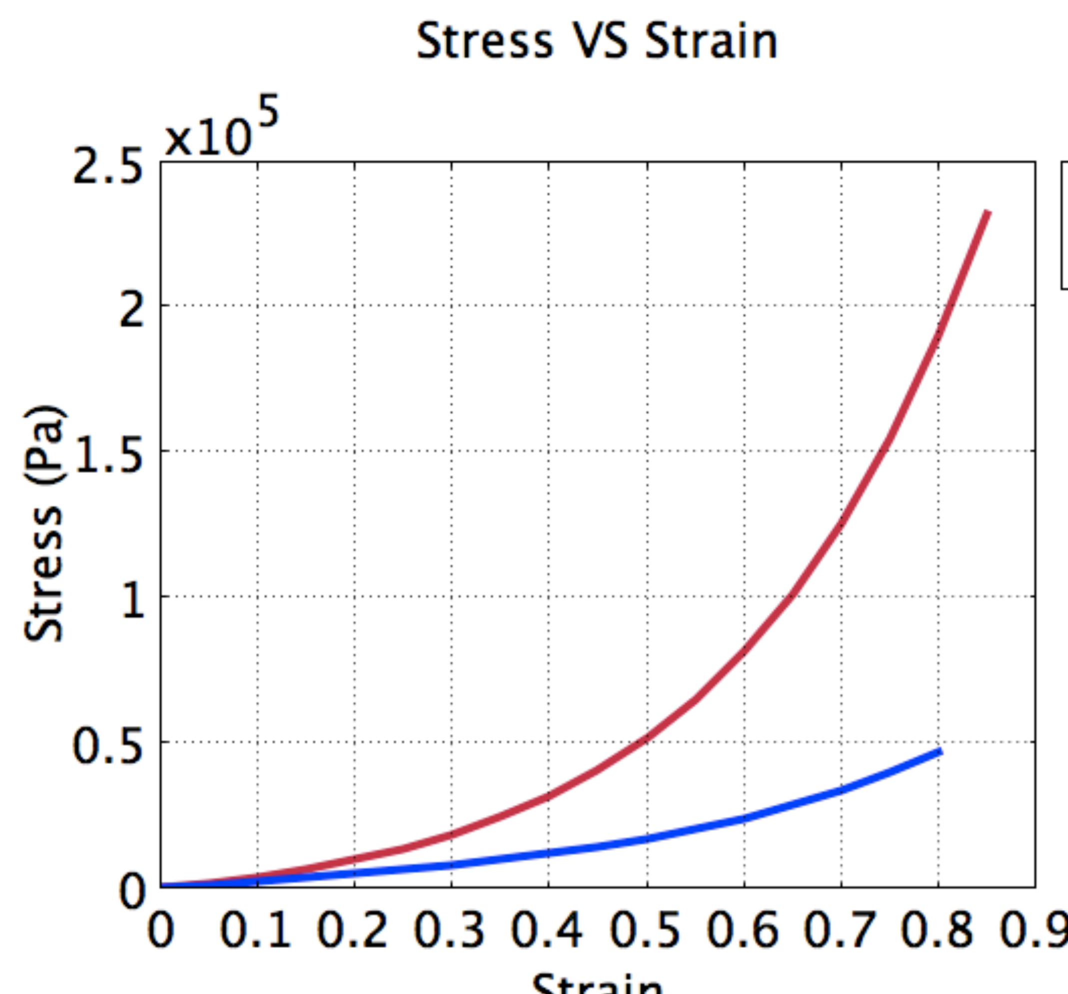
$$\frac{1}{2}\mu\Big((I_1(\mathbf{C}_e) - 3) + \gamma_4 \, (I_4(\mathbf{C}_e) - 1)^2 + \gamma_5 \, (I_5(\mathbf{C}_e) - 1)^2 \Big)$$





