Sensitivity Plots Using COMSOL 5.1 Multiphysics; A Tool for Optimizing Geophysical Field Survey

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Abstract: The purpose of electrical surveys in geophysics is to determine the subsurface resistivity distribution by making measurements of potential on the ground surface. From these measurements, the apparent resistivity of the subsurface can be estimated. These apparent resistivities are further inverted to obtain the resistivity of the subsurface. The geophysical inference helps in the geological interpretation. Various types of electrode combinations such as Schlumberger, Wenner, Pole-pole and Dipole-dipole can be used to investigate a particular area. But different array gives different type of response over same sub-surface feature. Response of different array is determined by sensitivity of that array. The sensitivity function of an array describes about the degree to which a small change in the resistivity of a section of the subsurface can influence the potential measured by the particular array. The higher the value of the sensitivity function, the greater is the influence of the subsurface region on the measurement.

The sensitivity function has been computed using electric current interface of AC/DC module of COMSOL Multiphysics 5.1. The sensitivity calculation is simulated for a typical situation of surface and borehole measurements using Electrical Resistivity Tomography (ERT) techniques with 25 point electrodes spaced one meter from each other. The model is geometrically parameterized to be easily adaptable to various sizes. As a material, a homogeneous half-space of conductivity, σ = 0.01 S/m that often serves as a reference model for forward algorithms is used. The model solves for two current dipole situations with a common midpoint. This allow the calculation of the sensitivity of a different electrode configuration according to the equation given below,

S=with(1,ec.Jx)*with(2,ec.Jx)+with(1,ec.Jy)* with(2,ec.Jy) +with(1,ec.Jz)*with(2,ec.Jz))

The sensitivity plot shows that Wenner array has better sensitivity for vertical changes in the subsurface that is for horizontal structures. In case of Dipole-dipole array sensitivity plot. shows better response for horizontal changes in resistivity The Schlumberger array is sensitive for an area where sub-surface has both kind of horizontal and vertical geological features. But there is limitation of these surface arrays as resolution decreases with increasing depth. The only possible way to improve the resolution is to place the sensors in the borehole. Sensitivity plots for cross borehole arrays using bipole-bipole array with C1P1-C2P2 arrangement shows large positive sensitivity between two boreholes. The sensitivity plots generated through COMSOL 5.1 Multiphysics helps in planning in field survey which is an integral part of geological interpretation.

Keywords: Sensitivity, Resistivity arrays, Crosshole, COMSOL Multiphysics

1. Introduction

Electrical resistivity surveys have been used for many decades in hydro geological, mining and geotechnical investigations.

The fundamental physical law used in resistivity surveys is Ohm's Law that governs the flow of current in the ground. The equation for Ohm's Law in vector form for current flow in a continuous medium is given by

$$\mathbf{J} = \sigma \mathbf{E} \qquad (1.1)$$

Where σ the conductivity of the medium, **J** is is the current density and **E** is the electric field intensity. In practice, electric field potential is measured. In geophysical parlance the resistivity of the medium, which is equals to the reciprocal of the conductivity, is more commonly used. The relationship between the electric potential and the field intensity is given by

$$\mathbf{E} = -\nabla \Phi \qquad (1.2)$$

Thus, the potential obtained for the simplest case is obtained with one current electrode in the ground radiating current in homogeneous subsurface is given by,

$$\phi = \frac{\rho I}{2\pi r} \qquad (1.3)$$

Where r is the distance of a point in the medium (including the ground surface) from the electrode. Generally all resistivity surveys use at least two current electrodes, a positive current and a negative current source.
The potential value in the medium from such a pair is given by

$$\phi = \frac{\rho I}{2\pi}\left(\frac{1}{r_{C1}} - \frac{1}{r_{C2}}\right) \qquad (1.4)$$

Where r_{C1} and r_{C2} are distances of the point from the first and second current electrodes.

1.1 Definition

The sensitivity function of an array describes about the degree to which a small change in the resistivity of a section of the subsurface can influence the potential measured by the particular array. The higher the value of the sensitivity function, the greater is the influence of the subsurface region on the measurement. Different array gives different type of response over same sub-surface feature. Some array provides better response over vertical changes while other provides better resolution for horizontal changes. If we have any idea of geological structure to be delineated it is better to use the best suitable array, to get better result.
Different resistivity arrays used to calculate sensitivity are

- Wenner array
- Dipole-dipole
- Schlumberger
- Pole-pole
- Pole- Dipole
- Double Wenner
- Half Schlumberger
- Cross borehole

1.2 Objective

To detect geological features precisely it is better to use the proper array. Objective of the present work is to find out the sensitivity of commonly used electrode arrays using COMSOL Multiphysics 4.4 through 3D forward modeling and interpretation of the sensitivity plots to find out the best suitable array for a particular geological feature.

