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Modelling of Seismoelectric Effects

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Outline

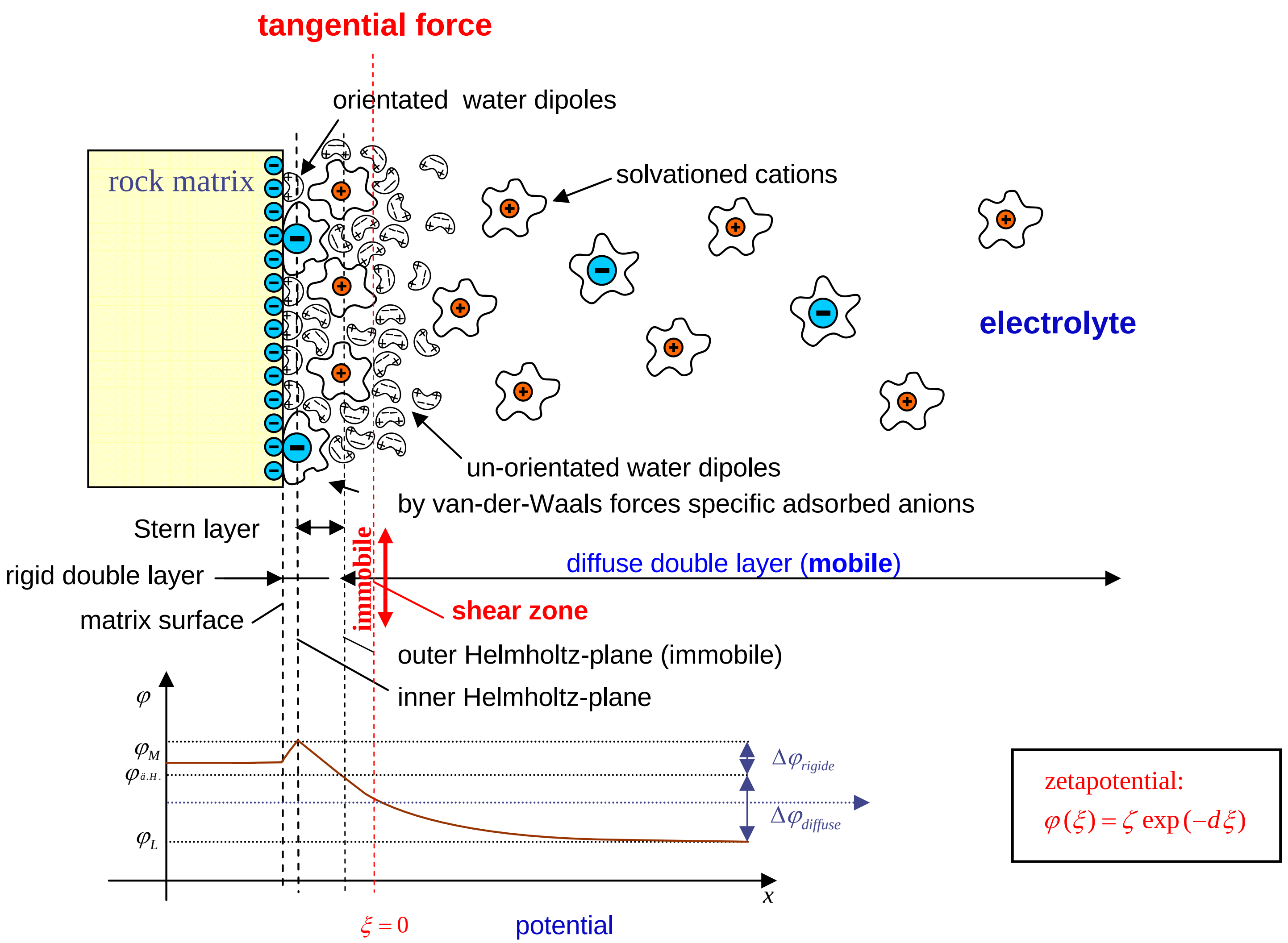
- **Electrical double layer / Electrokinetic phenomena**
- **Description of seismoelectric effects**
    - *Direct field, coseismic field and interface response*
- **Theoretical fundamentals**
    - *Governing equations („u-p formulation“)*
- **Numerical simulation**
    - *Model setup*
    - *Physical responses of the system*
    - *Anatomy of the interface response*
- **Conclusion & Outlook**

Motivation

Why numerical modelling of seismoelectric effects?

- *Seismoelectrics* is an energy transfer between seismic and electromagnetic wavefields occurring at the electrical double layer.
- Generation of seismoelectric signals in porous media is connected with properties such as *hydraulic permeability* and *porosity.*
- Seismoelectric method could be used in *hydrogeophysics* for determining these parameters *directly*.
- Numerical modelling in COMSOL with a view to an improved understanding of the *interactive processes* associated with seismoelectric effects.

