

Stress Distribution in Masonry Walls, Loaded in Plane, Simulated with Comsol.

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Abstract: The tensile strength of masonry is relatively low compared to its compressive strength and is affected by the direction of the joints and their filling. In masonry with modern thin layer mortar (joint thickness 3 to 5 mm) sometimes the head joints are left open. A total of 13 model-walls was built and for each model four general purpose mortar combinations and three thin layer mortar combinations were used. The linear elastic simulations showed the dependence of the stress distribution on height to length ratio. Tensile stresses were as expected and largest at the bottom of the model-walls. The properties of the head joints affected stress distribution. At the overall level, stresses follow the trend found with homogeneous material properties. Subsequent work will be performed to study the effect of cracking on stress distribution. Finally, numerical simulations and experiments should result in reliable design models and values.

Keywords: Shear stress, tensile stress, masonry, mortar, head joint.

1. Introduction

Masonry walls are structures with two relatively large dimensions in relation to their thickness. One of the main characteristics of masonry is its relatively high compressive strength compared to its low tensile strength. Tensile strength is affected by the direction of the joints and their filling. In masonry with general purpose mortar the mortar-brick bond is relatively weak and in modern masonry with thin layer mortar (joint thickness approximately 3 to 5 mm) sometimes, for practical reasons, the head joints are left open. Due to the dependence of direction, masonry codes like Eurocode 6 [1] recognize that strength may be different in three directions, denoted by: f_{x1} , f_{x2} and f_{x3} . In this paper we concentrate on f_{x3} , i.e. the stresses in the direction parallel to the bed joints. The goal is to establish the effects of head joint mortar properties on the tensile strength of masonry

walls to allow for the prediction of strength and eventually to prevent cracking. A numerical simulation program was established in which the head joint property is the main parameter. Especially when head joints are poorly filled the stresses may be higher than based on the assumption that they are fully filled and bonded. Expected behavior will be discussed first, followed by a description of the program and the way the models were built. The first highlights of the results will be presented and discussed. Further detailed analyses should reveal the focus of subsequent work in which the effect of cracking on stress distribution should also be taken into consideration. Finally, numerical simulations and experiments should result in reliable design models and values.

2. Expected behavior

2.1. A “masonry” chain in tension

To investigate the behavior of masonry in tension parallel to the bed joints, a chain of two layers of bricks, loaded in tension, is simulated. Figure 1 shows the model used in its deformed condition. When bricks in this chain are well bonded in the head joints, the stress distribution will be smooth and uniform with stress equal to the applied stress. This is shown in Figure 2a for a model with equal Young’s moduli for brick and mortar. However, the more the E values of brick and mortar differ the more the stress distribution will vary, Figure 2b, c and d. In the vertical section between two head joints, the stress distribution is more or less uniform over the full section.

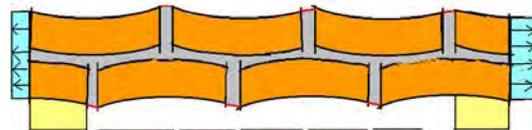


Figure 1 Scheme of a deformed “masonry” chain in tension. Horizontally loaded in tension, vertically supported at the ends.

In a section over a head joint (A), the stresses in the joint will be smaller than those in the brick when E_{mortar} is lower than E_{brick} . The stresses curl around the centre line of the chain, Figure 2, indicating higher tensile stresses in the centre.

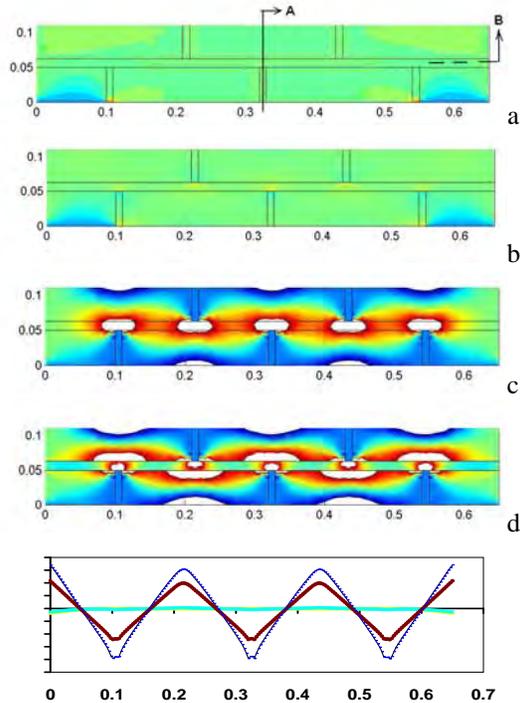


Figure 2 Four stress contour plots from COMSOL simulations of a “masonry” chain for decreasing E_{mortar} and, schematically, the vertical deformation of the centre lines of the bed joint.

