Full-wave Simulation of Light Propagation through Quarter-wave Plate

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Abstract: We present a full-wave simulation of light propagation through a quarter-wave plate based on finite element method. Cases of both linearly polarized and circularly polarized incident light were studied. Simulation results reveal a transformation of light from linearly polarization into circular polarization upon emergence from the plate. Vice versa for circularly polarized incident light. In addition, full-wave simulations offer an intuitive visualization for detailed light behavior at the submicron scale.

Keyword: Birefringence, full-wave simulation, quarter-wave plate.

Introduction

Advances in 3D display technology relies heavily on the birefringence of light as it propagates through various optical components such as the quarter-wave plate. While the Jones matrix formulation is commonly employed as a convenient means to study optical systems involving such optical components [1, 2], full-wave simulation provides an alternative theoretical approach which could offer more insight and intuitive understanding to the behavior of light through such components. In this paper, we study the behavior of monochromatic visible light as it propagates through a birefringent quarter-wave plate based on full-wave simulation using the Wave-Optics module.

Problem Set-up

We study the light propagation through a birefringent quarter-wave plate as shown in Figure 1 below.

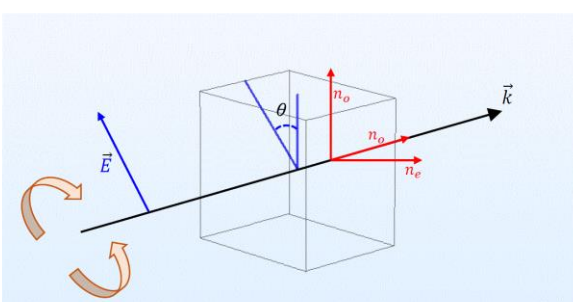

Figure 1. Light propagation through a birefringent quarter-wave plate. We consider different incident polarizations such as linear polarization and circular polarization.

From Figure 1, the normally directed incident light with different polarization configurations are analyzed. These includes (1) linearly polarized light making a polarization angle of θ = 45 deg, (2) linearly polarized light with θ = - 45 deg, (3) Right-hand circularly polarized incident light, and (4) Left-hand circularly polarized incident light.

Numerical Model

The sinusoidal steady-state wave propagating problem was solved using the wave optics module. COMSOL Multiphysics software solves the time-harmonic Maxwell's equations for electric field distribution $\vec{E}$ over the defined domains at the (angular) frequency ω.

$$\nabla \times \left(\mu_0 \mu_r{}^{-1} \nabla \times \vec{E}\right) - k_0^2 \left(\varepsilon_r - \frac{j\sigma}{\omega \varepsilon_0}\right) \vec{E} = 0 \quad (1)$$

where k_0 is the wave propagation constant in free space. μ_0, μ_r, ε_0, ε_r and σ are permeability in free space, relative permeability, permittivity in free space, relative permittivity and electrical conductivity respectively.

For many non-magnetic dielectric materials, their electromagnetic properties can also be expressed in terms of refractive indexes n. If the materials are also lossless, we can assume μ_r = 1, σ = 0, and

$$\varepsilon_r = n^2 \quad (2)$$

The uniaxial birefringent quarter-wave plate further exhibits anisotropy, defined by its optical axis as shown in Figure 1. Refractive index along the optical axis is called the extraordinary index, denoted by n_e. On the other hand, those along the remaining orthogonal axes are ordinary indexes, denoted by n_o. This results in different values of ε_r along the respective directions.

As a result, incident waves having polarization directions along the optical and ordinary axes travel at different speeds, given by

$$c = \frac{c_0}{n_i} \quad , i = o, e \quad (3)$$

For a linearly polarized incident wave entering the plate having electrical field components along both ordinary and extraordinary axes (e.g. at θ = 45 deg), the birefringent material would effectively split the wave into two waves with orthogonal polarizations along n_o and n_e respectively, each traveling at a different speed given by (3). Upon emergence from the plate, these two waves would have developed an optical bath difference (OPD) between them. For a plate of thickness d,

$$\mathrm{OPD} = (n_e - n_o)d \quad (4)$$

When OPD is equal to one quarter of a wavelength λ, or the thickness of the plate being equal to

$$d = \frac{\lambda}{4(n_e - n_o)} \quad (5)$$

the emerging waves have 90 deg phase difference between oscillations along n_o and n_e directions, resulting in a circularly polarized light. Such birefringent quarter-wave plates are often used as phase retarders to transform between linearly polarized light and circularly polarized light in 3D display technologies.

In this study we perform full-wave simulation to study wave propagation of normally incident light with the following configurations:

(1) linearly polarized with θ = 45 deg,
(2) linearly polarized with θ = - 45 deg,
(3) Right-hand circularly polarized, and
(4) Left-hand circularly polarized.

Wavelength used in simulation is λ = 550nm. For the quarter-wave plate, n_o=1.48 and n_e=1.65, μ_r = 1, σ = 0. For the remaining air domain ε_r = 1, μ_r = 1, σ = 0.