The electrical double layer in a porous media – „Stern model“



tangential force

orientated water dipoles

rock matrix

solvationed cations

electrolyte

un-orientated water dipoles

by van-der-Waals forces specific adsorbed anions

Stern layer

rigid double layer

diffuse double layer (**mobile**)

immobile

shear zone

matrix surface

outer Helmholtz-plane (immobile)

inner Helmholtz-plane

φ

φ_M

$\varphi_{d.H.}$

φ_L

$\Delta\varphi_{rigide}$

$\Delta\varphi_{diffuse}$

x

$\xi = 0$

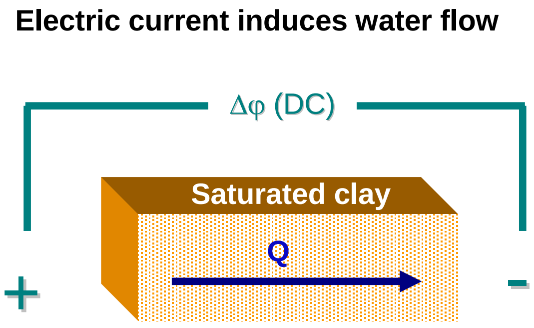
potential

zetapotential:

$\varphi(\xi) = \zeta \exp(-d\xi)$

The electrokinetic phenomena

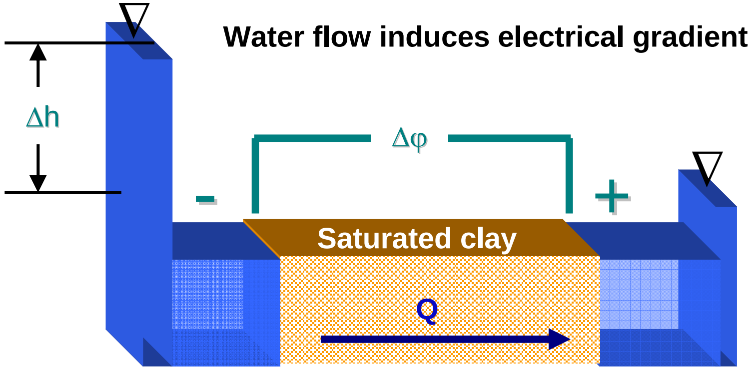
Effects caused by electrical gradient



- Electroosmosis -

- Imposed: electric potential gradient Δφ
- Measured: fluid flux Q

Effects caused by mechanical movement

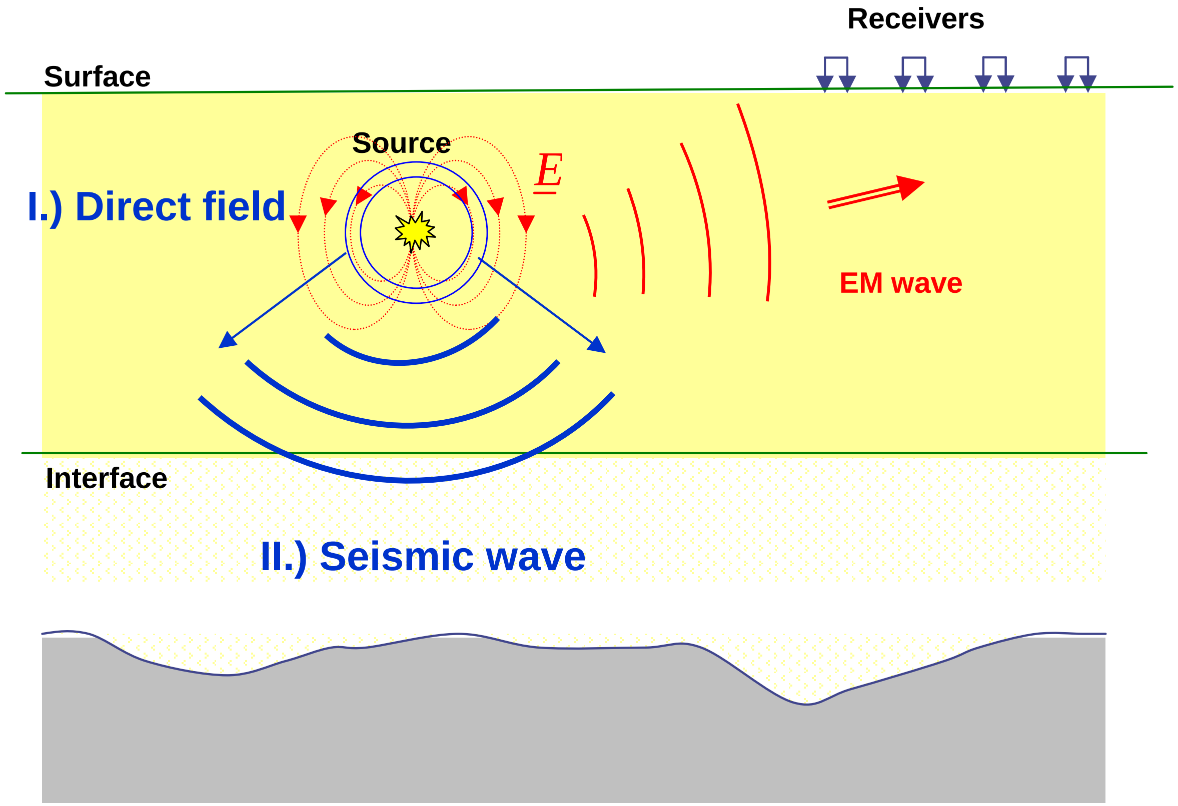