A consequence of the equal brick and mortar E-values of 4500 MPa is the uniform stress distribution of 0.1 MPa over the vertical mid section, Figure 4. In a uniform tensile stress situation, shear stresses are negligible [2], i.e. a horizontal line in Figure 5. A smaller E_{mortar} results in less uniformly distributed horizontal stresses and increasing shear and vertical stresses. The most extreme results are found for the model with open head joints (i.e. $E_{\text{brick}} = 4500$ MPa and $E_{\text{mortar}} = 0.50$ MPa).

Figure 3, 4 and 5 indicate the same effect. When the E_{mortar} is smaller, the bricks are loaded eccentrically and consequently they bend; the more difference between E_{mortar} and E_{brick} the larger the effect. From vertical stresses over a joint, Figure 4, it may be concluded that the outer edges of the brick-chain are in compression and these compressive strains have to be

compensated by (extra) tensile stresses. The deflections of the models also confirm that the units are bent. The effect of the vertical support at the chain’s end can be recognized in Figures 2 through 6

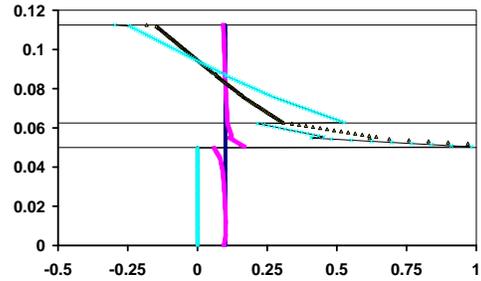


Figure 4 Horizontal stress distribution over height in “masonry” chain in a vertical section over a bed joint in the bottom layer for four different E_{mortar} values.

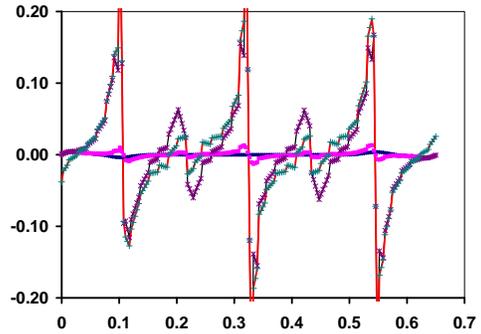


Figure 5 Shear stresses in the bed joint of the “masonry” chain, for different E_{mortar} values.

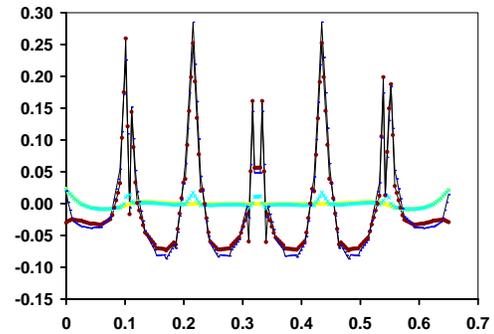


Figure 6. Vertical stresses in the bed joint of the “masonry” chain, for different E_{mortar} values.

2.2 Masonry wall in plane bending

Stress distributions in beams, deep beams and in plane loaded plates depend on the height to span ratio. Therefore, the height (H) of structures that span openings in relation to the span-length (L) is an important parameter. Beams have a relative small H/L ratio, e.g. 0.03 to 0.06, for deep beams this ratio goes to approximately 1.0. In structures with $H/L > 1$ plate action will occur.

A vertical section at mid span of a vertically downwards loaded beam or wall will have compressive stresses at the top and tensile stresses at the bottom. Near the supports stress concentrations may occur. Figure 7 shows the stress trajectories for the three main spanning structural types discussed in this paper. The trajectory models confirm that tensile stresses occur at the bottom of the structure.

For relatively slender beams the line of thrust is close to the top edge of the beam. The more squat the beam, the higher the position of the arched line of thrust will be. However, from a height/span ratio of approximately 0.6 the shape of the arch remains unchanged, [3] and [4].

