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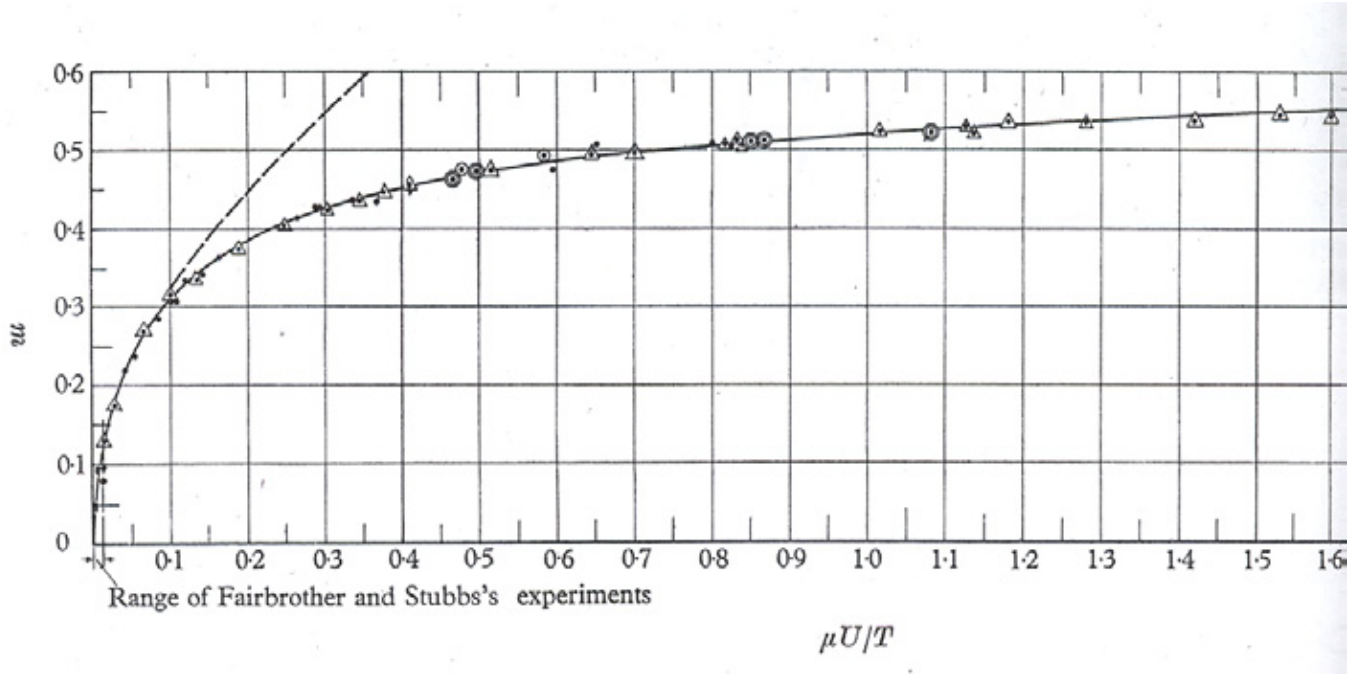
Flow and Mixing in the Liquid between Bubbles

Bruce A. Finlayson
Professor Emeritus, University of Washington
Seattle, WA 98195-1750
finlayso@u.washington.edu

G. I. Taylor, J. Fluid Mech. 10, 161 (1961)

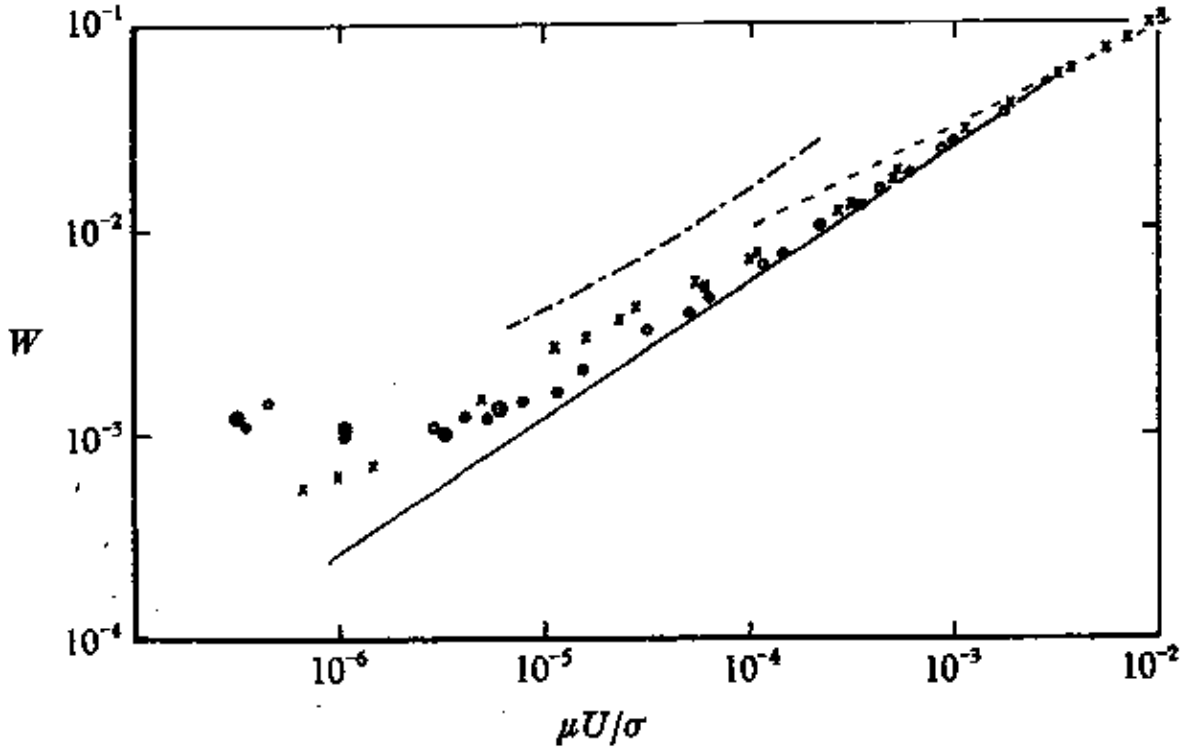
$$m = \frac{U_b - U}{U_b}, \quad m = 1.0(\eta U / \sigma)^{1/2} = 1.0\, Ca^{1/2}$$