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1.3 Mathematical representation of Sensitivity function

Mathematically, the sensitivity function is given by the Frechet derivative (McGillivray and Oldenburg 1990).

The 3D Frechet derivative is given by,

$$F_{3D}(x, y, z) = \frac{1}{4\pi^2} \frac{x(x-a)+y^2+z^2}{[x^2+y^2+z^2]^{1.5}[x(x-a)+y^2+z^2]}$$

This gives the Frechet derivative or sensitivity function for the pole-pole array consisting of just one current and one potential electrode. To obtain the Frechet derivative for a general four electrodes array, one needs to add up the contributions from the four current-potential pairs.

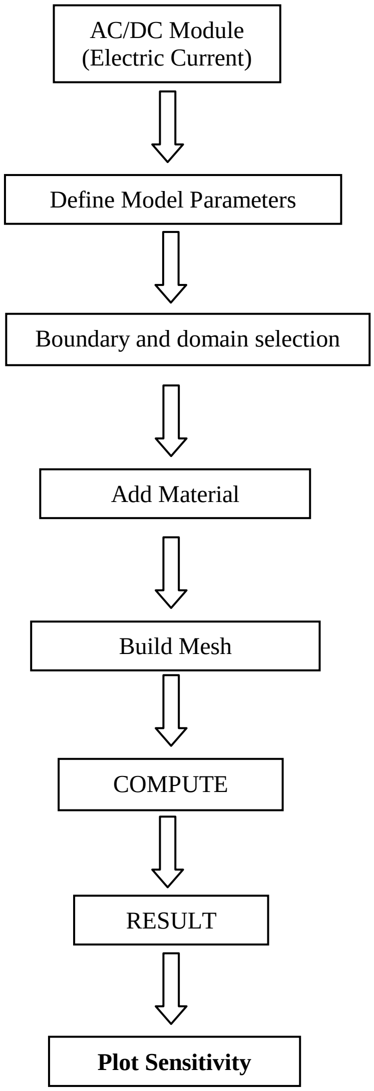
2. Use of COMSOL Multiphysics

The electric current interface of AC/DC module of COMSOL Multiphysics 4.4 was used in this work. The AC/DC Module provides a unique environment for simulation of AC/DC electromagnetic in 2D and 3D. The module is a powerful tool for detailed analysis of coils, capacitors, and electrical machinery. With this module you can run static, quasi-static, transient, and time-harmonic simulations in an easy-to-use graphical user interface.

3.1 Model Definition

The model illustrates the forward step and the sensitivity calculation for a typical situation of near-surface Electrical Resistivity Tomography (ERT) with 25 point electrodes spaced one meter from each other shown in Figure 1. The model is geometrically parameterized to be easily adaptable to various sizes. As a material, a homogeneous half-space that often serves as a reference model for forward algorithms is used. The electric conductivity is σ = 0.01 S/m, which corresponds to a resistivity of 100 Ω-m is used.
The model solves for two current dipole situations with a common midpoint. Together they allow the calculation of the sensitivity of a different electrode configuration according to the Equation given below,
S=with(1,ec.Jx)*with(2,ec.Jx)+with(1,ec.Jy)*with(2,ec.Jy) +with(1,ec.Jz)*with(2,ec.Jz))

3.2 Work flow in COMSOL Multiphysics



4. Results

Figs. 2-12 shows the sensitivity plots for various electrode arrangements

5. Discussion

5.1 Surface arrays

5.1.1 Wenner Array

The sensitivity section shows (Figure 2) large negative values near the surface between the C1 and P1 electrodes, as well as between the C2 and P2 electrodes. This means that if a small body with a higher resistivity than the background medium is placed in these negative zones, the measured apparent resistivity value will decrease. This phenomenon is also known as an "anomaly inversion". In comparison, if the high resistivity body is placed between the P1 and P2 electrodes where there are large positive sensitivity values, the measured apparent resistivity will increase. Thus, the Wenner array is good in resolving vertical changes (i.e., horizontal structures), but relatively poor in detecting horizontal changes (i.e. narrow vertical structures).