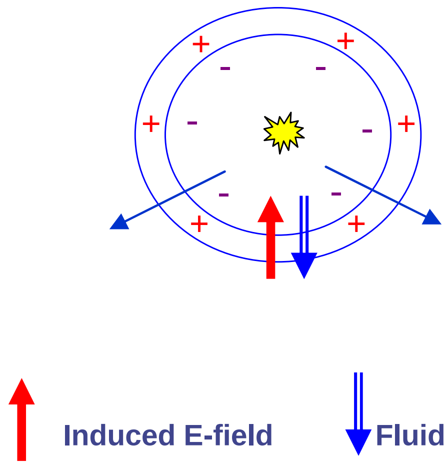
- Streaming potential -

- Imposed: pressure gradient Δh
- Measured: electric potential gradient Δφ

Generation of seismoelectric effects

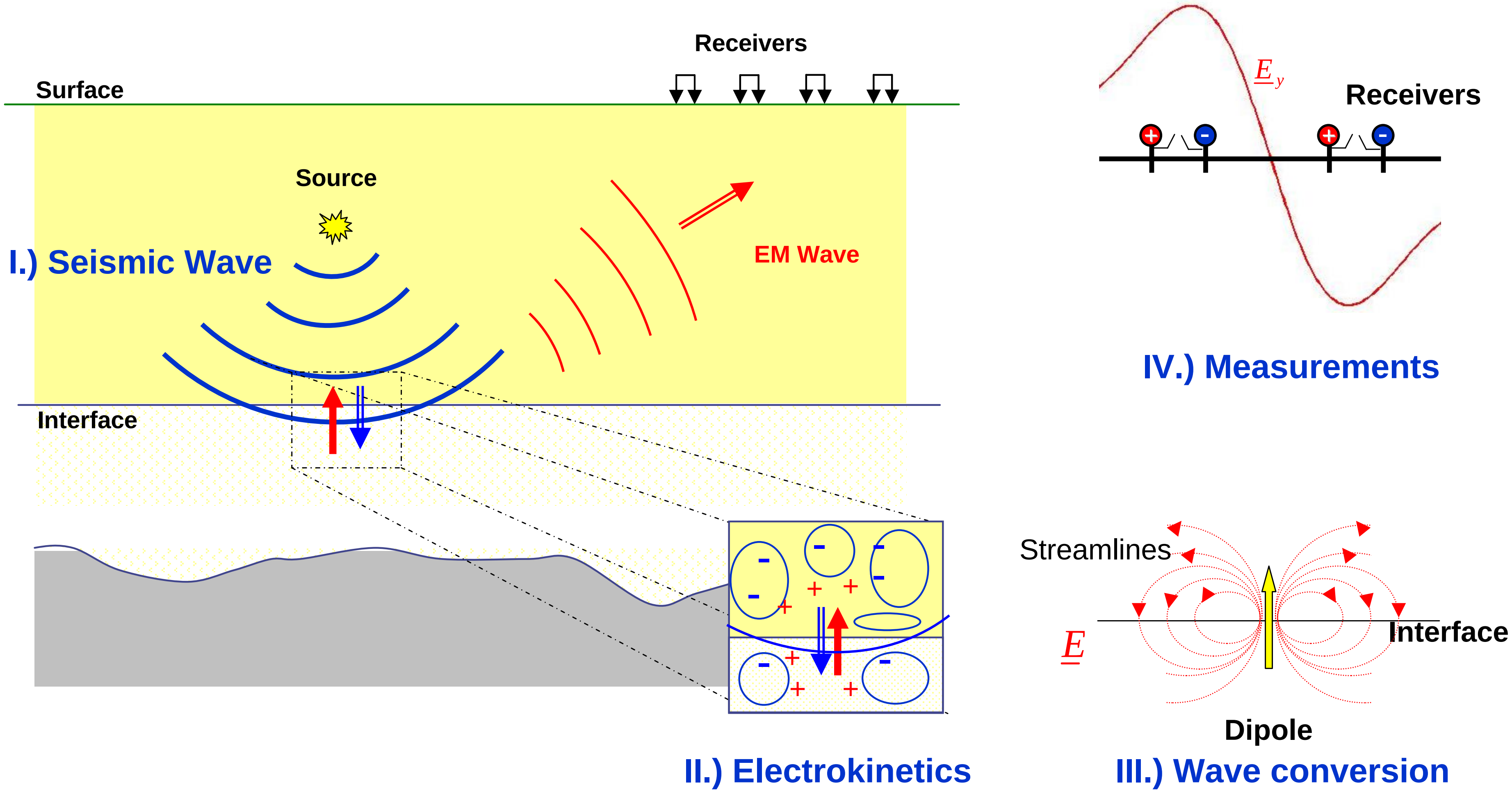
Electric and magnetic fields caused by deformation processes





II.) Coseismic wave

Generation of seismoelectric effects – interface response



Dynamic poroelasticity equations

Coupled processes of elastic deformation and pore fluid diffusion

- **Constitutive equations for linear poroelasticity (stress-strain relationship)**

$$\sigma_{ij} = 2G\varepsilon_{ij} + 2G\left(\nu/1-2\nu\right)\varepsilon_{kk}\,\delta_{ij}\ \boxed{-\alpha\, p\,\delta_{ij}}$$

Variation in the pore pressure

- **Dynamic equilibrium for the mixture („Biot formulation“)**

$$\sigma_{ij,j} = \rho\,\ddot{u}_i + \rho_f\,\ddot{w}_i \qquad \text{with} \qquad \boxed{w_i := \phi\left(u_i^{\,f} - u_i^{\,s}\right)}$$