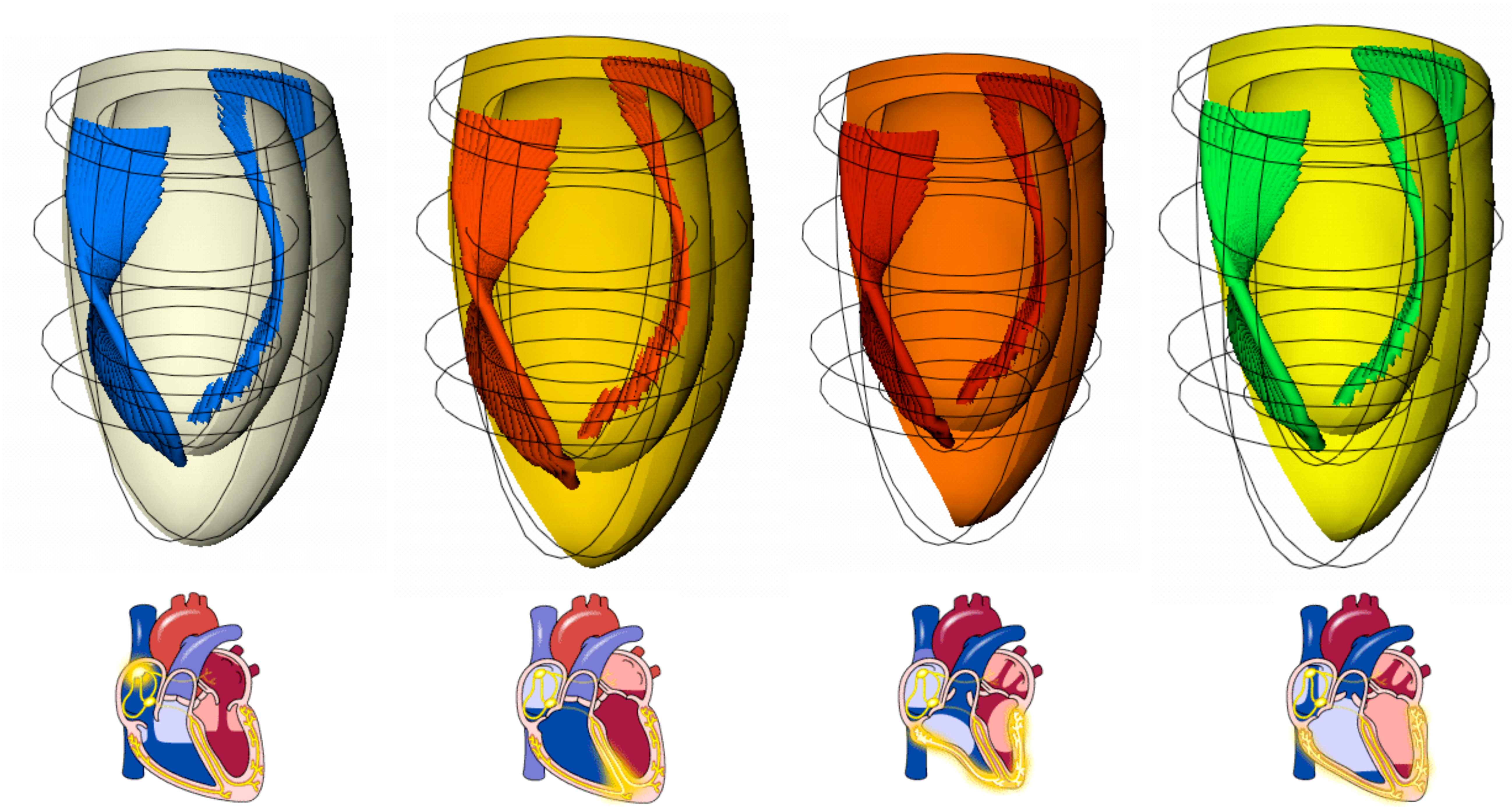
Longitudinal

Circumferential