5.1.2 Schlumberger Array

The sensitivity contours (Figure 3) has a slight vertical curvature below the center of the array. The high positive sensitivity lobe beneath the P1-P2 electrodes are more separated from the high positive sensitivity values near the C1 and C2 electrodes. This means that this array is moderately sensitive to both horizontal and vertical structures. In areas where both types of geological structures are expected, this array might be a good compromise between the Wenner and the dipole-dipole array.

5.1.3 Dipole- Dipole Array

The largest sensitivity values are generally located between the C2-C1 dipole pair, as well as between the P1-P2 pair (Figure 4). This means that this array is most sensitive to resistivity changes below the electrodes in each dipole pair. As the factor "**n**" is increased, the high sensitivity values become increasingly more concentrated beneath the C1-C2 and P1-P2 dipoles, while the sensitivity values beneath the center of the array between the C1-P1 electrodes decreases. For "**n**" greater than 2, the sensitivity values become negligible. The sensitivity contour pattern becomes almost vertical for "**n**" values greater than 2. Thus, the dipole-dipole array is very sensitive to horizontal changes in resistivity, but relatively insensitive to vertical changes in the resistivity. That means that it is good in mapping vertical structures, such as dykes and cavities, but relatively poor in mapping horizontal structures such as sills or sedimentary layers.

5.1.4 Pole-Pole Array

The sensitivity plot shown (Figure 5) indicates large spacing indicating deepest depth of investigation but poorest resolution. To approximate the pole-pole array, the second current and potential electrodes (C2 and P2) must be placed at a distance that is more than 20 times the maximum separation between C1 and P1 electrodes used in the survey.

5.1.5. Pole- Dipole Array

Figure 6 shows the sensitivity section with the greatest sensitivity lies beneath P1-P2 dipole pair. Thus, similar to the dipole-dipole array, this array is probably more sensitive to vertical structures. Note also the zone with negative sensitivity values between the C1 and P1 electrodes, as well as the smaller zone of high positive values to the left of the C1 electrode.

5.1.6 Double Wenner Array

The sensitivity section (Figure 7) show high negative value of sensitivity between current and potential electrodes (C1-P1) and high positive between two potential electrodes. This array resolves vertical changes in subsurface resistivity i.e. horizontal structure. This array is not a conventional array.

5.1.7 Half Schlumberger Array

The sensitivity section (Figure 8) shows that the large negetive sensitivity below C1-P1 is more horizaontal but below P1-P2 sensitivity contours are vertical. So this array is used sensitive to both horizaontal and vetical structure. This array is configured by keeping potential electrode spacing very small.

5.2 Cross borehole

Resolution of the arrays discussed above decreases with the incresing depth. So, this problem can be solved by placing the electrodes inside borehole. Cross borehole can improve the resolution significantly.

Sensitivity has been calculated using two configuration, using three electrode (Pole-bipole) and two electrode(Bipole-bipole).

5.2.1 Pole- Bipole

There are six possible configuration using three electrode in two borehole. Out of this six, two basic combination are used, with one current and one potential in one borehole and other potential electrode in second borehole and current electrode in one borehole and two potential elctrode in another borehole.

Sensitivity plot (Figure 9) for the first configuration shows positive sensitivity in the vicinity of two borehole which makes this configuration to be significantly useful for delineating structures between two boreholes.

For the second arrangment sensitivity plot (Figure 10) produces high negetive sensitivity in between two boreholes and positive sensitivity values between C1P1 and C1P2.

5.2.2 Bipole-bipole

Sensitivity plots are computed for two arrangements, one is the positive current and potential electrodes C1 and P1 are located in one borehole, while the negative current and potential electrodes C2 and P2 are located in the second borehole.

Sensitivity plot (Figure 11) for first configuration shows large positive sensitivity values in the area between the two boreholes which implies that this array can map the material between the two boreholes. The large negative sensitivity values are confined to the region along the boreholes between the C1 and P1 (and C2 and P2 electrodes).

Sensitivity for the later one shows large region of positive sensitivity values between the two boreholes which is flanked by two zones with large negative sensitivity values shown in the sensitivity plot (Figure 12). This property makes this configuration less desirable and complicated than the first one.