Relative displacement

- **Balance law for the solid equilibrium („dynamical behaviour of the system“)**

$$\left(G\,\nabla^2\underline{u} + G\,/\left(1-2\nu\right)\nabla\left(\nabla\cdot\underline{u}\right)\right) = \alpha\nabla p + \rho\,\underline{\ddot{u}} + \rho_f\,\underline{\ddot{w}}$$

- **Fluid mass balance equation, i. e. continuity equation**

Increment of fluid content := kind of volumetric strain

$$\dot{\zeta} = -\nabla\cdot\underline{q} \quad \text{with} \quad \underline{q} = \phi\,\underline{w} \quad \wedge \quad \boxed{\zeta} = \alpha\,\nabla\cdot\underline{u} + S_\alpha\, p$$

fluid flux

Maxwell equations – electrokinetic coupling equations

Maxwell`s electromagnetic field equations

- **Faraday's law**

$$\nabla \times \underline{E} = -\frac{\partial}{\partial t}\mu \underline{H}$$

- **Ampère's law**

$$\nabla \times \underline{H} = \underline{J} = \sigma \underline{E} + \boxed{L\nabla p}$$

Streaming electric current

The electrokinetic coupling equations

- **Fluid transport modeled with generalized Darcy 's law**

$$\nabla \cdot q = k_f / \eta \left(-\nabla p + \rho_f \underline{\ddot{u}}\right) + \boxed{L\,\underline{E}}$$

Electroosmosis

- **Electric balance modeled with generalized Ohm 's law (no external sources!)**

$$\nabla \cdot \underline{J} = \nabla \cdot \left(\boxed{L\left(-\nabla p + \rho_f \underline{\ddot{u}}\right)} + \sigma\, \underline{E} \right) = 0$$

Streaming electric current

- **Coupling coefficient**

$$L := -\varepsilon_0 \varepsilon_r \zeta / F_0 \eta \quad \approx 10^{-9}\ [m^2 / Vs]$$

Set of equations for all responses of the system

Governing equations – „u-p formulation“ *

(i) $$\frac{E}{2(1+\nu)}\left(\nabla^2\underline{u}+\frac{1}{(1-2\nu)}\nabla(\nabla\cdot\underline{u})\right)=\rho\frac{\partial^2}{\partial t^2}\underline{u}+\boxed{\frac{\partial^2}{\partial t^2}\rho_f\,\underline{w}}+\alpha\nabla p$$

Neglecting acceleration of relative displacement *

(ii) $$\nabla\cdot\left(\frac{k_f}{\eta}\left(-\nabla p+\rho_f\frac{\partial^2}{\partial t^2}\underline{u}\right)+L\,\underline{E}\right)=-\alpha\frac{\partial}{\partial t}\nabla\cdot\underline{u}-S_\alpha\frac{\partial}{\partial t}p$$

(iii) $$\nabla\cdot\left(L\left(-\nabla p+\rho_f\frac{\partial^2}{\partial t^2}\underline{u}\right)\right)=-\sigma\nabla\cdot\underline{E}$$

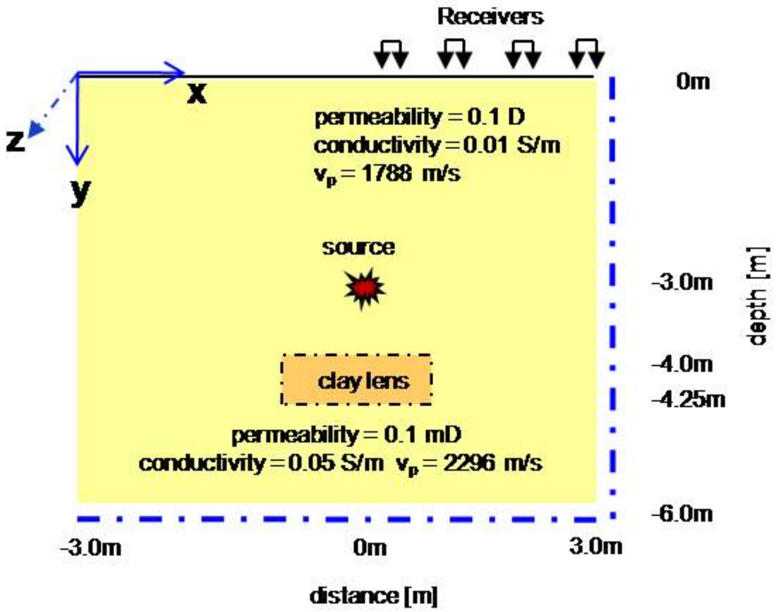
(iv) $$\nabla\times\left(\frac{1}{\sigma}\nabla\times\underline{H}-L\nabla p\right)=-\frac{\partial}{\partial t}\mu\underline{H}$$