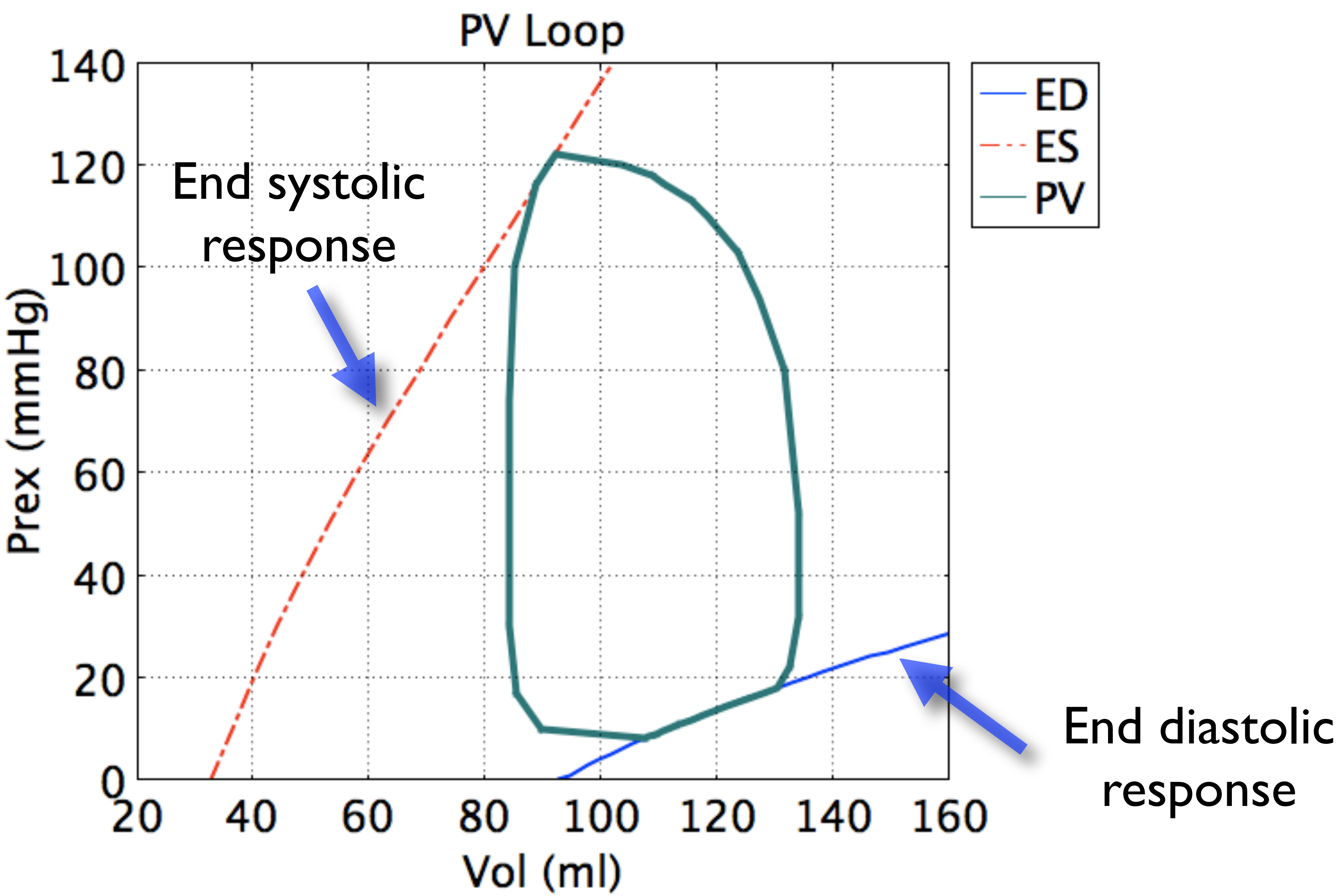


The LV Cycle

