One-Sided Magnet and Plate

Introduction

Permanent magnets with a one-sided flux are used to attach posters and notes to refrigerators and notice boards but can also be found in advanced physics applications like particle accelerators. The one-sided flux behavior is obtained by giving the magnet a magnetization that varies in the lateral direction (Ref. 1). As no currents are present, it is possible to model a permanent magnet using a scalar magnetic potential formulation. This model shows this technique to model a cylindrical one-sided permanent magnet. A special technique to model thin sheets of high permeability material was used to model a thin μ-metal plate next to the magnet. This circumvents the difficulty of volumetric meshing of thin extended structures in 3D.

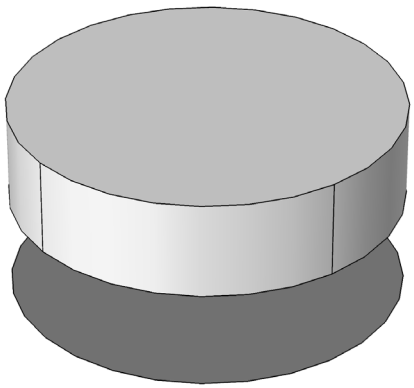

Figure 1: A cylindrical magnet above a μ-metal plate is modeled.

Model Definition

In a current free region, where

$$\nabla \times \mathbf{H} = \mathbf{0}$$

you can define the scalar magnetic potential, V_m, from the relation

$$\mathbf{H} = -\nabla V_m$$

This is analogous to the definition of the electric potential for static electric fields.

Using the constitutive relation between the magnetic flux density and magnetic field

$$\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M})$$

together with the equation

$$\nabla \cdot \mathbf{B} = 0$$

you can derive the following equation for V_m:

$$-\nabla \cdot (\mu_0 \nabla V_\mathrm{m} - \mu_0 \mathbf{M}) = 0$$

It can be shown that applying a laterally periodic magnetization of

$$\mathbf{M} = (M_\mathrm{pre}\sin(kx), 0, M_\mathrm{pre}\cos(kx))$$

results in a magnetic flux that only emerges on one side of the magnet.

BOUNDARIES

Along the exterior boundaries, the magnetic field should be tangential to the boundary as the flow lines should form closed loops around the magnet. The natural boundary condition from the equation is

$$\mathbf{n} \cdot (\mu_0 \nabla V_m - \mu_0 \mathbf{M}) = \mathbf{n} \cdot \mathbf{B} = 0$$

Thus the magnetic field is made tangential to the boundary by a Neumann condition on the potential. On the interior boundary representing the μ-metal plate, you apply a special boundary condition for thin sheets of highly permeable material. Such plates are often used for the purpose of magnetic shielding.

MODELING USING A NONLINEAR MAGNETIC MATERIAL IN THE PLATE

Magnetic saturation effects are important in many applications. In a second version of this model, the instructions show how to include a nonlinear magnetic material with saturation in the plate. The relative permeability μ_r is a function of the magnitude, $|\mathbf{H}|$ of the magnetic field as described by the following equation:

$$\mu_\mathrm{r} = 1 + \left[(\mu_\mathrm{rmax} - 1)^{-1} + \left(\frac{\mu_0 |\mathbf{H}|}{\mathrm{B_{sat}}} \right)^{\frac{1}{n}} \right]^{-1}$$

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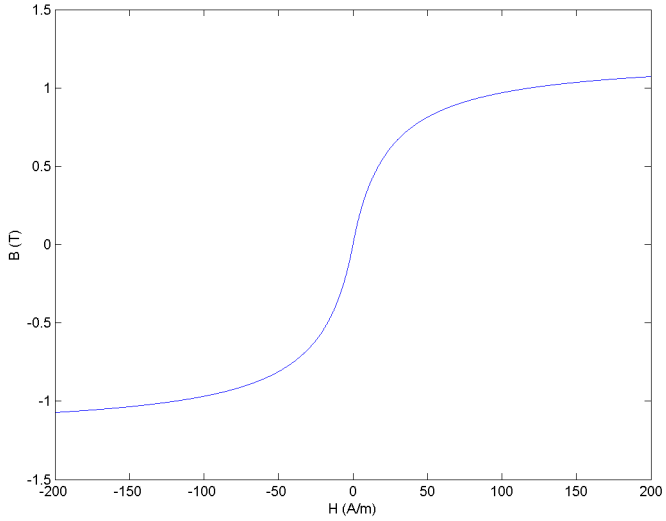
B_{sat} is the magnetic saturation flux, which for μ-metal is about 1.2 T.