$U = \text{velocity of liquid}, \quad U_b = \text{velocity of bubble}$

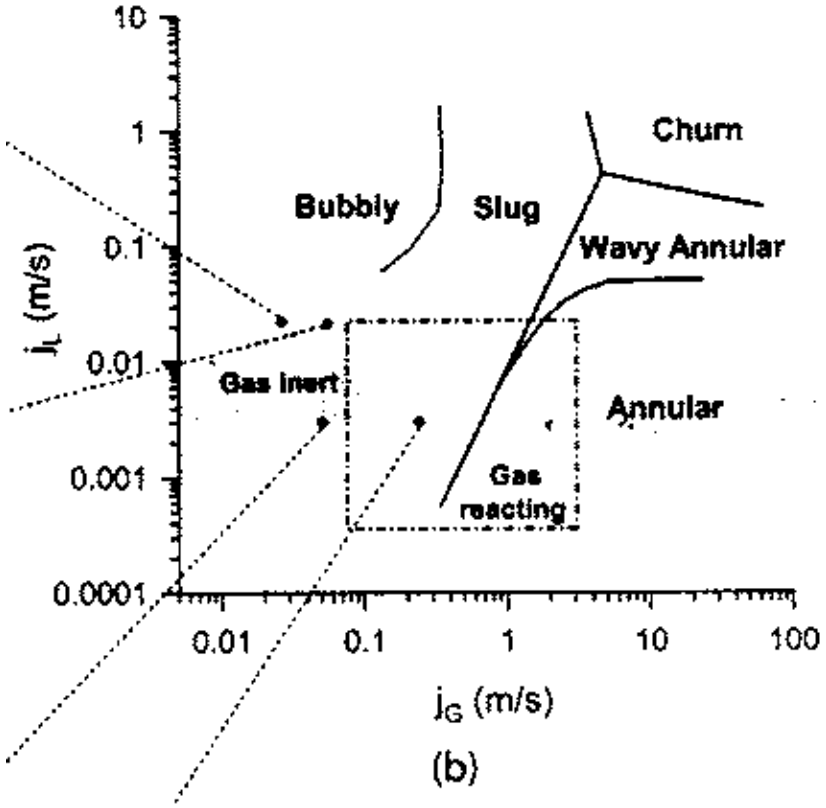


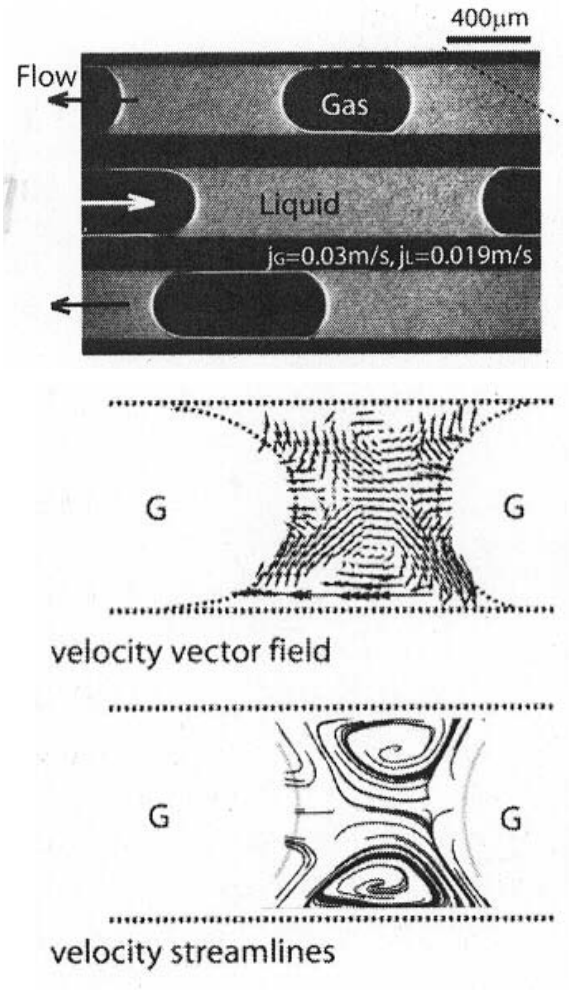
F. P. Bretherton, J. Fluid Mech. 10, 161 (1961)

$$m = 1.29(3Ca)^{2/3}$$

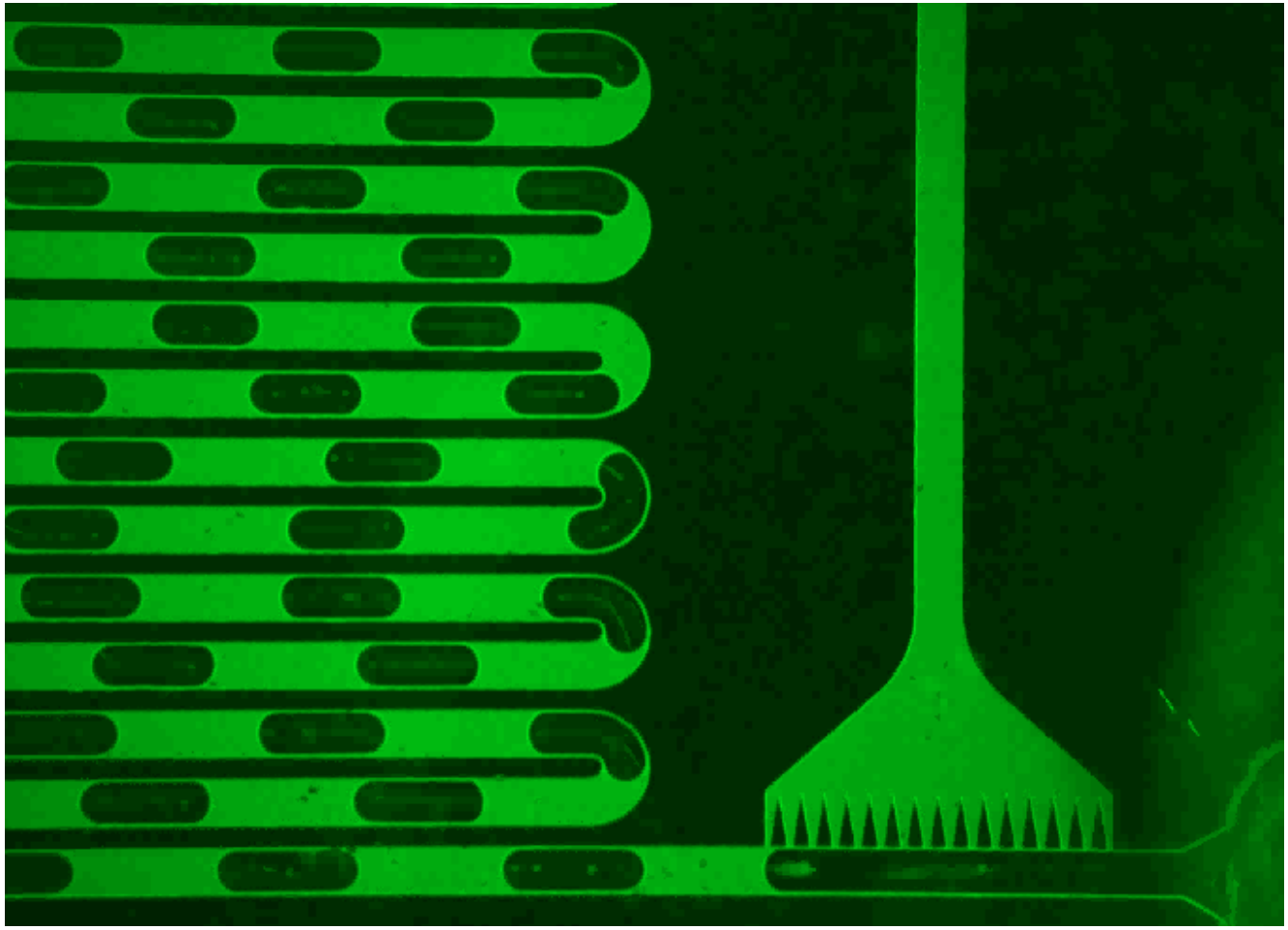


Gunther,Khan, Thalmann, Trachsel, Klavs F. Jensen, Lab Chip, 4, 278 (2004)