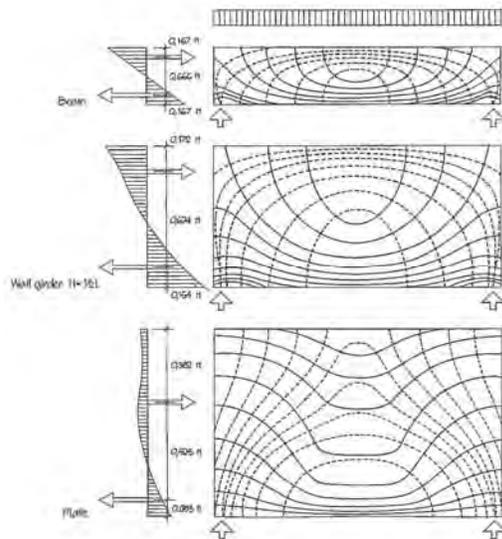


Figure 7 Stress trajectories for a beam, a deep beam and a plate. Redrawn after Heino Engel [3]

At mid section, the maximum values of the tensile and compressive stresses can be found using the hypotheses of Bernoulli, i.e. a linear strain distribution over the height of the beam. Consequently, strains vary linearly over beam

height and when linear elastic material behavior is assumed stresses also. Maximum tensile stress (s) and mid span deflections (d) can be calculated with:

$$s = 6qL^2 / 8bH^2 \quad \text{and} \quad [1]$$

$$d = 5qL^4 / 32EbH^3 \quad [2]$$

respectively, with q = load; L is span length, H is beam height. However, for deep beams, with a height/span ratio higher than approximately 0.6, the strain distribution becomes non-linear.

3. Set-up simulation program

3.1 Parameters.

Based on the ideas about in plane loaded masonry discussed above, the following main parameters for the simulations were chosen: span-length, height, properties of bed joint and head joint mortar and position of head joints in layers in a stretcher bond pattern. In the numerical simulations linear elastic material behavior was assumed. The modeling was performed on a unit and joint scale.

A total of 13 models was built and for each model four general purpose mortar combinations and three thin layer mortar combinations were used. Three wall lengths, i.e. four, six and nine units were used in combination with four heights, i.e. four, eight, twelve and sixteen layers of masonry. This resulted in relatively slender, beam-like walls and in squat walls that act like deep beams and plates. Seven combinations of material properties (Table 1) and thirteen geometries (Table 2) resulted in ninety one simulations.

3.2 Features

The main feature of the models is their height-length ratio. A second feature is the joint thickness for which two values were used in the simulations, 12.5 mm and 3 mm for the bed joints and 10 mm and 3 mm for the head joints, respectively. In common practice general purpose mortar requires a joint thickness of 10 to 15 mm, modern thin layer mortar requires a joint thickness of 3 to 5 mm.

As a third feature the mortar stiffness (modulus of elasticity) was varied. The E -value of the brick units was not varied, it was $E_{brick} =$

4500 MPa for all models. The mortar E-values were $E_{\text{mortar}} = 4500$ MPa (equal to E_{brick}) or $E_{\text{mortar}} = 1500$ MPa for all joints. Head joint mortar was assigned an even lower E-value to simulate poor mortar-brick bond and open head joints ($E_{\text{mortar}} = 0.50$ MPa). In practice, thin layer mortar is usually stiffer than the bricks used, therefore $E_{\text{mortar}} = 6000$ MPa was used for TL mortar.

Table 1. Material properties used in simulations. Type of masonry, E-values of materials used and joint dimensions. Data based on [5] and [6].

Type of masonry		E modulus			thickness	
		brick	bed joint	head joint	bed joint	head joint
		MPa	MPa	MPa	mm	mm
1	MM	4500	4500	4500	12.5	10
2	MM		4500	1500	12.5	10
3	MM		4500	50	12.5	10
4	MM		1500	50	12.5	10
1	TL	4500	6000	6000	3	3
2	TL		6000	1500	3	3
3	TL		6000	50	3	3

MM : Masonry with general purpose mortar

TL : Masonry with Thin Layer mortar

Table 2. Geometry of walls.