4. Figures

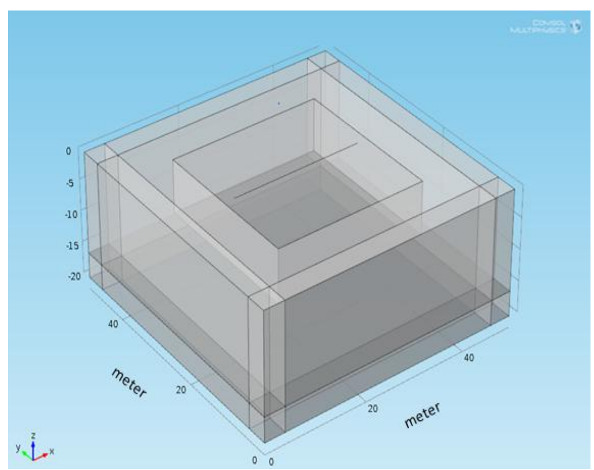


Figure 1 Computational domain. A line of 25 electrodes is placed at the top of a 50-by-50-by-20 meters box bounded by infinite element regions.

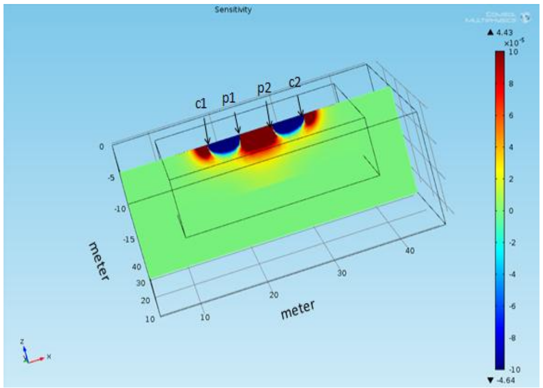


Figure 2 Sensitivity Plot of Wenner Array

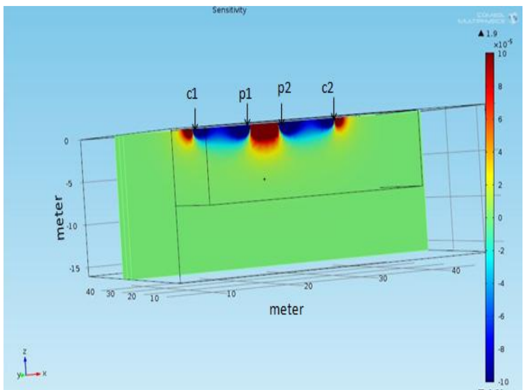
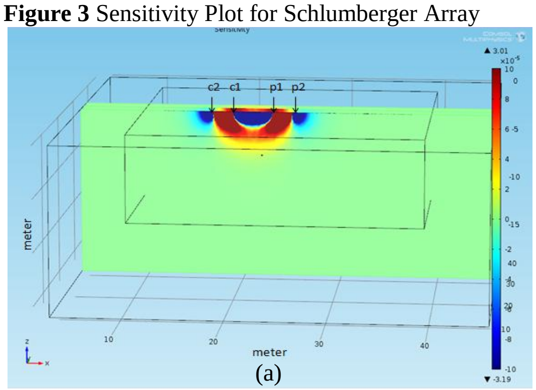
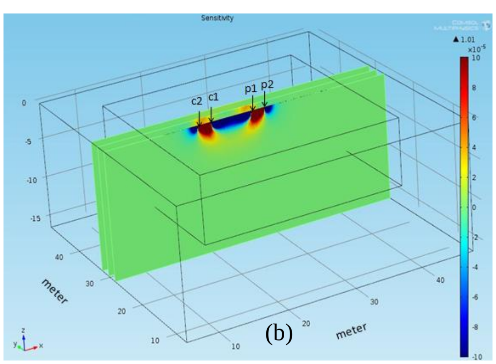