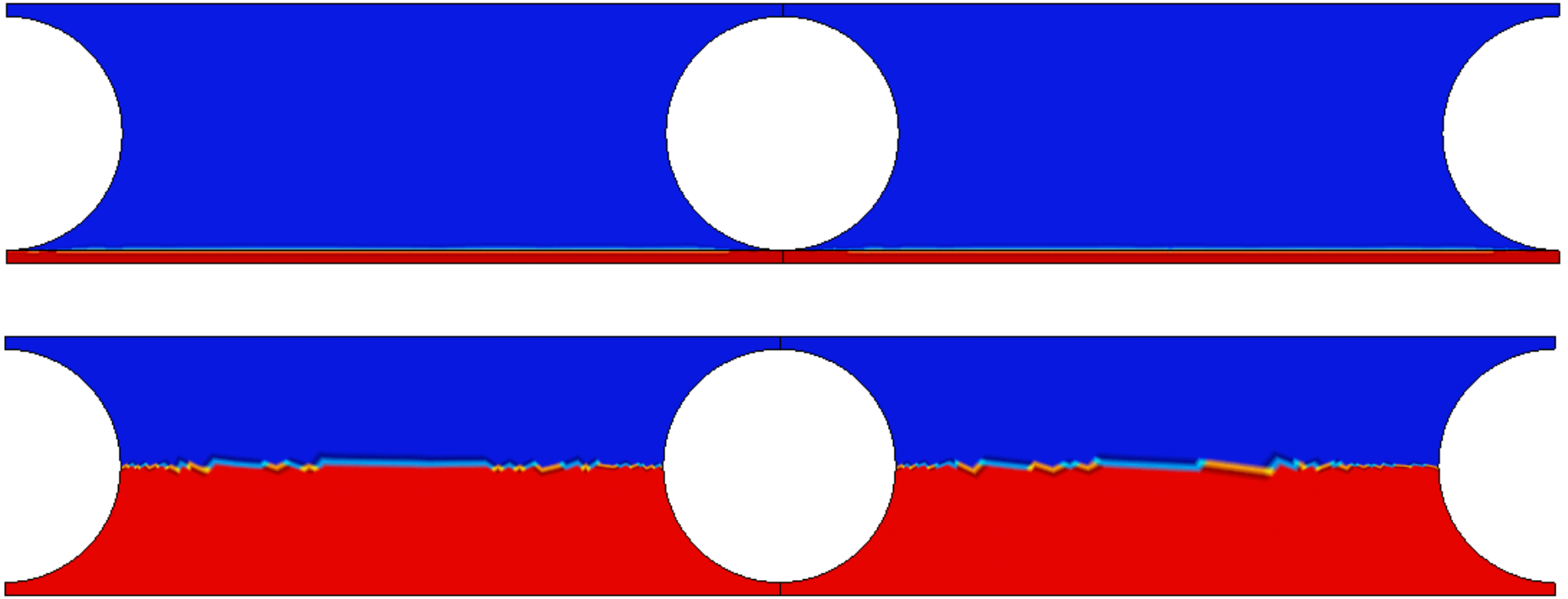


http://www.rsc.org/suppdata/lc/b4/b403982c/



Goal: quantify the mixing that occurs between the bubbles

- Case 1 – thin layer at bottom
- Case 2 – half and half



Flow – key assumption: assume a reasonable bubble shape, but don't solve for it; flow is steady

- Solve in a coordinate system moving with the bubble (velocity U_b)
- Non-dimensional equations

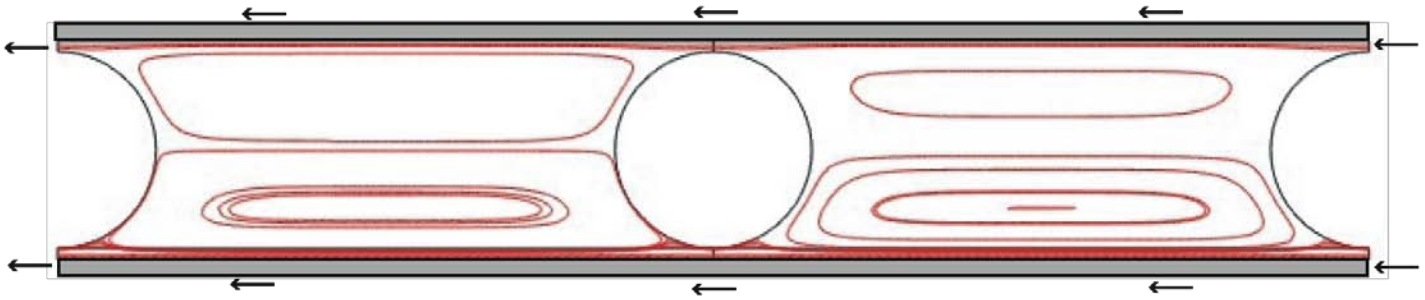
$$Re\,\mathbf{u}\bullet\nabla\mathbf{u} = -\nabla p + \nabla^2\mathbf{u}, Re = 1$$

$\mathbf{u} = -1$ on solid boundary

zero stress on gas - liquid interface (perfect slip)

velocity specified at inlet to match these conditions
and have a specified flow rate

flow out the end



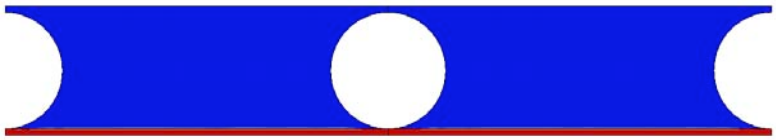
Unsteady Convection and Diffusion

- Concentration = 0 at one inlet, = 1 at other
- Zero flux on all bubble surfaces and wall
- Convective flux out
- Initial concentration for Case 1 and Case 2

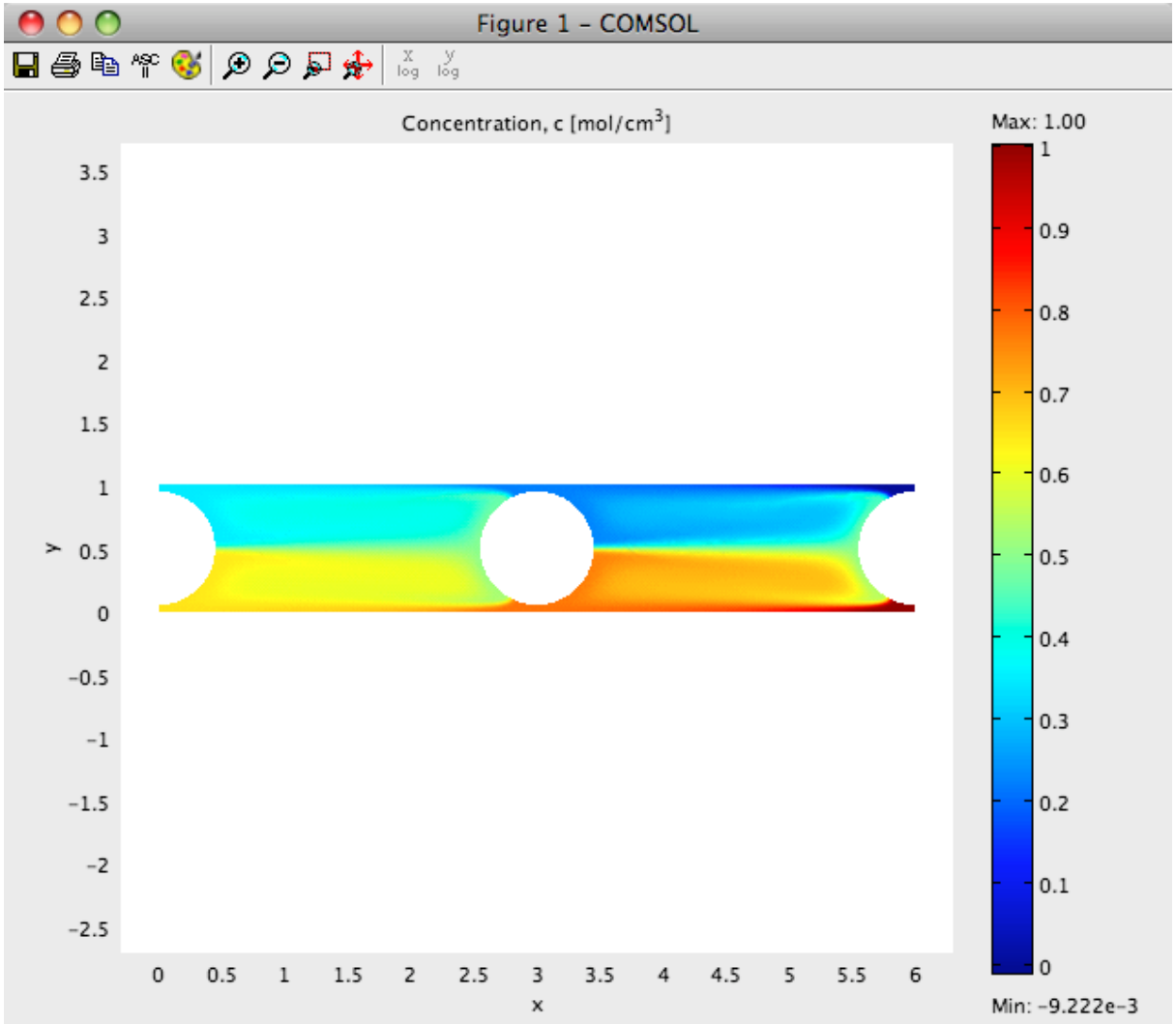
$$\frac{\partial c}{\partial t} + \mathbf{u} \bullet \nabla c = \frac{1}{Pe} \nabla^2 c$$