FORCE CALCULATION

To calculate the force on the plate, use the surface stress tensor

$$\mathbf{n}_1 T_2 = -\frac{1}{2}(\mathbf{H}\cdot\mathbf{B})\mathbf{n}_1 + (\mathbf{n}_1\cdot\mathbf{H})\mathbf{B}^T$$

where $\mathbf{n}_1$ is the boundary normal pointing out from the plate and T_2 the stress tensor for air.

In this model the $\mathbf{H}$ and $\mathbf{B}$ fields are discontinuous across the plate, which makes it necessary to evaluate the fields on both sides of the plate

Results and Discussion

Figure 2 shows the calculated magnetic flux density and direction for the nonlinear version of the one-sided magnet. The force on the plate is considerably higher for the case with the one-sided magnetization compared to the case with uniform magnetization of the same amplitude.

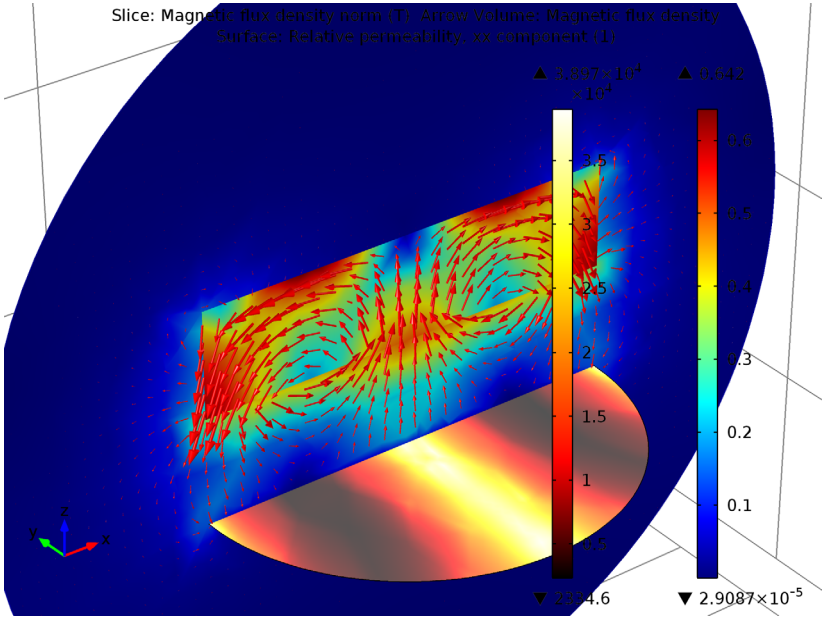


Figure 2: The magnetic flux density and direction is plotted in a cross section of the geometry. The tangential flux density in the plate is shown on a separate scale. The one-sided behavior is apparent, as the flux does not emerge on the top of the magnet.

Reference

1. H.A. Shute, J.C. Mallinson, D.T. Wilton, and D.J. Mapps, "One-Sided Fluxes in Planar, Cylindrical and Spherical Magnetized Structures," *IEEE Transactions on Magnetics*, vol. 36, no. 2, pp. 440–451, 2000.

Model Library path: `ACDC_Module/Magnetostatics/one_sided_magnet`

Modeling Instructions

MODEL WIZARD

1 Go to the **Model Wizard** window.

2 Click **Next**.

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3 In the **Add physics** tree, select **AC/DC>Magnetic Fields, No Currents (mfnc)**.

4 Click **Next**.

5 Find the **Studies** subsection. In the tree, select **Preset Studies>Stationary**.

6 Click **Finish**.

GLOBAL DEFINITIONS

Parameters

1 In the **Model Builder** window, right-click **Global Definitions** and choose **Parameters**.

2 In the **Parameters** settings window, locate the **Parameters** section.

3 In the table, enter the following settings:

Name	Expression	Description
k	pi/10[mm]	Wave number in x direction
M_pre	5e5[A/m]	Magnetization amplitude in magnet