Figure 3 Sensitivity Plot for Schlumberger Array





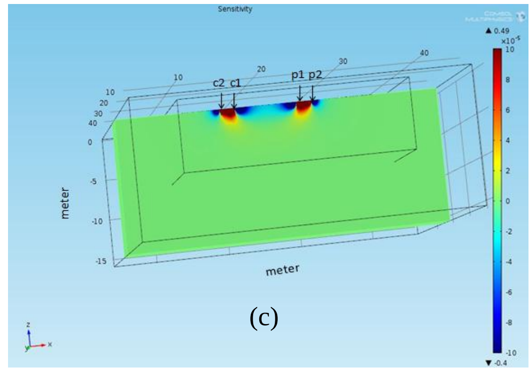


Figure 4 Sensitivity Plot for Dipole-Dipole array (a) n=2 (b) n=3 (c) n=4

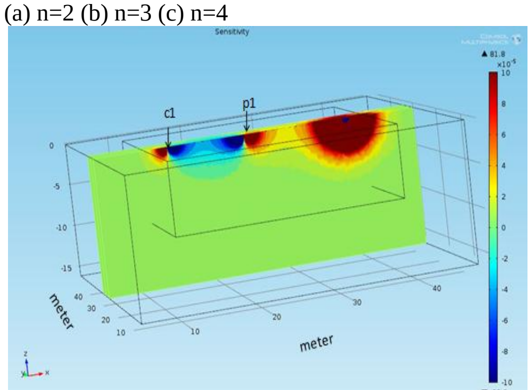


Figure 5 Sensitivity Plot for Pole-pole Array

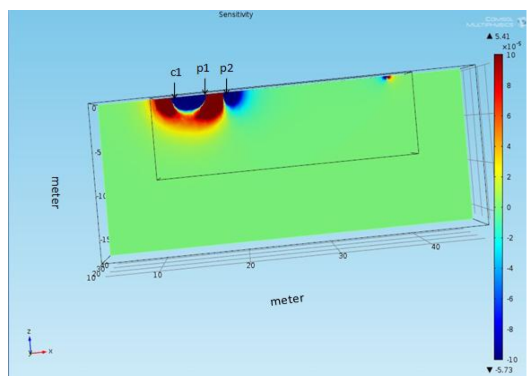


Figure 6 Sensitivity Plot for Pole-dipole Array

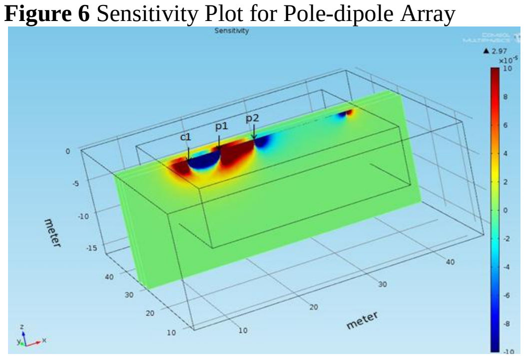


Figure 7 Sensitivity Plot for Double wenner Array

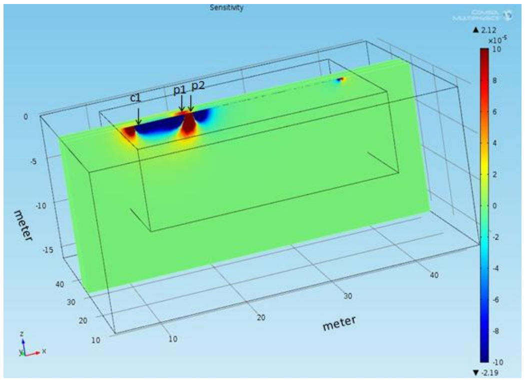


Figure 8 Sensitivity Plot for Half Schlumberger

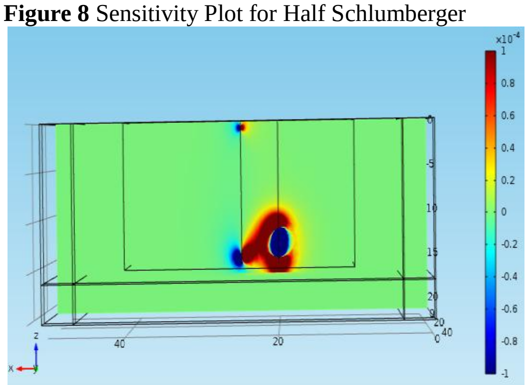


Figure 9 Sensitivity plot for Pole-bipole for C1P1 in one borehole and P2 in another borehole

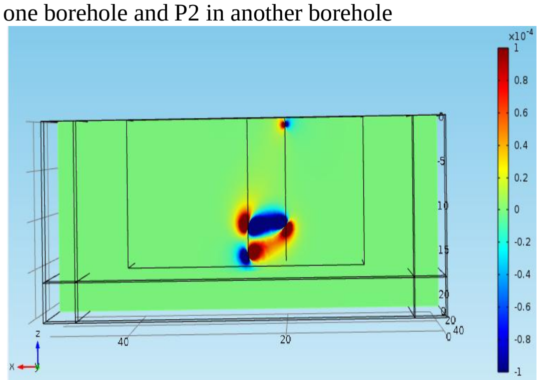


Figure 10 Sensitivity plot for Pole- bipole array for C1 in one borehole and P1P2 in another