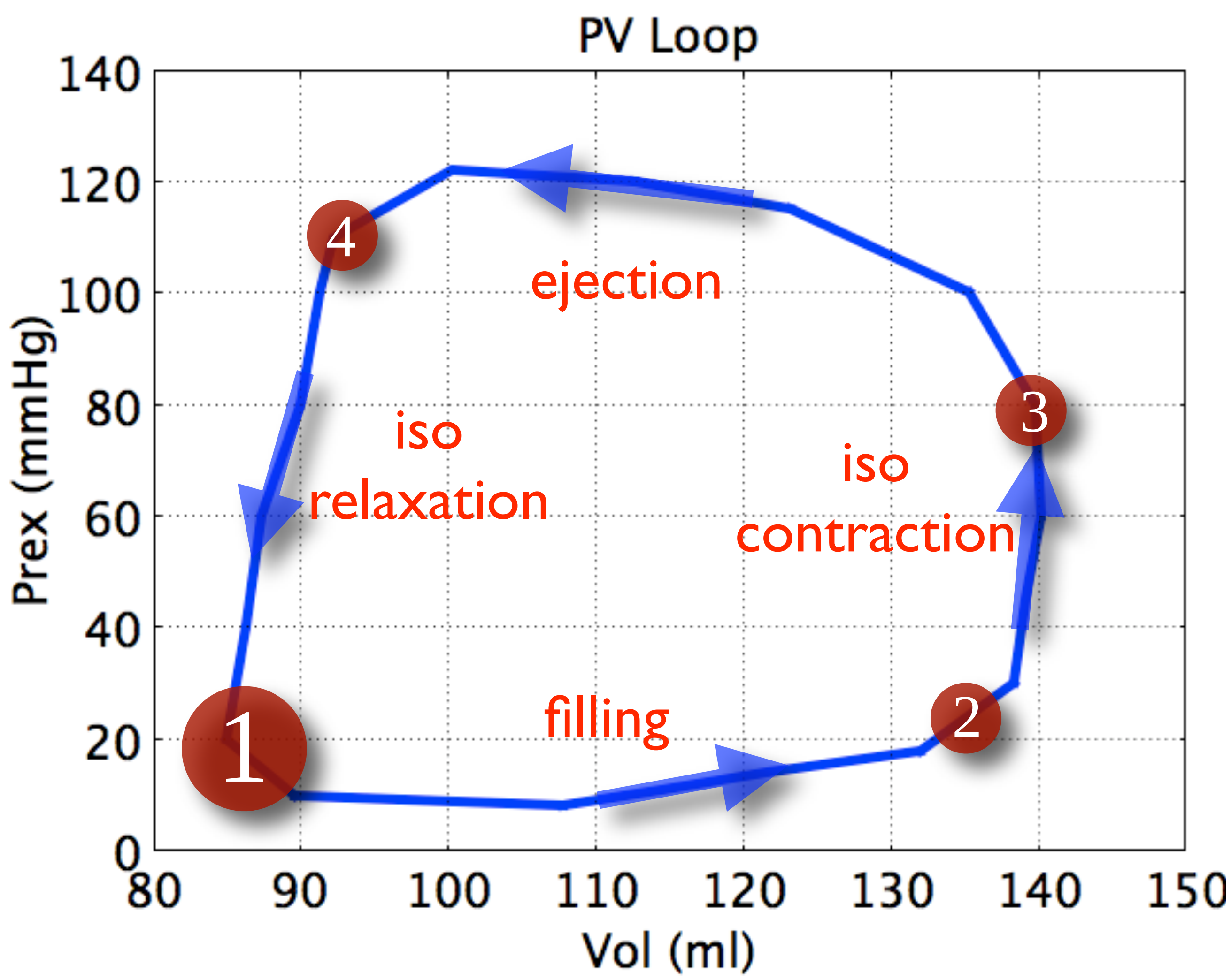
Filling | Contraction | Ejection | Relaxation