GEOMETRY 1

1 In the **Model Builder** window, under **Model 1** click **Geometry 1**.

2 In the **Geometry** settings window, locate the **Units** section.

3 From the **Length unit** list, choose **mm**.

Cylinder 1

1 Right-click **Model 1>Geometry 1** and choose **Cylinder**.

2 In the **Cylinder** settings window, locate the **Size and Shape** section.

3 In the **Radius** edit field, type 10.

4 In the **Height** edit field, type 5.

Sphere 1

1 In the **Model Builder** window, right-click **Geometry 1** and choose **Sphere**.

2 In the **Sphere** settings window, locate the **Size and Shape** section.

3 In the **Radius** edit field, type 20.

Work Plane 1

1 Right-click **Geometry 1** and choose **Work Plane**.

2 In the **Work Plane** settings window, locate the **Work Plane** section.

3 In the **z-coordinate** edit field, type -5.

Circle 1

1 In the **Model Builder** window, under **Model 1>Geometry 1>Work Plane 1** right-click **Plane Geometry** and choose **Circle**.

2 In the **Circle** settings window, locate the **Size and Shape** section.

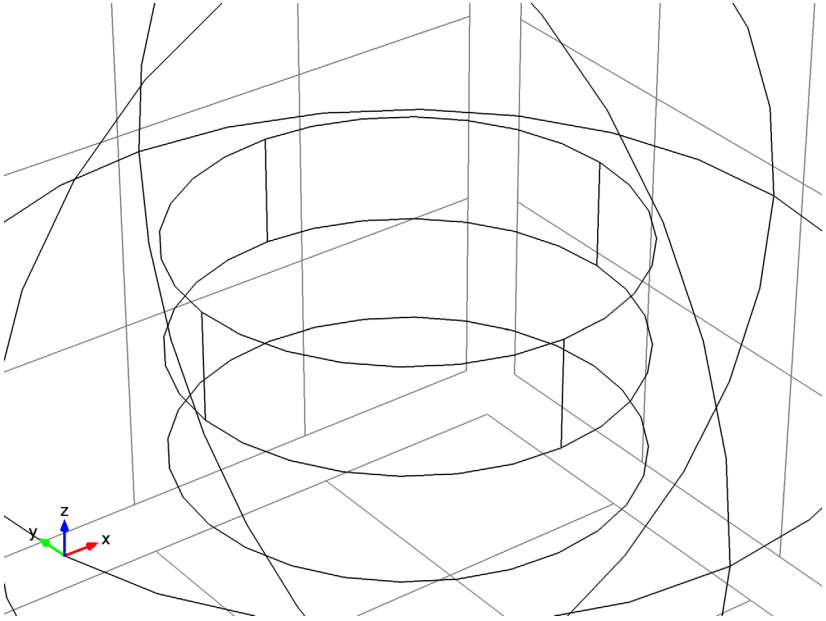
3 In the **Radius** edit field, type `10`.

Form Union

1 Click the **Wireframe Rendering** button on the Graphics toolbar.

2 In the **Model Builder** window, under **Model 1>Geometry 1** click **Form Union**.

3 In the **Settings** window, click **Build All**.



MATERIALS

Material 1

1 In the **Model Builder** window, under **Model 1** right-click **Materials** and choose **Material**.

2 Right-click **Material 1** and choose **Rename**.

3 Go to the **Rename Material** dialog box and type `Air` in the **New name** edit field.

4 Click **OK**.

5 Select Domain 1 only.

6 In the **Material** settings window, locate the **Material Contents** section.

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7 In the table, enter the following settings:

Property	Name	Value
Relative permeability	mur	1

MAGNETIC FIELDS, NO CURRENTS

Magnetic Flux Conservation 2

1 In the **Model Builder** window, under **Model 1** right-click **Magnetic Fields, No Currents** and choose **Magnetic Flux Conservation**.

2 In the **Magnetic Flux Conservation** settings window, locate the **Magnetic Field** section.

3 From the **Constitutive relation** list, choose **Magnetization**.

4 Select Domain 2 only.

5 In the $\mathbf{M}$ table, enter the following settings:

M_pre*sin(k*x)	x
0	y
M_pre*cos(k*x)	z

This premagnetization will result in a magnetic flux that only emerges on the lower side of the magnet.