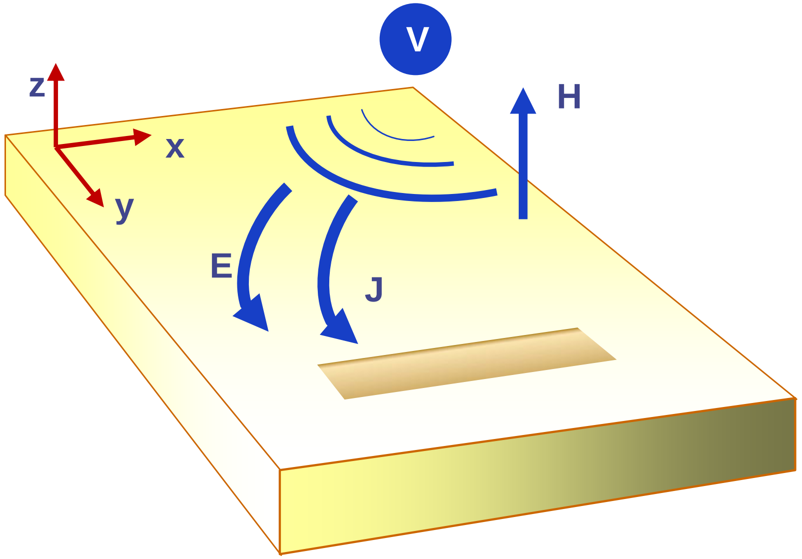
- **Valid for a low-frequency range! – Modelling is performed by *COMSOL Multiphysics*.**

*Zienkiewicz et al. Computational Geomechanics, 1999

Model geometry

The model features a thin clay lens in a sand background





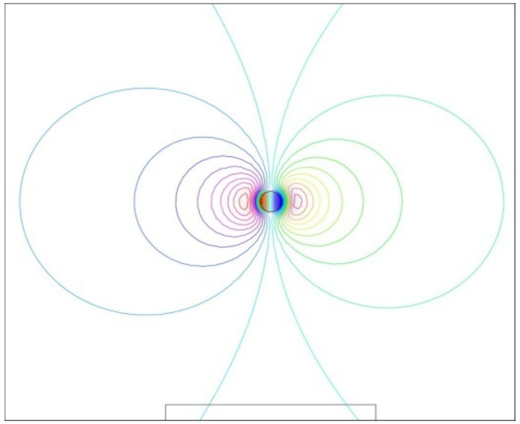
In-plane induction currents

Responses illustrated as (i) snapshots at different times and (ii) magnetograms recorded by a surface receiver line.

Signal input is a Ricker wavelet with a centre frequency of 380 Hz.

Snapshots at different times - seismoelectric responses

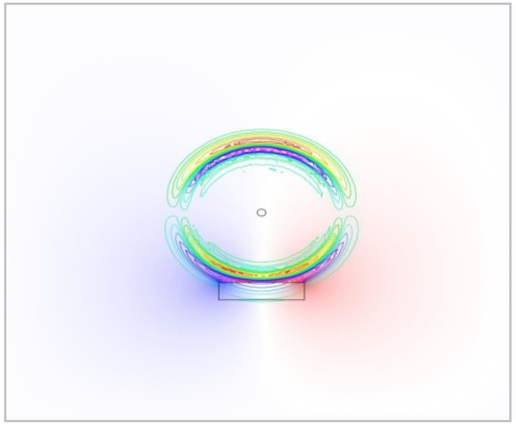
Direct field



Surface: ux, Contour: Electric potential [V]

Electric field due to charge distribution at impact source with reversed polarity on opposite sides of the shotpoint

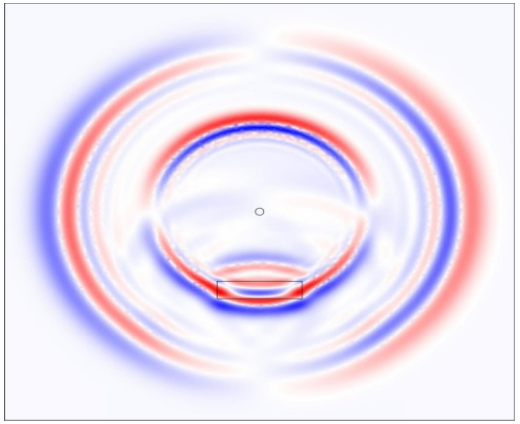
Interface response



Surface: vy, Contour: Magnetic field, z-component [A/m]

Conversion from seismic-to-electromagnetic waves at the interface – SV-wave generates a transversal polarized magnetic (TM-) wave