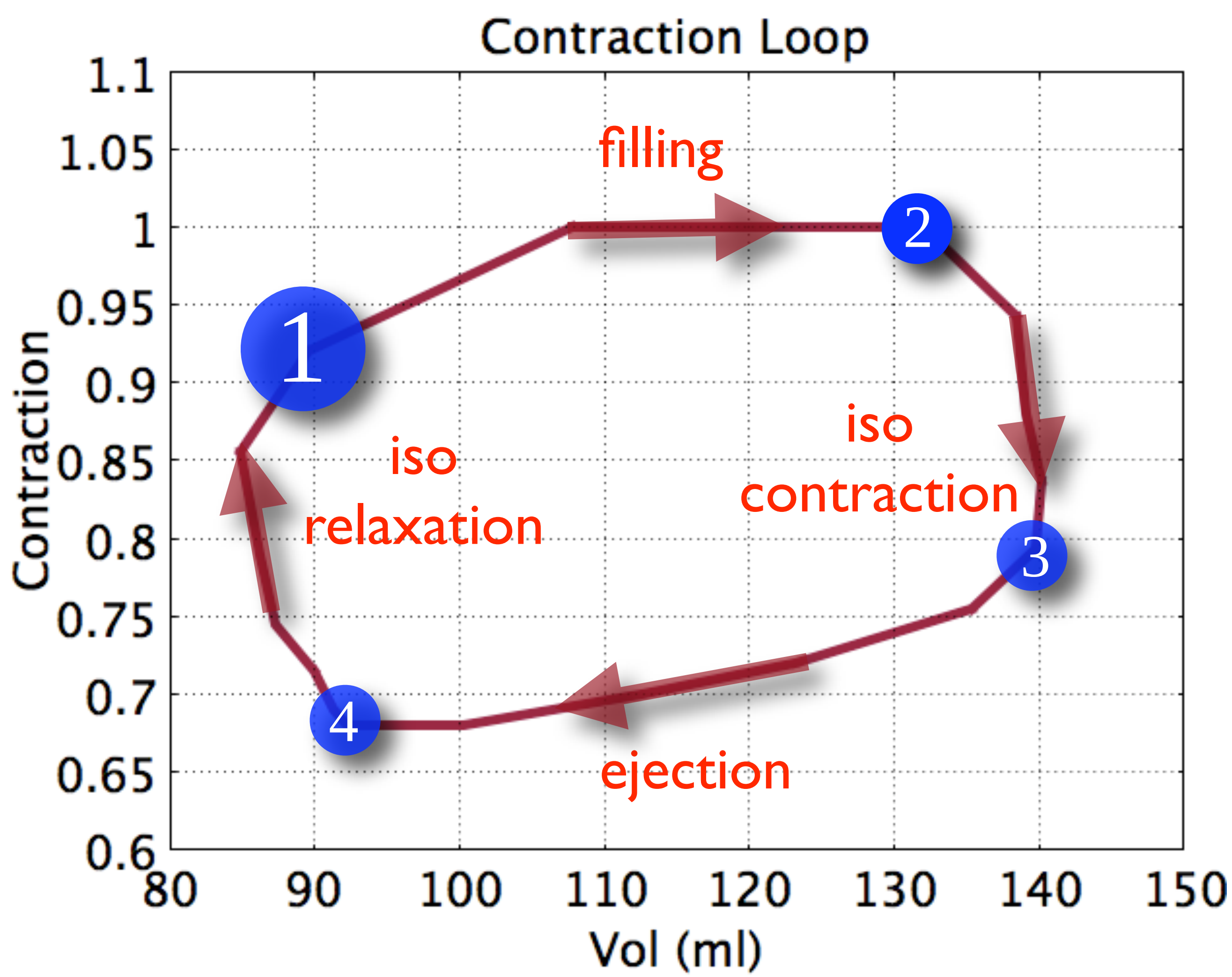


End Diastolic & End Systolic Pressure-Volume Relationships



Pressure-Volume Loop
&
Contraction - Volume Loop





Acknowledgements

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