#	Length of wall		Height of wall		First unit	Span
	units	mm	layers	mm		
44a*)	4	870	4	237.5	whole	770
44b*)			4	237.5	half	770
48a*)	4	870	8	487.5	whole	770
48b*)			8	487.5	half	770
412a*)	4	870	12	737.5	whole	770
412b*)			12	737.5	half	770
64	6	1310	4	237.5	whole	1210
68	6	1310	8	487.5	whole	1210
612	6	1310	12	737.5	whole	1210
620	6	1310	20	1237.5	whole	1210
912	9	1970	12	737.5	whole	1870
920	9	1970	20	1237.5	whole	1870
930	9	1970	30	1862.5	whole	1870

*) model starts with a) a whole brick and b) a half brick in the bottom layer.

Finally, the configuration of the bricks in the bottom layer, and consequently through the rest of the wall, was varied. Usually, a mason starts from the left with a full length brick. Therefore, four brick long models were made either starting

with a full brick or with a halve brick, a and b in Table 2. In this way, the mid section is either through a head joint or through a whole brick. For the longer models stresses were obtained at two vertical sections, one section was made at mid span and one 100 mm further.

3.3 Structural scheme

The structures were modeled on a detailed level. Both brick, bed joint and head joint mortar was recognized. To allow for a symmetric structure two vertical roller supports were modeled, Figure 8. For horizontal equilibrium, the point at the roof edge at mid span was supported horizontally by a vertical roller support. The walls, 100 mm in thickness, were vertically loaded by a uniformly distributed load of 0.1 MPa. The dimensions of the bricks used were: $50 \times 100 \times 210$ mm³, wall thickness was 100 mm. The wall height depends on the number of layers (nl). With general purpose (MM) mortar each layer is 62.5 mm high as in common practice in The Netherlands; 16 layers are one meter high. The bottom bed joint is usually not made which means that wall height equals the number of layers (nl) minus the thickness of one joint, i.e. $H = nl * 62.5 - 12.5$ mm for MM-mortar. Similar for TL mortar, wall height is $H = nl * 53 - 3$ mm.

Each support is represented by a block of steel of half brick size: $100 \times 100 \times 50$ mm³ and supported in the middle by an axis (in plane this is represented as a point support). Consequently span length is wall length minus 100 mm. Wall length is the number of bricks used times $(210 + hj)$ minus hj in which hj is head joint thickness.

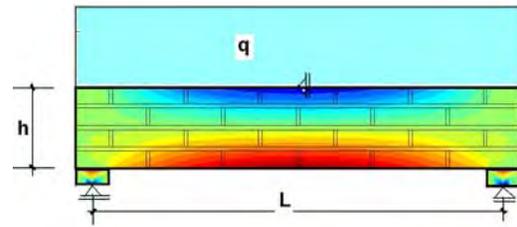


Figure 8. Example of a model of a simulated wall, Length 6 units, height 4 layers, uniformly distributed load ($q = 0.1$ MPa), vertical roller support at mid span, horizontal roller supports at both ends.

4. Results and discussion

Based on the ideas about in plane loaded masonry discussed above the following results were established: deflection of the bottom bed joint at mid span, horizontal stresses in vertical section at mid span, shear and vertical stresses in the bottom bed joint.

Results presented in this paper are all related to the load of 0.1 MPa applied at the roof edge of the model, i.e. a uniformly distributed load of 10 N/mm. The results are used to investigate critical situations in each model where combinations of tensile and shear stresses are highest. The ratio between stresses found and material strength gives an indication of the capacity of the structure. Comparison of the capacity of the simulated walls is part of further analyses and not discussed in this paper.

4.1 Deflection

The deflections depended on the mortar properties, joint thickness and filling of head joints. The downward displacement of a point at mid span in the bottom bed joint was used for comparison. Self evidently the two roller supports did not move. The displacements at mid span are given in the Appendix, Table 4. For a few cases the displacements from simulations were compared with values expected according to simple applied mechanic calculations [1], Table 5. These simple equations are not suitable for deep beams while shear effects and impression near the supports are not taken into account.

One main effect of the variation of mortar properties is the increase in stresses and deflection when the head joint properties decrease. A similar idea is obtained from the graphs of stresses plotted versus height or length. The larger the difference between E_{brick} and E_{mortar} the larger the bending and shear stresses. In open head joints, stresses are zero as expected.