Simulation Results and Discussion

Results for the above mentioned polarization configurations were generated.

(1) linearly polarized incident light with θ = 45 deg:

Incident waves with linear polarization turns into left-hand circularly polarized light as shown in Figure 2. For a full animation, please see [3]. Electric field vectors in the quarter-wave plate reveals a transition from linear polarization to circular polarization as the phase difference between the two effective orthogonally polarized waves (discussed above) increases from 0 to 90 deg.

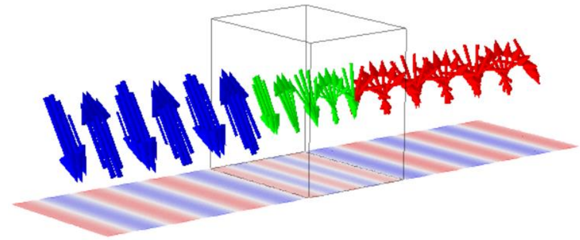

Figure 2. Linearly polarized light with θ = 45 deg turns into left-hand circularly polarized light.

It can also be noticed that the emergent circularly polarized wave exhibits constant amplitude, thus plane wave behavior. On the other hand, for the incident wave, the amplitude is apparently oscillatory in time. Furthermore, polarization direction in this region shows an oscillatory deviation from θ = 45 deg. This is attributed to the interference of incident light and reflected light from the plate. Our full-wave simulation offers an intuitive visualization of such details in light behavior at the submicron scale.

(2) linearly polarized incident light with θ = - 45 deg:

Similar to Case (1), the linear polarization turns into right-hand circularly polarized light upon emergence from the plate as shown in Figure 3. For a full animation, please see [4].

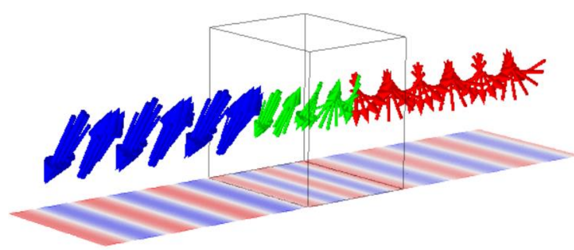

Figure 3. Linearly polarized light with θ = -45 deg turns into right-hand circularly polarized light.

The amplitude and polarization direction for the incident light also show oscillatory deviations, which are interpreted as results of interference between incident light and reflected light from the plate.

(3) Right-hand circularly polarized incident light:

Incident waves with right-hand circular polarization turns into θ = 45 deg linearly polarized light as shown in Figure 4. For a full animation, please see [5].

Electric field vectors in the quarter-wave plate reveals a transition from circular polarization to linear polarization, as the phase difference between the two effective orthogonally polarized waves decreases from 90 deg to 0.

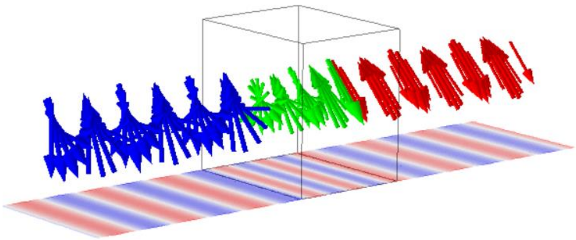

Figure 4. Right-hand circularly polarized light turns into linearly polarized light with θ = 45 deg.

The interference between incident and reflected wave from the plate results in apparent oscillations in amplitude in the incident light region. On the other hand, the emergent linearly polarized light exhibits plane wave behavior.

(4) Left-hand circularly polarized incident light:

Incident waves with left-hand circular polarization turns into θ = -45 deg linearly polarized light as shown in Figure 5. For a full animation, please see [6].

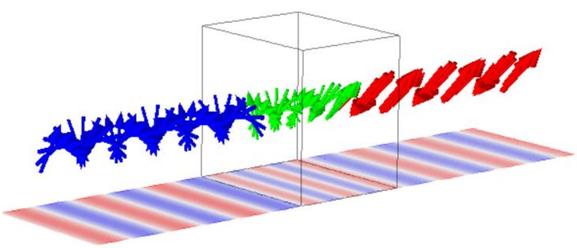

Figure 5. Light-hand circularly polarized light turns into linearly polarized light with θ = -45 deg.

The observed oscillations in amplitude in the incident light region can also be attributed to interference effect between incident and reflected waves.

Conclusions

Full-wave simulations of light propagation through a quarter-wave plate were carried out based on finite element method. Cases with different incident wave polarization configurations including linearly polarization and circular polarizations are studied. Simulation results offers an intuitive visualization of details in light behavior at the submicron scale as linearly polarized light is transformed into circularly polarized light and vice versa. In particular, for the linearly polarized incident wave, both the amplitude and polarization direction in the incident light region show oscillatory deviations. Such simulation result provides visual details in the interference of incident light and reflected light from the plate.

References

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