$$\text{cavg} = \int_V c dV / V$$

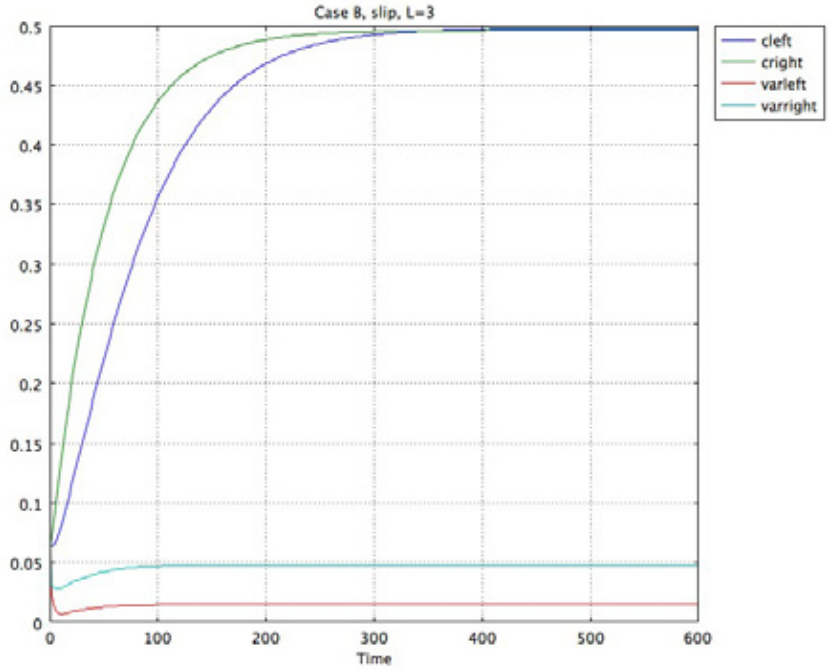
$$\text{variance} = \int_V (c - cavg)^2 dV / V$$