The deflection of the a and b models (4-4, 4-8 and 4-12) hardly differs. The relatively small differences may be caused by the fact that in the b model stresses near the support are forced to a higher level than in the a model, i.e. the net height is smaller.

Thin layer models deflect more than similar general purpose mortar models while their height is smaller due to the joint thickness of 3 mm

instead of 12.5 mm. This effect is less for higher walls and also compensated by the higher E -value for TL mortar.

Only for relatively slender walls (4-4 and 6-4) the deflection calculated with equation [2] is of the same magnitude as the one from Comsol simulation. For all other models, equation [2] underestimates the mid span deflection. One of the reasons is the fact that shear deformation is not taken into account in equation [2]. For deeper walls also the non uniform stress distribution over the wall will contribute to a more optimal flexural behavior.

4.2 Contour-plots

In the contour-plots, arching effects, i.e. concentration of highest compressive stresses along a curved arch, are clearly visible. Peak stresses near places where a head joint meets a brick are observed. Figure 9 shows two examples.

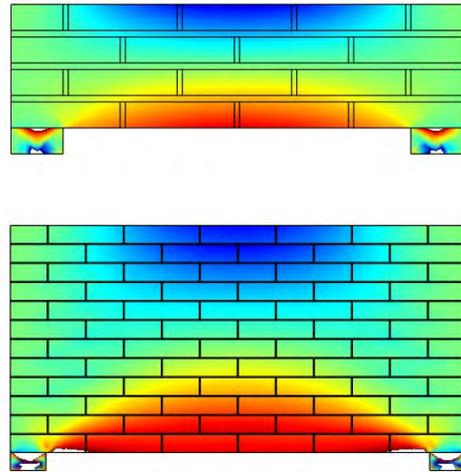


Figure 9 Example Contour plots of wall 4-4 and wall 6-12 both simulations with material model MM4

4.3 Horizontal stresses at mid span (σ_x)

Table 3 gives the stresses calculated with equation [1] and from Comsol simulations for the models with uniform material properties. Differences are only a few per cent except for model 6-12 which is the most squad of the five compared models.

In Figures 10 and 11 the stress distribution over the height of several walls is shown. For

equal E-values for brick and mortar a straight line is found for the beam-models and a slightly curved, smooth line for the deep beam and plate models. The effects of different E_{mortar} are visible. The stresses follow the same trend as for equal E-values, but with peak stresses at the brick-mortar interfaces. Naturally, in “open” head joints the stresses are zero.

Table 3 Comparison of stresses from equation [1] and from Comsol simulations

model	4-4	6-4	6-8	6-12	9-12
Equation [1]	7.88	19.47	4.62	2.02	4.82
Comsol	7.95	20.00	4.80	2.35	5.00
ratio C/E[1]	1.01	1.03	1.04	1.16	1.04

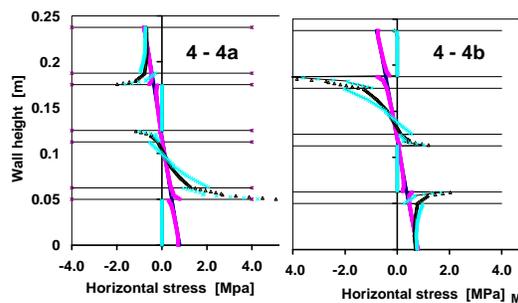


Figure 10 Stresses at mid span. Compare section over head joint (4-4a) and section over a brick (4-4b). Zero stresses in “open” head joints.

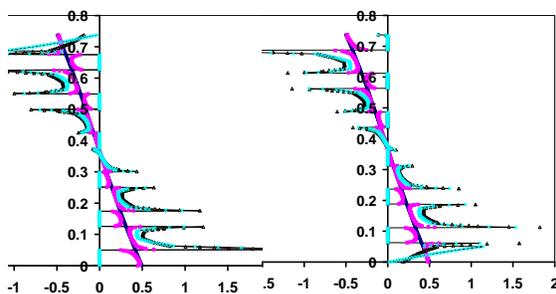
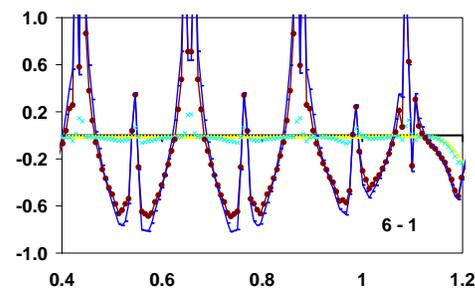


Figure 11 Stresses at mid span, span 1870 mm. Compare section over head joint and section over a brick. Uniform E-values result in a straight line. “Open” head joints reduce stresses at the bottom edge.