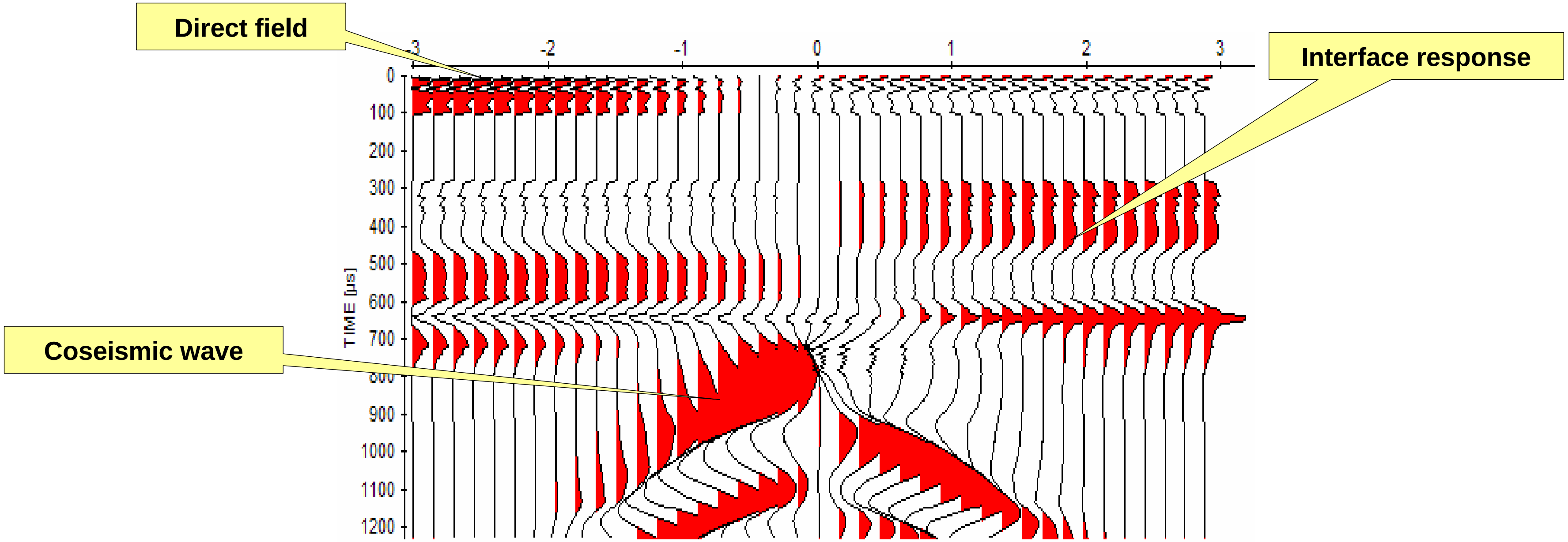
Coseismic field



Surface: Mises stress, Contour: Electric potential [V]

Electric field travelling with seismic wave because of charge accumulations due to streaming currents

Seismoelectrogram – the different responses collected



„Direct field“ and „interface response“:

- The waves reach all receivers at virtually the same time due to the high EM-velocity!
  - Signals change sign on opposite side of shotpoints

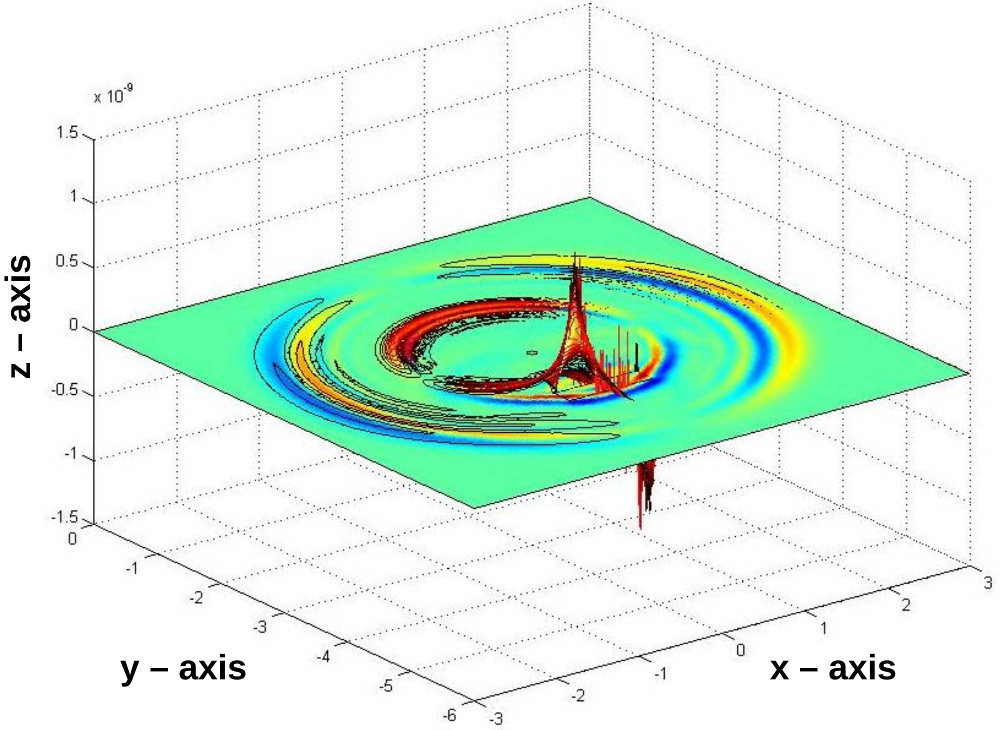
„Coseismic field“:

- Coseismic wave with the same waveform as seismic wave (hyperbolic structure)
  - Signals change on opposite side of shotpoints

Generation of electromagnetic TM – mode caused by SV - wave

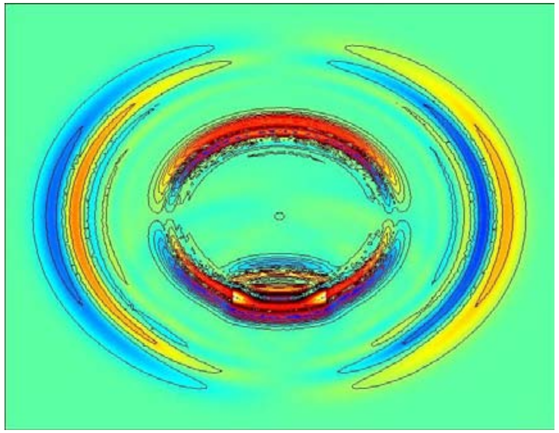
Figures from different views at the same time (t = 1.07 ms)