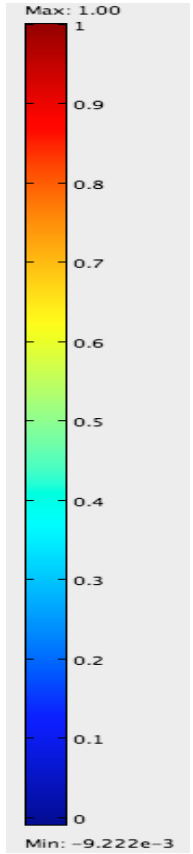


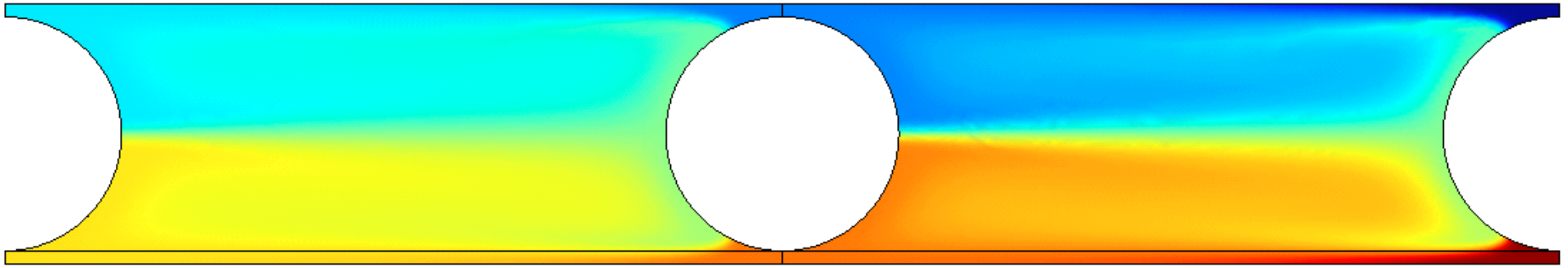
Concentration Development



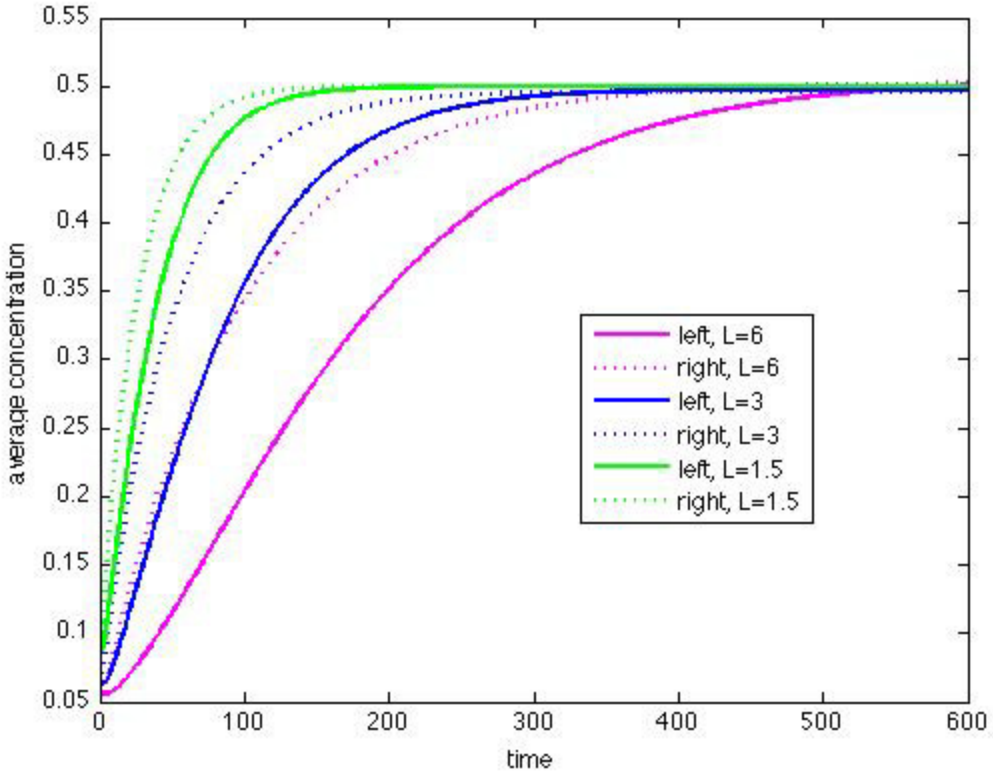
Case 1 – B – base case







Effect of distance between bubbles

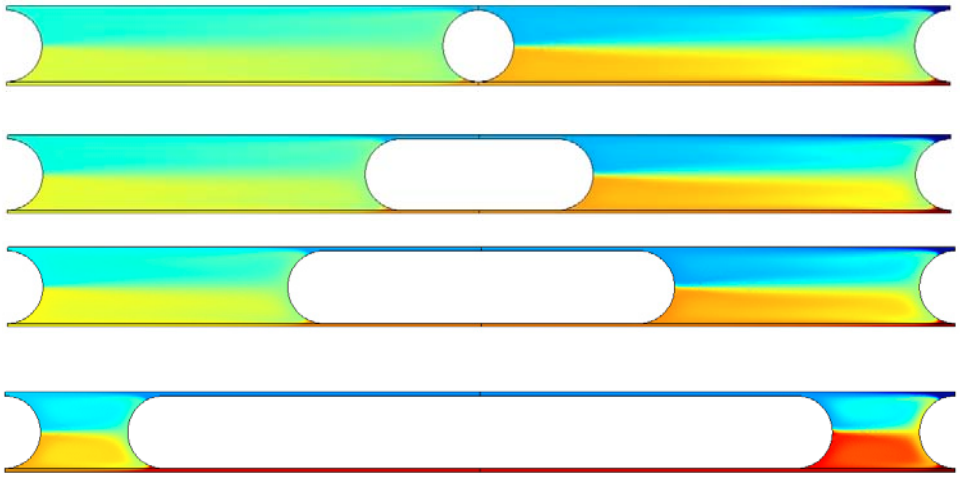


Variances
L = 6: 0.0049/.028
L = 3: 0.014/.048
L = 1.5: 0.055/.094

- For bubbles closer together, the variance is higher but it doesn't take as long to reach steady state

Effect of elongated bubbles

- Little effect on the approach to steady state but the variance increases as the length of the bubble increases



Bubble Length –

Left Variance/Right Variance

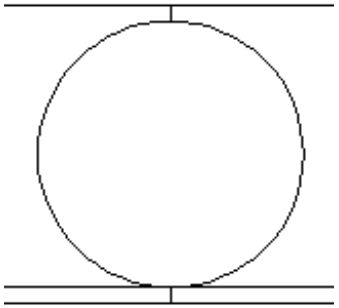
1 - .0049/.028

2 - .0073/.032

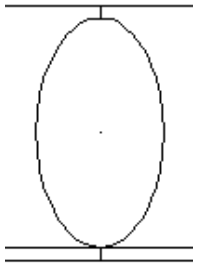
4 - .012/.039

6 - .043/.071

Effect of bubble shape



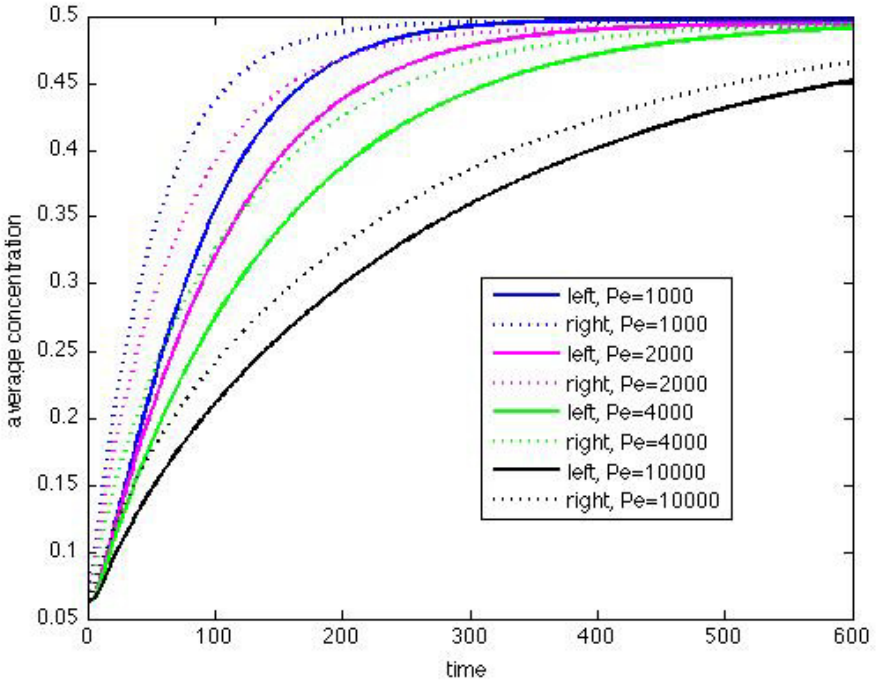
Variance = 0.014/0.048



Variance = 0.011/0.041

Shape makes little difference.

Effect of Peclet number



Peclet No. - Variance

1000 - .014/.048

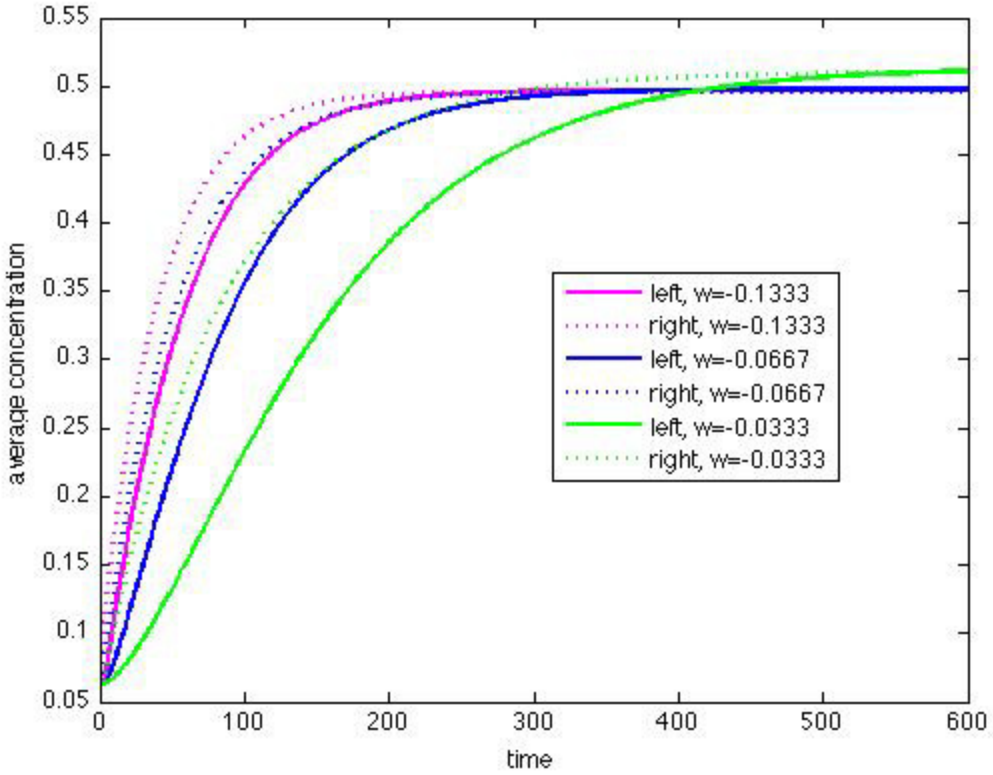
2000 - .024/.058

4000 - .034/.063

10000 - .041/.063

- As the Peclet number increases, it takes longer to reach steady state and the steady variance is larger