Magnetic Shielding 1

1 In the **Model Builder** window, right-click **Magnetic Fields, No Currents** and choose **Magnetic Shielding**.

2 Select Boundary 5 only.

3 In the **Magnetic Shielding** settings window, locate the **Magnetic Shielding** section.

4 From the μ_r list, choose **User defined**. In the associated edit field, type 40000.

5 In the d_s edit field, type 0.5[mm].

Zero Magnetic Scalar Potential 1

So far the magnetic potential is not constrained anywhere and the problem has an infinite number of solutions. Thus, you must supply a condition fixing it to a specific value somewhere.

1 Right-click **Magnetic Fields, No Currents** and choose **Points>Zero Magnetic Scalar Potential**.

2 Select Point 1 only.

MESH 1

Free Tetrahedral 1

1 In the **Model Builder** window, under **Model 1** right-click **Mesh 1** and choose **Free Tetrahedral**.

2 In the **Settings** window, click **Build All**.

STUDY 1

1 In the **Model Builder** window, click **Study 1**.

2 In the **Study** settings window, locate the **Study Settings** section.

3 Clear the **Generate default plots** check box.

4 Click the **Compute** button.

RESULTS

3D Plot Group 1

1 In the **Model Builder** window, right-click **Results** and choose **3D Plot Group**.

2 Right-click **3D Plot Group 1** and choose **Slice**.

3 In the **Slice** settings window, click **Replace Expression** in the upper-right corner of the **Expression** section. From the menu, choose **Magnetic Fields, No Currents>Magnetic>Magnetic flux density norm (mfnc.normB)**.

4 Locate the **Plane Data** section. From the **Plane** list, choose **zx-planes**.

5 In the **Planes** edit field, type `1`.

6 In the **Model Builder** window, right-click **3D Plot Group 1** and choose **Arrow Volume**.

7 In the **Arrow Volume** settings window, click **Replace Expression** in the upper-right corner of the **Expression** section. From the menu, choose **Magnetic Fields, No Currents>Magnetic>Magnetic flux density (mfnc.Bx,...,mfnc.Bz)**.

8 Locate the **Arrow Positioning** section. Find the **x grid points** subsection. In the **Points** edit field, type `50`.

9 Find the **y grid points** subsection. In the **Points** edit field, type `1`.

10 Find the **z grid points** subsection. In the **Points** edit field, type `50`.

11 Click the **Plot** button.

The arrow plot shows the magnetic flux density and the surface plot shows its norm. Zoom in to get a better view.

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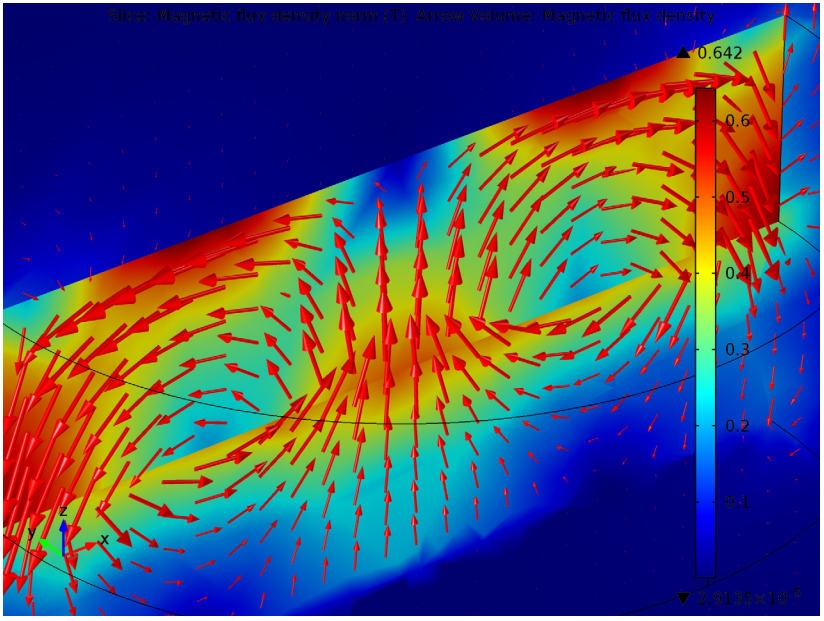
12 Click the **Zoom In** button on the Graphics toolbar.



Having plotted the magnetic flux density, proceed to visualize the magnetic field on the plate.