Overall view

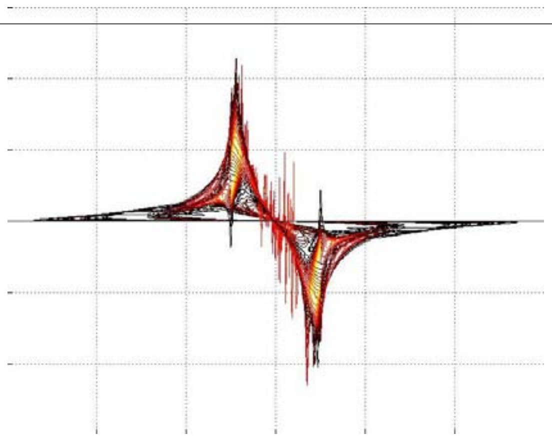


Surface: Electric potential [V]   Contour: Mises Stress

Heigth: Magnetic Field, z-component [A/m]



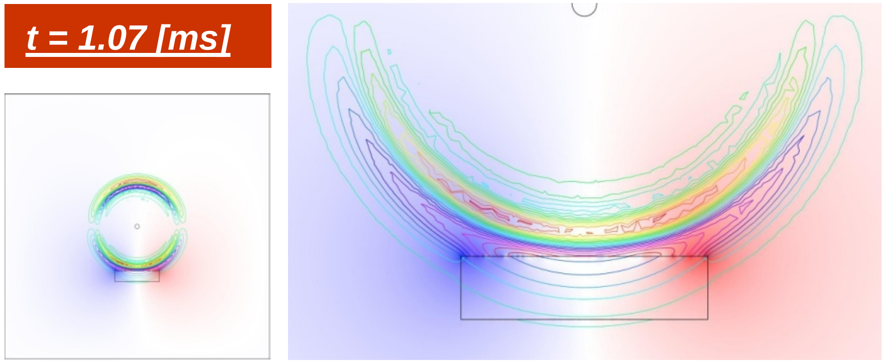
Topview x–y plane



View from the front in x–z plane

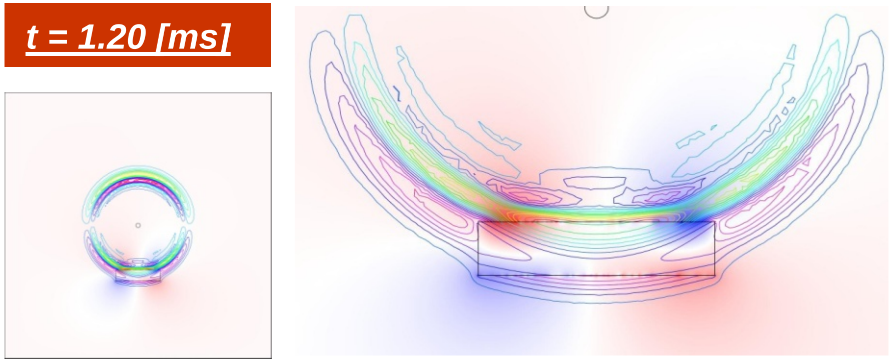
Snapshots - magnetic dipoles at the interface caused by SV-waves

t = 1.07 [ms]



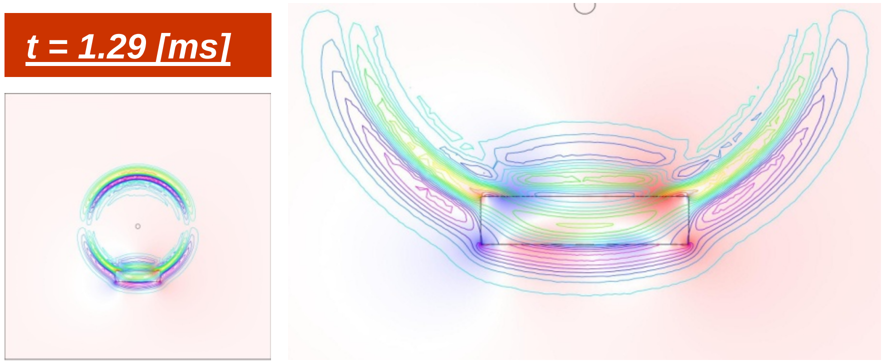
Peak of the wave at 1st interface

t = 1.20 [ms]



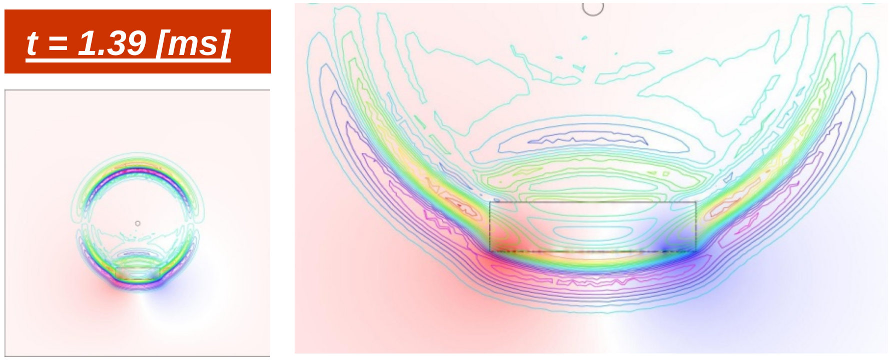
Peak at 2nd interface and trough at 1st interface

t = 1.29 [ms]



Multipole generation

t = 1.39 [ms]