Effect of leakage rate



Leakage Rate/Variance

-.13 - .046/.077

-.07 - .014/.048

-.03 - .0026/.021

-.01 - 4e-4/.01

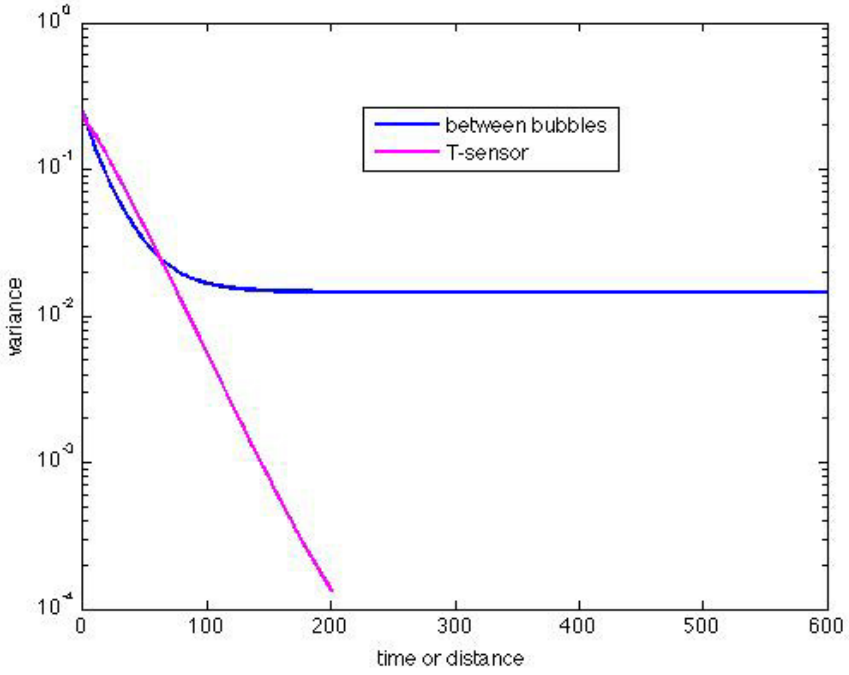
- As the leakage rate increases, the time to steady state decreases, and the variance increases

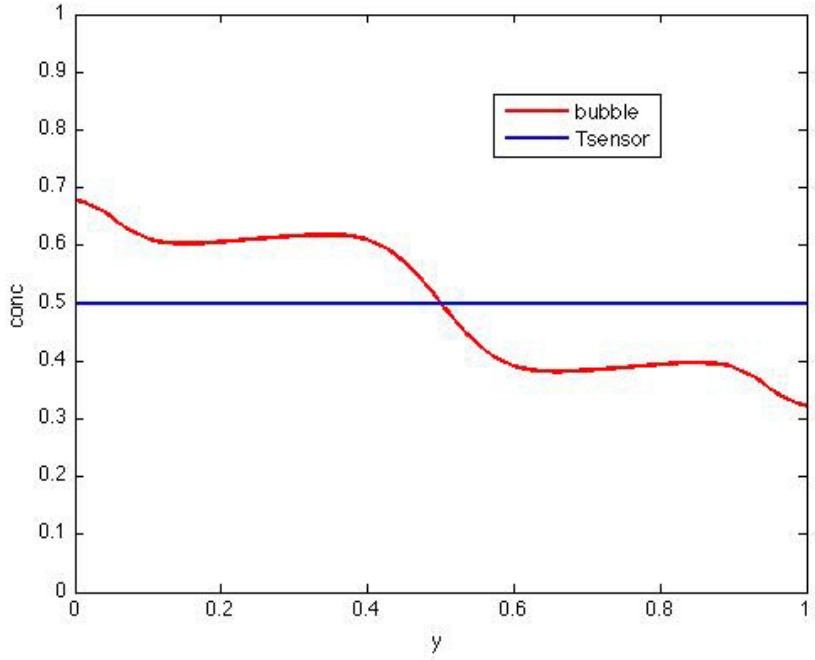
Effect of gap thickness

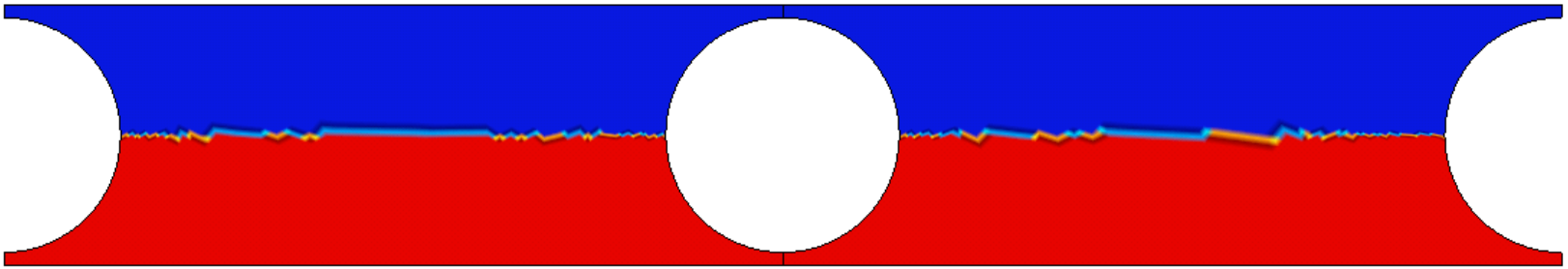
- The smallest variances are for smaller gaps and smaller leakage rates
- A leakage rate of -0.01 is an average of what is predicted by Taylor (-0.012) and Bretherton (-0.0072) for a Capillary number of 0.00014 (air/water)

Leakage rate = -0.0667	Leakage rate = -0.01
Gap thickness/Variance	Gap thickness/Variance
0.05 - .014/.048	0.05 - .0044/.027
0.03 - 4e-4/.01	0.03 - 3e-4/.0043

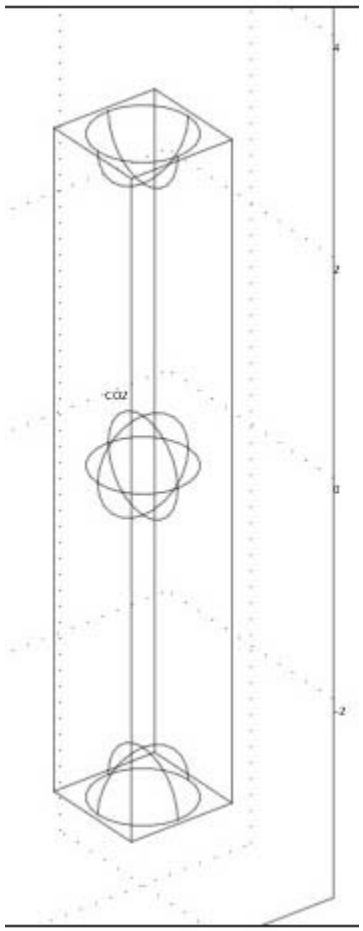
Results for Case 2; comparison with T-sensor