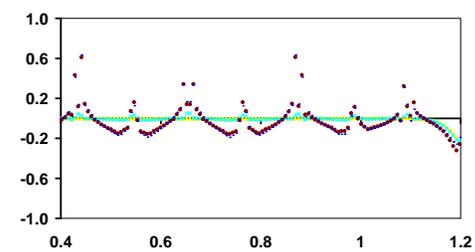
4.4 Vertical stresses (σ_Y)

The distribution of vertical stresses in the bed joint are plotted in Figure 12. In short span walls the vertical stresses in the bed joint near the supports are of the same order of magnitude

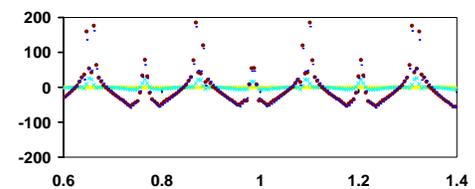
of those at mid span. In long span, higher walls, stresses near the supports are of another magnitude than the stresses at mid span. However, in all cases, these vertical stresses are not critical, while masonry is relatively strong in compression. In the b-type walls that start with a half brick, the bottom layer contributes less to the load bearing capacity, especially when low mortar E-values were applied for the head joint. Mortar at the corner of bricks in the bottom layer in the central part of the wall is in tension.



6-4



6-8



9-30

Figure 12 Vertical stresses in the bottom bed joint. For 6-4 and 6-8 stresses in MPa, for 9-30 in kPa.

4.5 Shear stresses in the bottom bed joint

The bricks in the bottom layer of a wall are in a similar situation as the bricks in the masonry chain discussed earlier. When bonded via the head joints, hardly shear nor vertical stresses develop. Then the only vertical stresses occur near the supports due to the reaction force while all vertical load concentrates near both supports. With small or hardly any bond in the head joints the tension in the bottom part has to be

transmitted from one bottom brick to the next one via shear. A more or less linear shear distribution over each separate brick can be observed in nearly all models.

Like in the brick-chain, stresses concentrate around the bed joint causing bending in each separate brick. Due to this bending effect, tensile stresses perpendicular to the bed joint develop. The occurring tensile stresses decrease the shear capacity considerably, [1] and [7].

Bricks in the bottom layer in the central part of the wall are partly in tension and shear which is a truly critical situation. Actually, the principle tensile stresses should be evaluated.

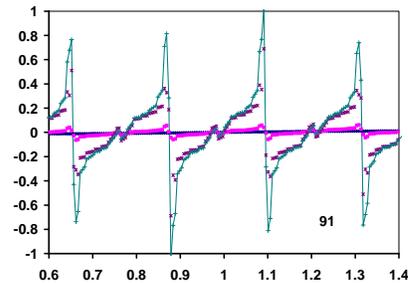
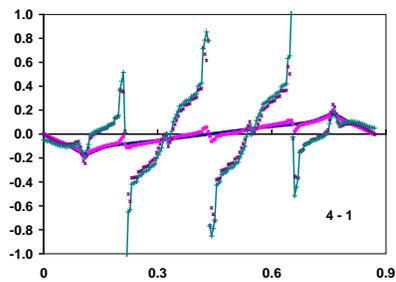


Figure 13 Shear stresses at central part of wall 4-4a and 9-12.

4.6 Thin layer versus general purpose mortar

Comparison of results of simulations with TL properties (Thin layer mortar) are compared to those with general purpose mortars shows that they are similar. Vertical stresses are plotted versus height for the 6-1 model with MM mortar in Figure 14 and for TL mortar in Figure 15. Models of other geometries show that results with MM and TL mortar properties are similar. Peak stresses in TL mortar simulations are higher while joints are thinner.