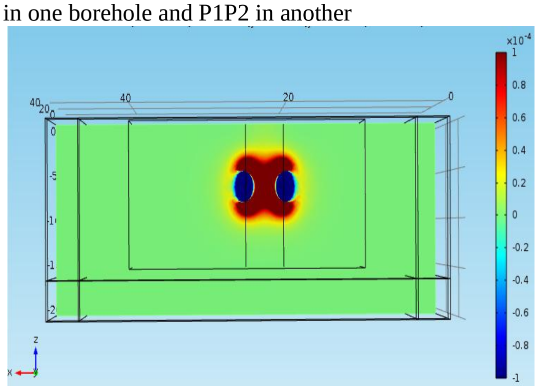


Figure 11 Sensitivity plot for Bipole-bipole array with C1P1 at same borehole and P2 at another borehole

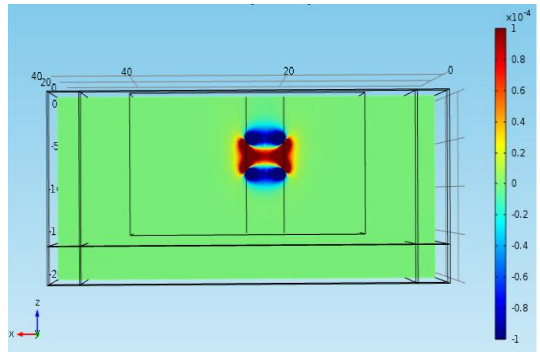


Figure 12 Sensitivity plot for Bipole-bipole array with C1 C2 at same bore hole and P1 at another borehole.

5. Conclusions

For all the arrays, the highest sensitivity values are found near the electrodes. At larger distances from the electrodes, the contour patterns are different for the different arrays. The difference in the contour pattern of the sensitivity function plot helps to explain the response of the different arrays to different types of structures. Pole-pole array gives the widest horizontal coverage, while the coverage obtained by the Wenner array decreases much more rapidly with increasing electrode spacing. Wenner and Schlumberger array both resolves the vertical resistivity changes, but out of these two Wenner is more effective for resolving horizontal structures. The Dipole-Dipole array is very sensitive to horizontal changes in resistivity, but relatively insensitive to vertical changes in the resistivity. Pole-pole array gives the widest horizontal coverage, while the coverage obtained by the Wenner array decreases much more rapidly with increasing electrode spacing. Similar to the dipole-dipole array, Pole Dipole array is probably more sensitive to vertical structures. The Wenner array is an attractive choice for a survey carried out in a noisy area (due to its high signal strength) and also if good vertical resolution is required. The dipole-dipole array might be a more suitable choice if good horizontal resolution and data coverage is important (assuming your resistivity meter is sufficiently sensitive and there is good ground contact). The Schlumberger array (with overlapping data levels) is an acceptable if both good and vertical resolutions are needed, particularly if good signal strength is also required. If there were a limited number of electrodes, the pole-dipole array with measurements in both the forward and reverse directions might be a viable choice. For surveys with small electrode spacing and require a good horizontal coverage, the pole-pole array might be a suitable choice.

In case of crosshole survey the bipole-bipole array can give high resolution image in the vicinity of the two boreholes than pole-bipole. So, bipole-bipole configuration is more desirable for mapping region between two boreholes. Thus, the sensitivity plots are very useful in planning in field survey.

6. References

Friedel, S., "Resolution, Stability and Efficiency of Resistivity Tomography Estimated from a Generalized Inverse Approach," *Geophys. J. Int*., vol. 153, pp. 305– 316, 2003.

Loke, M.H. and Barker, R.D. "Practical techniques for 3D resistivity surveys and data inversion," *Geophysical Prospecting*, vol. 44, pp. 499–523, 1996.

Loke, M. H., 2000, Tutorial : 2-D and 3-D electrical imaging surveys,22-35.

Telford, W.M, L.P. Geldart, and R.E. Sheriff. 1990. Applied Geophysics. 2nd ed. Cambridge Univ. Press, Cambridge, UK

7. Acknowledgements

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