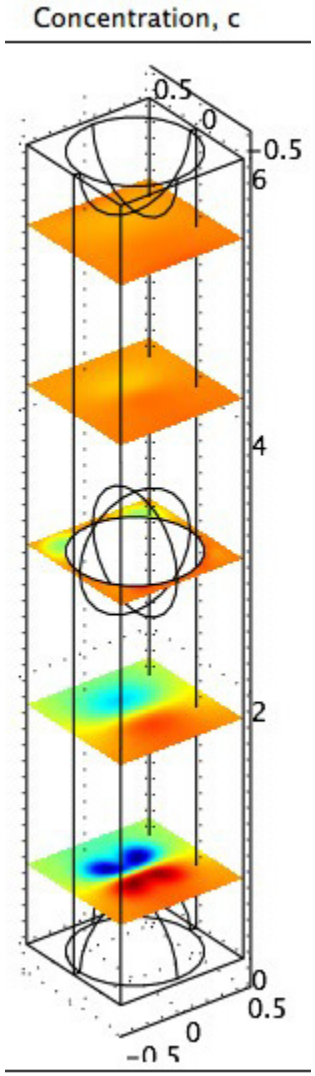


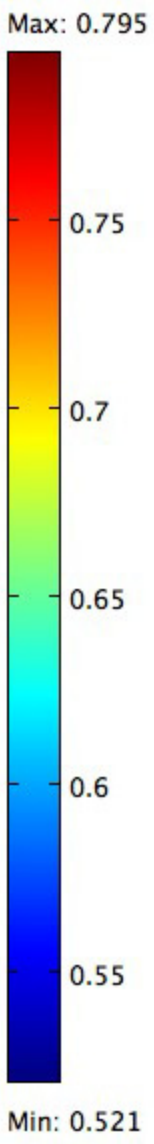


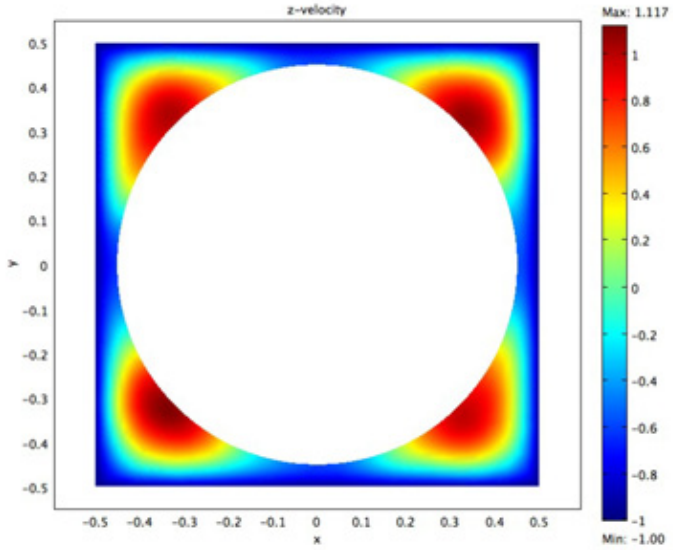


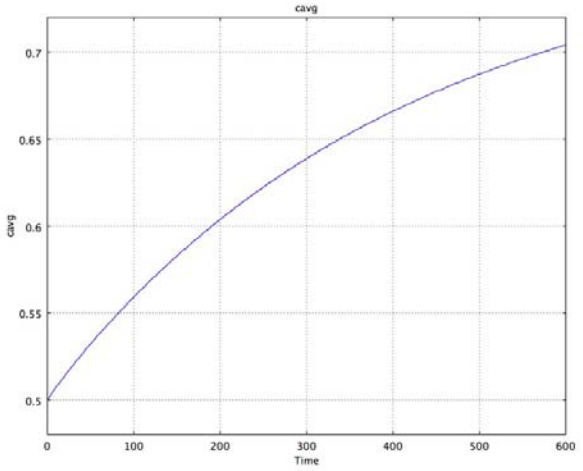
3D results


cc2

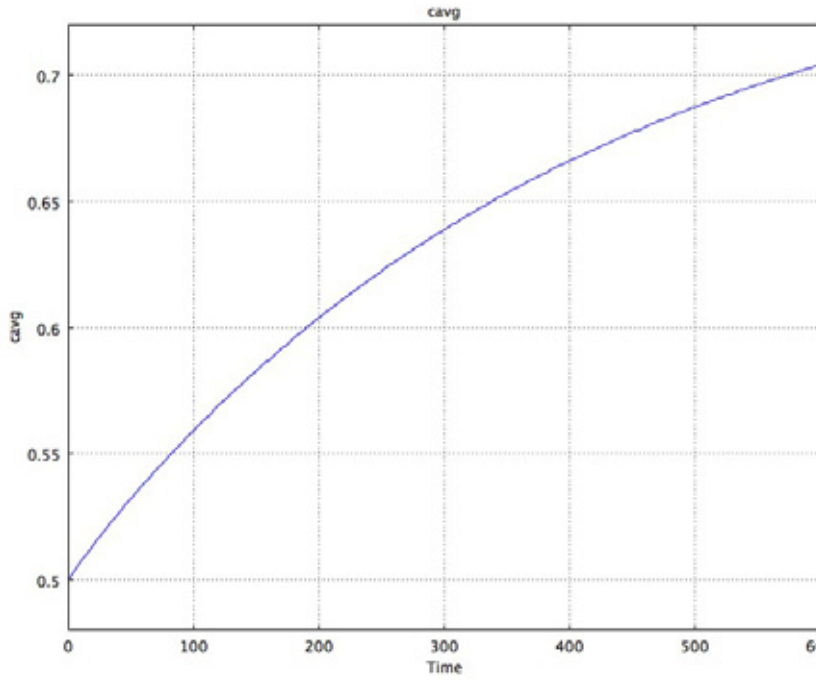

Concentration, c
0.5
0
-0.5
6
4
2
0
0.5
0
-0.5

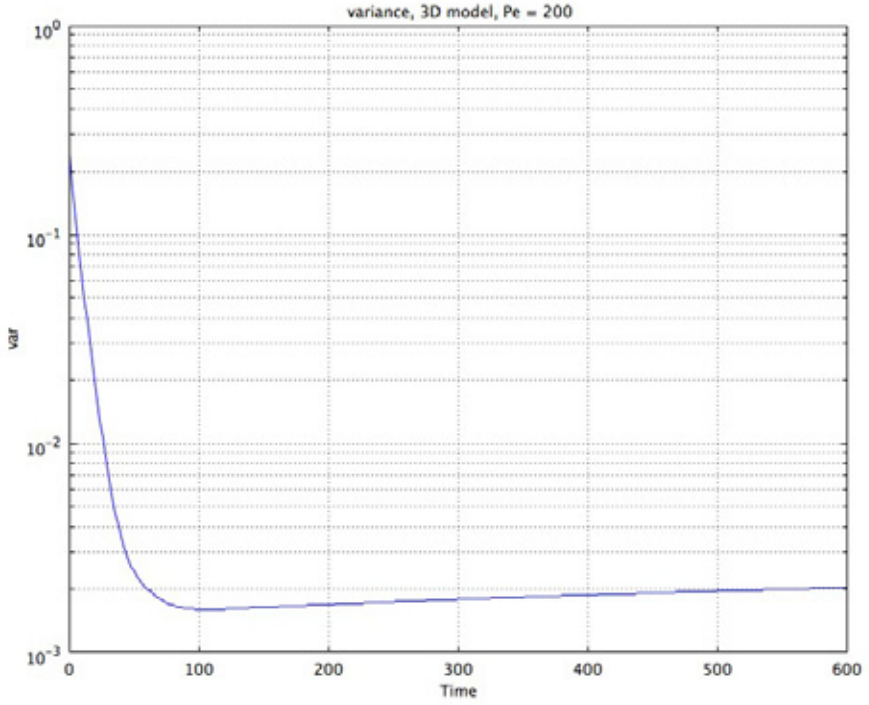

Max: 0.795
0.75
0.7
0.65
0.6
0.55
Min: 0.521


z-velocity
Max: 1.117
Min: -1.00
x
y

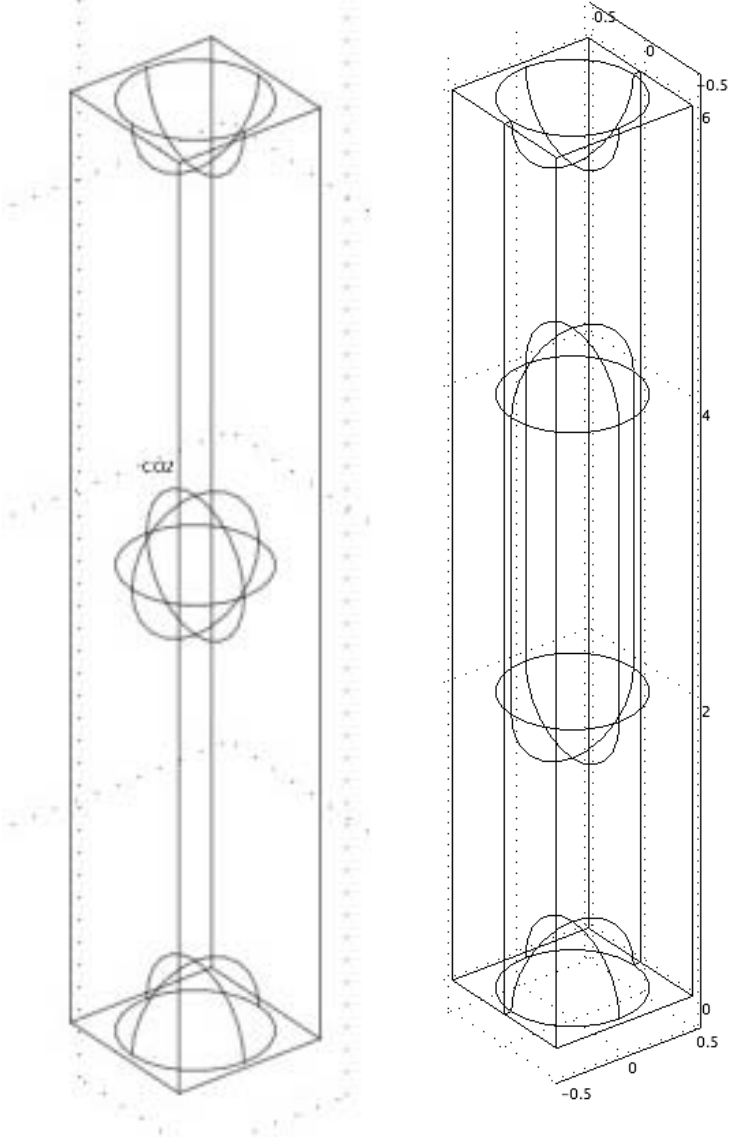

cavg
Time

Cavg and variance in 3D



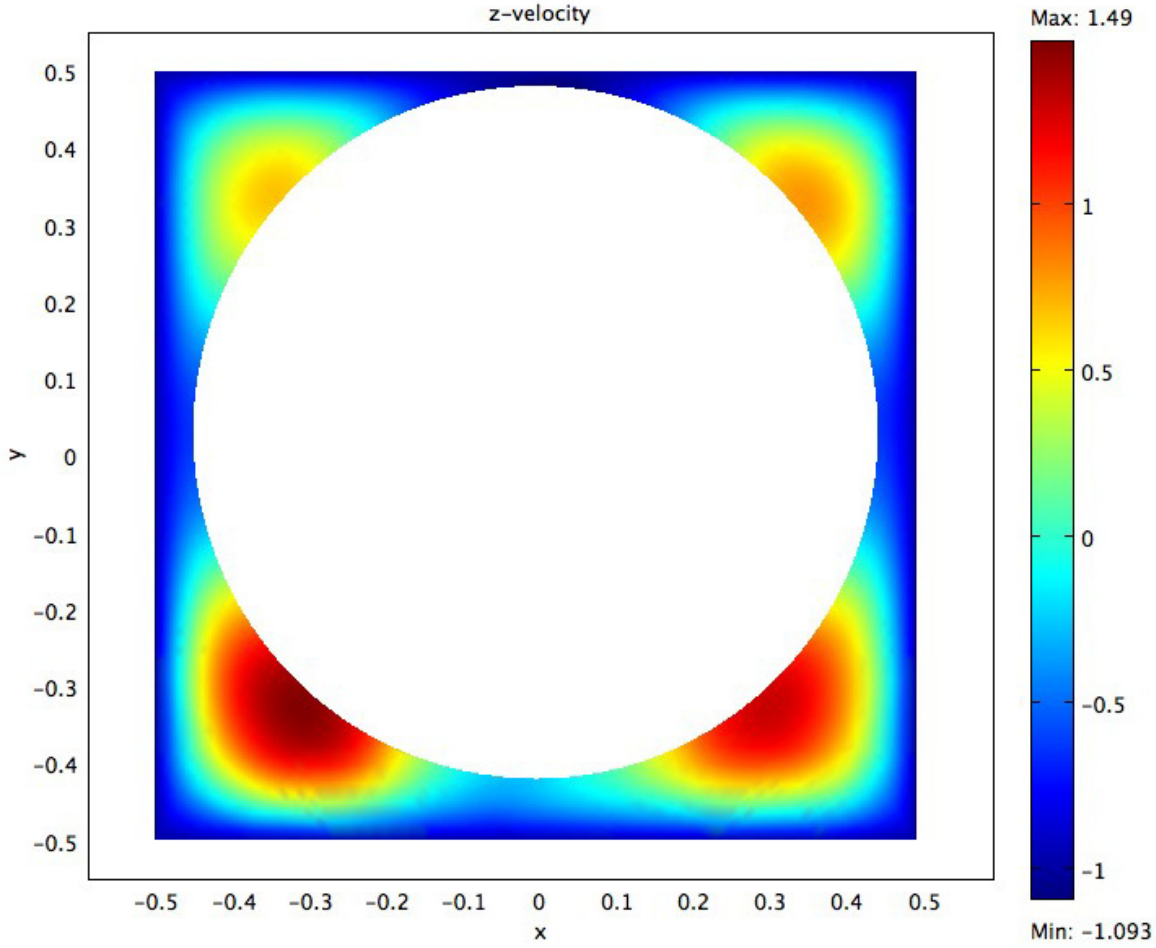


Effect of longer bubble in 3D – same as in 2D – less mixing



- Variance – 0.002 0.019

Effect of bubble not centered



- Variance
- Centered - 0.002
- Offset - 0.0014
- improved mixing

Conclusions

- The variance always approaches an asymptotic value.
- While the geometry is assumed, the qualitative results are independent of the geometry.
- Better mixing is achieved by:
  - Bubbles further apart
  - Spherical bubbles, not elongated ones
  - Low leakage rates
  - Small gap thicknesses