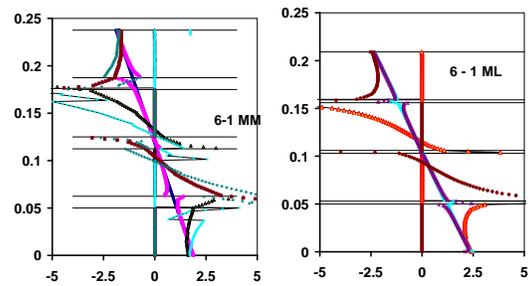


Figure 14 Stress distribution over height for geometry 6-4 with MM mortar and model 6-1 with TL mortar. Sections over head joint show higher stresses at the brick mortar interface. In the section over a brick stresses are maximal at the edges.

5. Conclusions

With 13 geometric models and 7 material combinations an impression was obtained from the behavior of walls loaded in plane in bending. Main parameters were: span length, height and joint thickness. The results showed the dependence of the stress distribution on height to length ratio.

As expected, tensile stresses were largest at the bottom of the walls, and the property of the head joint affected stress distribution. Critical situations in each model where combinations of tensile and shear stresses are highest were established. In combination with material strength, the load bearing capacity of the simulated walls is part of further analyses.

Head joints with small E-values force the flow of stresses from the bottom layer to the layer above causing uneven stress distribution at detail-level. At the overall level stresses follow the trend found with homogeneous material (i.e. the same E-values for brick and mortar).

In all models the effect of each separate brick was clearly visible in contour plots and stress distribution graphs. Models with more units followed the general trend expected for homogeneous material. Tendencies for stress distribution over the height and the length of the wall for uniform E-values and different E-values are the same. Different E-values result in peak stresses in the brick-mortar interface.

Walls with relatively small H/L ratio act like beams. Bending stresses can be estimated using the section modulus of the full section. When head joints are poorly filled, the section-height reduces with one layer and consequently, stresses

increase accordingly. This increase is more critical for walls with a relative small number of layers.

At the corner of the bricks both tensile stresses perpendicular to the bed joint and shear stress in the bed joint occur. This combination is expected to be more critical than the horizontal tensile stresses in a vertical section over a brick. Further research into these critical stress combinations in combination with non linear material behavior is suggested. This research should focus on the brick in the bottom layer at mid span.

Simulations were performed assuming linear elastic material behavior. Subsequent work will be performed to study the effect of cracking on stress distribution. Finally, numerical simulations and experiments should result in reliable design models and values.

6. References

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7. Appendix

Table 4 Displacements in mm at mid span of the bottom bed joint for 13 geometry and 7 material models

span height	M4						M6				M9		
	770	770	770	770	770	770	1210	1210	1210	1210	1870	1870	1870
	4	4	8	8	12	12	4	8	12	20	12	20	30
MM1	1.09	1.10	0.32	0.32	0.27	0.27	5.82	1.02	0.60	0.52	1.76	1.02	0.95
MM2	1.18	1.17	0.33	0.33	0.28	0.28	6.26	1.08	0.63	0.53	1.84	1.06	0.98
MM3	2.61	2.45	0.47	0.43	0.34	0.32	14.63	1.69	0.81	0.62	2.56	1.24	1.10
MM4	3.82	3.54	0.65	0.60	0.47	0.44	21.53	3.87	1.68	1.09	5.97	2.47	1.96
TL1	1.35	1.35	0.34	0.26	0.26	0.27	7.36	1.10	0.58	0.46			
TL2	1.39	1.39	0.35	0.26	0.27	0.27	7.58	1.13	0.59	0.47			
TL3	3.35	3.16	0.53	0.29	0.34	0.31	19.03	1.82	0.78	0.55			

span: centre to centre distance of vertical supports

height: n.o.l. = number of layers; in mm: (n.o.l.*(50+bj))-bj with bj is bed joint thickness, 3 mm or 12.5 mm.

Table 5 Deflection at mid span of the bottom bed joint calculated with Equation [2] and from Comsol simulations. For deeper walls the deflection is underestimated with Equation [2] because this equation neglects shear deformation.

model	4-4a	4-8a	4-12a	6-4	6-8	6-12	6-20	9-12	9-20	9-30
Comsol	1.10	0.32	0.27	5.82	1.02	0.60	0.52	1.76	1.02	0.95
Equatio [2]	0.91	0.11	0.03	5.56	0.64	0.19	0.04	1.06	0.22	0.07
C/eq.[2]	1.21	3.04	8.87	1.05	1.59	3.23	13.24	1.66	4.55	14.46