Data Sets

1 In the **Model Builder** window, under **Results>Data Sets** right-click **Solution 1** and choose **Add Selection**.

2 In the **Selection** settings window, locate the **Geometric Entity Selection** section.

3 From the **Geometric entity level** list, choose **Boundary**.

4 Select Boundary 5 only.

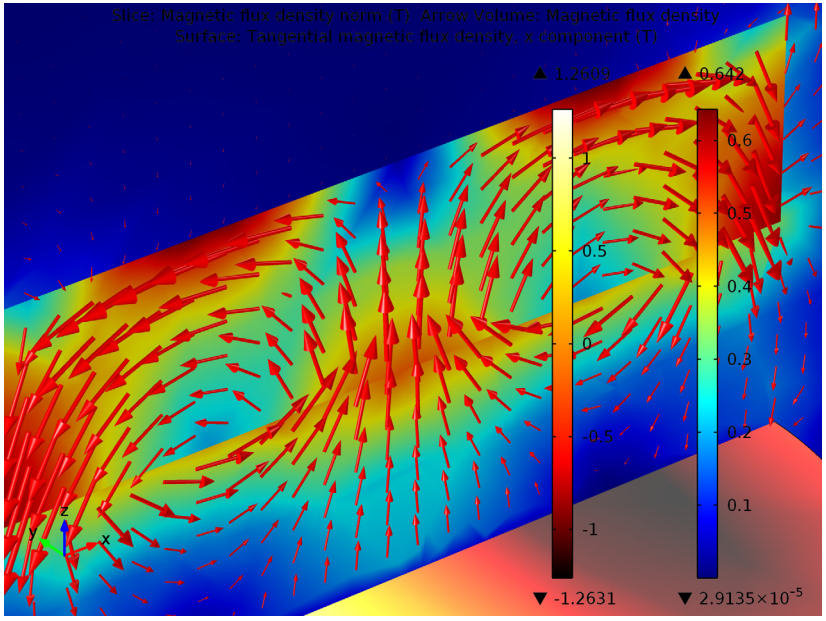
3D Plot Group 1

1 In the **Model Builder** window, under **Results** right-click **3D Plot Group 1** and choose **Surface**.

2 In the **Surface** settings window, click **Replace Expression** in the upper-right corner of the **Expression** section. From the menu, choose **Magnetic Fields, No Currents>Magnetic>Tangential magnetic flux density>Tangential magnetic flux density, x component (mfnc.tBx)**.

3 Locate the **Coloring and Style** section. From the **Color table** list, choose **Thermal**.

4 Click the **Plot** button.



This picture includes both the surface plot and the slice plot

To evaluate the force on the plate, integrate the surface stress tensor.

All surfaces have an up and a down side. To find out which side is up and which is down, you can make an arrow plot of the components of the up normal (unx, uny, unz) on the boundaries. The up normal is the outward normal from the up side. In this case, you evaluate the fields on both sides and there is no need to know the direction of the up normal. The physics interface defines variables for the surface stress tensor on the up and down side of the boundaries, for example, `mfnc.unTmz` and `mfnc.dnTmz` for the z component of the magnetic surface stress tensor.

Derived Values

1 In the **Model Builder** window, under **Results** right-click **Derived Values** and choose **Integration>Surface Integration**.

2 Select Boundary 5 only.

3 In the **Surface Integration** settings window, click **Replace Expression** in the upper-right corner of the **Expression** section. From the menu, choose **Magnetic Fields, No Currents>Mechanical>Maxwell upward magnetic surface stress tensor>Maxwell upward magnetic surface stress tensor, z component (mfnc.unTmz)**.

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4 In the **Expression** edit field, append +.

5 Click **Insert Expression (Ctrl+Space)** in the upper-right corner of the **Expression** section. From the menu, choose **Magnetic Fields, No Currents>Mechanical>Maxwell downward magnetic surface stress tensor>Maxwell downward magnetic surface stress tensor, z component (mfnc.dnTmz)**.

6 Click the **Evaluate** button. The result should be 1.2 N. As a comparison, if you set k to 0 and solve the model again, you get 0.4 N. The one-sidedness of the magnet increases the force by approximately a factor 3.

This concludes the part of the model where you use a linear mu-metal. The remaining instructions show how to use a nonlinear relative permeability for the mu-metal.

GLOBAL DEFINITIONS

Create additional parameters to be used in the nonlinear relation.