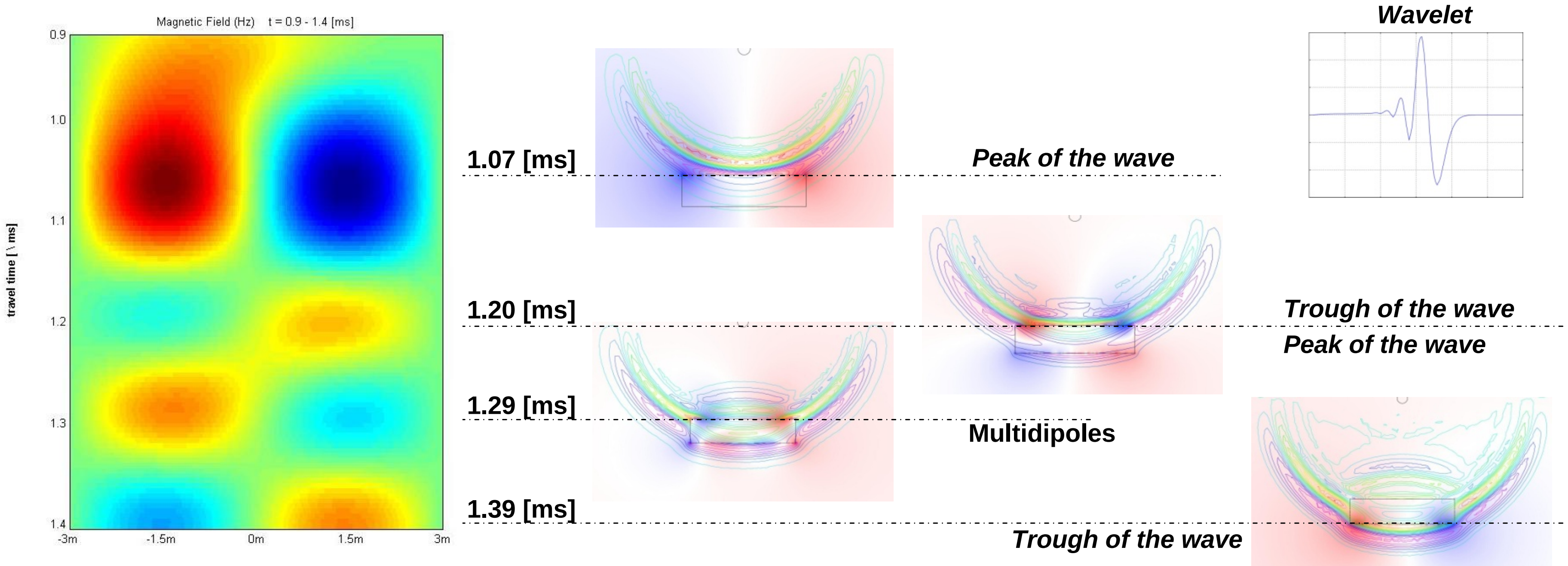


Trough of the wave at 2nd interface

Surface: Magnetic field, z-component (Hz) [A/m]   Contour: Vertical displacement SV-wave (vy)

Left figures: Full waveform   -   Right figures: Zoom at the interfaces

Magnetogram for z-component – correlation at different times



Magnetogram *Snapshots*

- *Magnetic dipoles are generated by vertical displacement of SV-wave*
- *Highest amplitudes of dipoles are correlated with peaks and troughs of the wavelet (SV-wave: Displacements perpendicular to direction of wave propagation!)*

Summary

What did we learn?

- Our finite-element algorithm („u-p formulation“) provides a reasonable method for *understanding the seismoelectric coupling*.
- Synthetic time sections of wave propagation show the *interaction of the different responses* in the system.
- The direction of the *streaming potential gradient* induced by the seismic wave *corresponds* with the direction of the generated *dipole response*.
- Our modelling results indicate the capability of the seismoelectric method to detect thin layers (*thickness smaller than wavelength*).

Outlook

What comes next?

- **Investigation of new geometries: *downhole and crosswell surveys*.**
- ***Quantitative analysis* of seismoelectric effects in 2.5D and 3D.**
- ***Validation* of the „u-p formulation“ with existing algorithms.**
- **Development of a seismoelectric inversion algorithm.**
- ***Application of the seismoelectric method* to determine permeabilites.**

References

[1] **Biot**, M., 1956a. Theorie of propagation of elastic waves in a fluid-saturated porous solid. I. Low frequency range, J. Acoust. Soc. Am., **28**, 168-178.

[2] **Garambois**, S. und **Dietrich**, M. 2002. Full waveform numerical simulations of seismoelectromagnetic wave conversions in fluid-sturated stratified porous media. Journal of Geophysical Research 107, ESE 5 1-18.

[3] **Haines**, S.S. 2004. Seismoelectric imaging of shallow targets. PhD thesis, Stanford University.

[4] **Kröger**, B. 2007. Modellierung und Sensitivitätsanalysen für Seismoelektrik mit Finiten Elementen. Diplomarbeit TU Berlin.

[5] **Pride**, S. R., 1994. Governing equations for the coupled electromagnetics and acoustics of porous media, Phy. Rev. B, Condens. Matter, **50**, 15678-15696.

[6] **Wang**, H.F. 2000. Theory of linear poroelasticity with applications to geomechanics and hydrology. Princeton University Press.

Thank you for your attention!