Parameters

1 In the **Model Builder** window, under **Global Definitions** click **Parameters**.

2 In the **Parameters** settings window, locate the **Parameters** section.

3 In the table, enter the following settings:

Name	Expression	Description
mur_max	40000	Maximum mur in nonlinear mu-metal
B_sat	1.2[T]	Saturation flux density in nonlinear mu-metal
N	1	Exponent for mur in nonlinear mu-metal

Specify the relation for the relative permeability in the nonlinear mu-metal as a model variable. Since this variable will be evaluated on a boundary, the tangential component of the magnetic field `mfnc.tHx`, `mfnc.tHy`, `mfnc.tHz` must be used.

DEFINITIONS

Variables 1a

1 In the **Model Builder** window, under **Model 1** right-click **Definitions** and choose **Variables**.

2 In the **Variables** settings window, locate the **Variables** section.

3 In the table, enter the following settings:

Name	Expression	Description
mur_mu	1-1/(1/(1-mur_max)-(mu0_const*(mfnc.tHx^2+mfnc.tHy^2+mfnc.tHz^2)^(1/2)/B_sat)^(1/N))	Relative permeability in mu-metal

MAGNETIC FIELDS, NO CURRENTS

Magnetic Shielding 1

1 In the **Model Builder** window, under **Model 1>Magnetic Fields, No Currents** click **Magnetic Shielding 1**.

2 In the **Magnetic Shielding** settings window, locate the **Magnetic Shielding** section.

3 In the μ_r edit field, type `mur_mu`.

Initial Values 1

The nonlinear solver works by performing iterations, linearizing and solving the model for a sequence of operating points that converge to the final solution. For this to work properly in the first iteration, you need to start from an initial guess with a non-zero gradient:

1 In the **Model Builder** window, under **Model 1>Magnetic Fields, No Currents** click **Initial Values 1**.

2 In the **Initial Values** settings window, locate the **Initial Values** section.

3 In the Vm edit field, type `1[A/m]*sqrt(x^2+y^2+z^2)`.

STUDY 1

Solver 1

In the **Model Builder** window, under **Study 1>Solver Configurations** right-click **Solver 1** and choose **Compute**.

RESULTS

3D Plot Group 1

The nonlinear permeability results in a lower field norm in the plate. To study how far the material is brought into saturation, you can plot the permeability.

1 In the **Model Builder** window, under **Results>3D Plot Group 1** click **Surface 1**.

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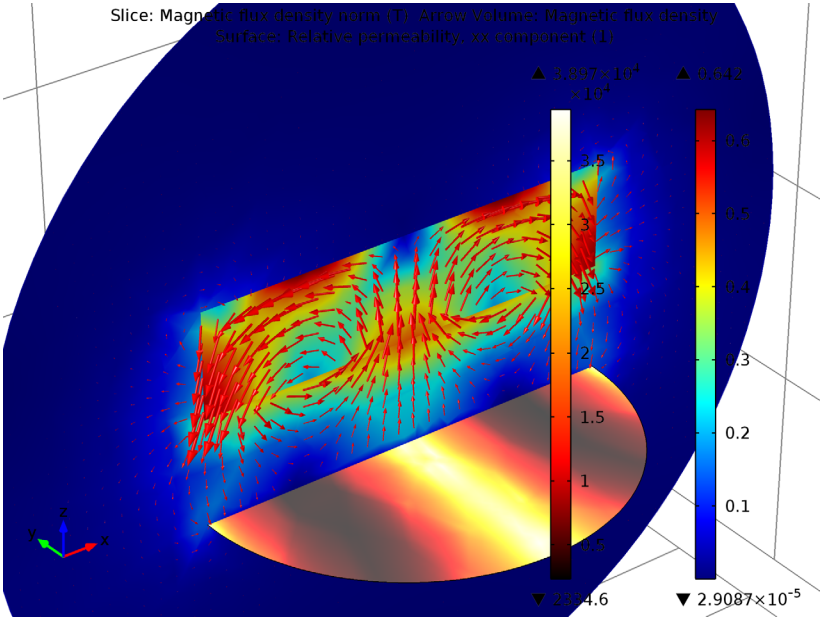
2 In the **Surface** settings window, click **Replace Expression** in the upper-right corner of the **Expression** section. From the menu, choose **Magnetic Fields, No Currents>Material properties>Relative permeability>Relative permeability, xx component (mfnc.murxx)**.

3 Click the **Plot** button.

4 Click the **Zoom Out** button on the Graphics toolbar.



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